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Abstract

This paper examines the contributions of population aging, mortgage innovation and historically low interest rates to the sharp rise in U.S. house prices and mortgage debt between 1994 and 2005. I construct an overlapping generations general equilibrium housing model and find that these three factors together account for over half of the increase in house prices and most of the increase in mortgage debt during this period. Population aging contributes to rising house prices and mortgage debt, but it accounts for only a small portion of their observed changes. Meanwhile, mortgage innovation significantly increases the mortgage borrowing of various age cohorts, but it has a trivial effect on house prices because interest rates rise due to higher demand for mortgage loans. This increases households' savings in financial assets and leaves their housing assets nearly unchanged. The observed run-up in house prices can, however, be justified in an open-economy setting where interest rates fall due to a global saving glut. Declining interest rates force households at prime saving ages to reallocate their wealth from financial assets to housing assets, which dramatically drives up house prices.

JEL classification: E21, E44, G11, R21
Bank classification: Asset pricing; Credit and credit aggregates; Economic models

Résumé

L'auteur examine l'effet du vieillissement de la population, des innovations en matière de prêts hypothécaires et des taux d'intérêt historiquement bas sur l'escalade des prix des maisons et de l'endettement hypothécaire aux États-Unis entre 1994 et 2005. Il construit un modèle d'équilibre général dynamique à générations imbriquées du marché du logement et conclut qu'ensemble, les trois facteurs susmentionnés contribuent à plus de la moitié de la hausse des prix des maisons et à la majeure partie de l'augmentation de l'endettement hypothécaire durant la période étudiée. Si le vieillissement a une incidence sur la poussée des prix des maisons et de l'endettement hypothécaire, il ne compte que pour une faible proportion dans les variations observées. Quant aux innovations en matière de prêts hypothécaires, elles causent un important accroissement du crédit hypothécaire dans les divers groupes d'âge, mais elles ont un effet négligeable sur les prix des maisons, car les taux d'intérêt montent lorsque la demande de prêts hypothécaires progresse. Cette situation conduit à une hausse de l'épargne des ménages affectée aux actifs financiers et n'a pratiquement aucune influence sur leurs actifs immobiliers. Dans une économie ouverte, l'envolée des prix des habitations observée peut toutefois s'expliquer : la surabondance de l'épargne à l'échelle mondiale y fait reculer les taux d'intérêt. Cette baisse des taux pousse les ménages du groupe d'âge ayant la plus forte propension à l'épargne à réaménager leur richesse en se tournant vers les actifs immobiliers, ce qui fait bondir les prix des maisons.

Classification JEL : E21, E44, G11, R21
Classification de la Banque : Évaluation des actifs; Crédit et agrégats du crédit; Modèles économiques

1 Introduction

The late-2000s financial crisis in the United States is widely considered the worst financial crisis since the Great Depression. A broad consensus among economists has emerged on the central role the collapse of house prices and the default of mortgage debt played in this crisis. The housing boom between 1994 and 2005 saw a striking rise in both house prices and mortgage debt, as shown in Figure 1.[1] During this period, average national real house prices increased by 45 percent. While consumer credit rose only slightly from 14 percent of GDP in 1994 to 18 percent in 2005, home mortgage debt increased more quickly from 45 percent of GDP in 1994 to 70 percent in 2005. Examining the formation of this housing boom is important for both economic researchers and policy-makers to better understand the sources of the financial crisis. This paper investigates whether population aging, mortgage innovation and historically low interest rates might account for the sharp rise in house prices and mortgage debt during this period.

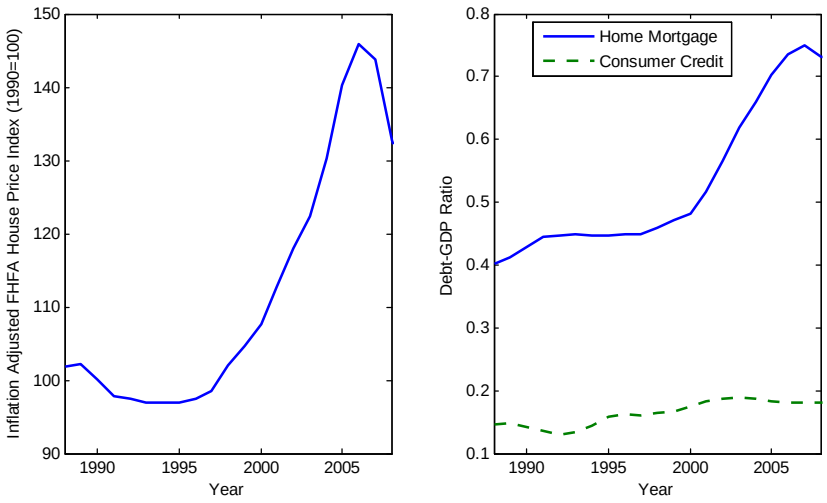

Figure 1: House Price Index and Debt-GDP Ratio

Population aging is one of the notable demographic changes in the United States over the past two decades. Figure 2 plots the age distribution for three selected years using data from the U.S. Census Bureau. It is evident that the population distribution in 2005 dominates its counterpart in 1994 in the cohort shares of middle-aged and older households. Factors causing this demographic change include declining fertility rates, falling mortality rates at older ages, the aging of baby boomers and declining immigration.[2] Changes in the demographic structure

[1] The house price data and mortgage debt data are sourced, respectively, from the Federal Housing Finance Agency (FHFA) U.S. House Price Index and the Federal Reserve Board U.S. Flow of Funds Accounts.

[2] Slowing immigration contributes to population aging, since immigrants are, on average, younger than the natives.

could be responsible for rising house prices. As shown in previous studies (e.g., Fernandez-Villaverde and Krueger, 2005; Yang, 2009), the life-cycle profile of housing demand increases monotonically early in life and remains relatively flat afterward. A rise in the share of households at older ages would thus increase aggregate housing demand and drive up house prices. Mankiw and Weil (1989) argue that the upward trend of house prices in the late 1970s is due to the entry of baby boomers into their primary house-purchasing ages.

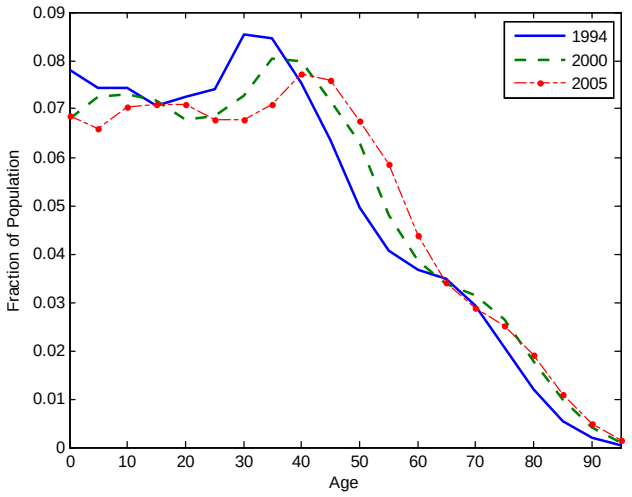

Figure 2: U.S. Population Distribution by Age

Since the early 1990s, U.S. mortgage markets have undergone considerable changes. In particular, a number of new types of mortgage loans were introduced into mortgage markets by government-sponsored mortgage agencies in the late 1990s. These loans, known as combo or piggyback loans, became popular in private mortgage markets in the early 2000s, especially among those who wanted to avoid large downpayments and private mortgage insurance associated with traditional fixed-rate mortgage (FRM) loans. The first advantage of combo loans over FRM loans lies in the tax deduction available for paying piggyback loan interest versus paying a mortgage insurance premium that is not tax deductible on a single loan. The second advantage is that the total payment on a combo loan is often much lower than the payment on an FRM loan requiring mortgage insurance. The insurance premium for a traditional FRM loan is based on the full loan value, whereas it is only on the secondary loan for a combo loan. Mortgage innovation could be responsible for pushing up the housing demand, since it relaxes financial restrictions and makes the housing market more accessible to more households. Chambers, Garriga and Schlagenhauf (2009a) show that mortgage innovation is the major driver of the U.S. homeownership boom during the same period. They find that it accounts for between 56 percent and 70 percent of the observed change in homeownership, which rose from 64 percent in 1994 to 69 percent in 2005. The expanded borrowing induced by mortgage innovation might

be particularly important in explaining the run-up in both house prices and mortgage debt.

Since the early 2000s, interest rates have declined considerably in the United States. The federal fund rates fell from about 6.5 percent to just 1 percent. This in turn lowered mortgage interest rates which are set according to 10-year treasury bond yields. Between 2000 and 2003, interest rates for 30-year FRM loans fell from 8 percent to about 5.5 percent, and interest rates for one-year adjustable-rate mortgages dropped from 7 percent to about 4 percent. Declining interest rates could be responsible for surging house prices and mortgage debt. They reduce returns on financial assets and induce investors to reallocate their wealth toward housing assets. The expanding entry into the housing market is further stimulated by the falling mortgage cost.

In this paper, I construct an overlapping generations (OLG) general equilibrium incomplete markets model to quantify the contributions of population aging, mortgage innovation and historically low interest rates to the sharp rise in house prices and mortgage debt between 1994 and 2005. The model specifies demographic characteristics and housing market arrangements. In the model, population dynamics are explicitly decomposed into the component driven by natives and that driven by immigrants; housing has the triple roles of consumption good, saving asset and borrowing collateral; and house purchases are financed with long-term mortgage loans from a finite menu of mortgage contracts. I employ the parameterized model to disentangle the individual contribution of these three factors to the rise in house prices and mortgage debt using three counterfactual experiments, where each experiment varies one factor at a time but holds others constant relative to the benchmark scenario. The corresponding contribution of this varying factor is estimated by comparing the associated model-generated changes in house prices and mortgage debt to those observed in the data. I also measure the combined effects of the three factors by allowing their changes to occur simultaneously.

My main findings in this paper are as follows. First, population aging, mortgage innovation and declining interest rates together account for over half of the observed increase in house prices and most of the observed increase in mortgage debt during this period. Second, the historically low interest rates are the major driver of the run-up in house prices. They are also responsible for rising mortgage debt. Falling interest rates lower returns on financial assets and the cost of mortgage borrowing. This induces households at the prime saving ages of 35-65 to reallocate their wealth from financial assets to housing assets as an alternative to save for their retirement, which consequently drives up house prices. I find that it is mainly those middle-aged households who increase their housing assets and mortgage borrowing when interest rates decline. Third, mortgage innovation significantly increases the aggregate mortgage debt, where the lower downpayment requirement and higher tax deduction associated with the newly introduced combo loan encourage the mortgage borrowing of various age cohorts. However, surprisingly, the innovation has barely any effect on house prices due to a general equilibrium effect on interest rates. Indeed, mortgage innovation is found to have two offsetting effects

on aggregate housing demand. On the one hand, it relaxes borrowing constraints for house purchases and thus drives up housing demand. On the other hand, it also raises interest rates due to households' rising demand for mortgage loans. Rising interest rates increase returns on financial assets and the cost of mortgage borrowing, which in turn induces many middle-aged households to increase their savings in financial assets. This substitution effect lowers housing demand. My model shows that the two opposite forces neutralize each other in equilibrium, which explains why mortgage innovation alone barely affects house prices. Fourth, population aging contributes to rising house prices and mortgage debt, but it accounts for only a small portion of their observed changes during this period. Finally, I also find a negative effect of immigration on house prices through its impact on the age distribution of the population. Since immigrants are younger than natives, increasing arrivals of immigrants lower the average age of the population in the long run. Due to the life-cycle profile of housing consumption, the aggregate demand for housing falls, which then lowers house prices.

This paper is first related to the strand of literature studying the implications of housing for the macroeconomy.[3] It builds upon the Aiyagari-Bewley-Huggett type model with heterogeneous agents and incomplete markets. Studies most closely related to this paper are Chambers, Garriga and Schlagenhauf (2009a, b) in terms of the model, and Nakajima (2005), Waldron and Zampolli (2010), Sommer, Sullivan and Verbrugge (2011) and Favilukis, Ludvigson and Van Nieuwerburgh (2011) in terms of the topic.

My modelling of mortgage choices is similar to that of Chambers, Garriga and Schlagenhauf (2009a). The authors document the U.S. homeownership boom and considerable mortgage innovation between 1994 and 2005. They build a model that takes into account mortgage structures and household-supplied rental properties, and find that mortgage innovation is the major driver of the observed run-up in homeownership. However, they do not consider price movement during the period, and house prices are not determined endogenously in their model.

My topic of studying the relationship between house prices and economic fundamentals using a structural model is similar to that of Nakajima (2005), Waldron and Zampolli (2010), Sommer, Sullivan and Verbrugge (2011), and Favilukis, Ludvigson and Van Nieuwerburgh (2011). Nakajima (2005) studies the effects of rising earnings inequality on the portfolio allocation and asset prices, and shows that widening earnings dispersion can account for 40 percent of the increase in U.S. house prices during the period between the late 1960s and the mid-1990s, due to a negative effect of rising precautionary savings on interest rates. However, Nakajima's model predicts a decline in the total amount of secured debt, in contrast to the U.S. data. Sommer,

[3]Examples include Gervais (2002), Fernandez-Villaverde and Krueger (2005), Glaeser, Gryourko and Saks (2005), Ortalo-Magné and Rady (2006), Davis and Heathcote (2007), Jeske and Krueger (2007), Li and Yao (2007), Diaz and Luengo-Prado (2008), Chambers, Garriga and Schlagenhauf (2009a, b, c), Nakajima (2005, 2010), Yang (2009), Chatterjee and Eyigungor (2009), Waldron and Zampolli (2010), Corbae and Quintin (2011), Sommer, Sullivan and Verbrugge (2011), and Favilukis, Ludvigson and Van Nieuwerburgh (2011).

Sullivan and Verbrugge (2011) develop a model with fully specified markets for homeownership and rental properties to study the effects of rising incomes, lower interest rates and relaxing downpayment requirements on rising house prices. They show that these factors can account for over half of the increase in the U.S. house price-rent ratio between 1995 and 2005. However, interest rates are exogenously given throughout their analysis. Thus, their paper does not consider the equilibrium interaction between financial markets and housing markets. As my paper shows, this interaction is very important for revealing the exact contribution of certain factors to rising house prices and mortgage debt. Similarly, assuming exogenously given interest rates, Waldron and Zampolli (2010) find that falling interest rates since the late 1990s can account for rising household debt and house prices in the United Kingdom during the past decade. Using a two-sector RBC model, Favilukis, Ludvigson and Van Nieuwerburgh (2011) show that fluctuations in the U.S. price-rent ratio since the late 1990s are driven by changing risk premia in response to aggregate shocks and financial market liberalization. However, their focus is on studying house price dynamics, whereas I study the observed run-up in both house prices and mortgage debt.

This paper is also related to the strand of literature studying the economic impact of demographic change on asset prices.[4] It builds upon the simple intuition that demography matters for asset markets as asset demand varies over the life cycle. Mankiw and Weil (1989) study empirically the relationship between demographics and house prices. They find that the entry of the baby-boom generation into its house-buying years is the major cause of the increase in real house prices in the 1970s. Poterba (2001) builds a two-period OLG model with a constant saving rate and fixed asset supply, and shows that a baby boom that increases the size of the young-worker cohort drives up the prices of financial assets. When it is followed by a baby bust, asset prices will first increase and then decline. Poterba's intuitive model has been further developed in a number of related studies (e.g., Yoo, 1997; Brooks, 2002; Abel, 2003; Geanakoplos, Magill and Quinzii, 2004), which provide a more realistic characterization of saving behavior and asset price determination. These authors find that demographic shocks could generate substantial swings in asset values. The papers mentioned above, however, mainly focus on the effects of demographics on the pricing of financial assets, and do not consider the interaction between financial markets and housing markets as emphasized in this paper.

The remainder of this paper is organized as follows. Section 2 describes the model and defines the equilibrium. Section 3 calibrates the benchmark model. Section 4 measures the contributions of population aging, mortgage innovation and historically low interest rates to rising house prices and mortgage debt. Section 5 concludes.

[4]Examples include Mankiw and Weil (1989), Yoo (1997), Poterba (2001, 2004), Brooks (2002), Abel (2003), and Geanakoplos, Magill and Quinzii (2004).

2 The Model

This section constructs an OLG general equilibrium model with uninsurable idiosyncratic earning and mortality risks. The economy consists of households, a financial intermediary and a government. There are two assets in the model: a financial asset and a housing asset. House purchases are financed with long-term mortgage loans provided by the financial intermediary.

2.1 Demographics

The economy is populated by life-cycle individuals who are ex-ante heterogeneous. Let $j \in \left\{1, ..., \bar{J}\right\}$ denote the age of an individual, where $\bar{J}$ stands for the maximum number of periods an individual can live. In each period, an individual faces an exogenous mortality risk. The survival probability to age $j+1$, conditional on being alive at age j, is denoted by $\psi_j \in [0,1]$, with $\psi_0 = 1$ and $\psi_{\bar{J}} = 0$. The survival probability is assumed to be identical across individuals of the same age. There does not exist an annuity market for the mortality risk, so individuals cannot insure themselves against the longevity risk. The model assumes that the total amount of wealth left over by individuals who die in a given period is redistributed uniformly among the living.

The evolution of the population is partitioned into natural population increase and immigration. Denote $\mathcal{P}_t = \left(\mathcal{P}_{t,1}, ..., \mathcal{P}_{t,\bar{J}}\right)'$ as the population vector of $\bar{J}$ age cohorts at time $t \geq 0$, where $\mathcal{P}_{t,j}$ represents the size of the age j cohort. The evolution of $\mathcal{P}_t$ can be written as

$$\mathcal{P}_{t+1} = \Gamma \mathcal{P}_t + \Omega \mathcal{P}_t, \tag{1}$$

where

$$\Gamma = \begin{bmatrix} \kappa_1 & \kappa_2 & \kappa_3 & . & . & \kappa_{\bar{J}} \\ \psi_1 & 0 & 0 & . & . & 0 \\ 0 & \psi_2 & 0 & . & . & 0 \\ 0 & 0 & \psi_3 & . & . & 0 \\ . & . & . & . & . & . \\ 0 & 0 & 0 & . & \psi_{\bar{J}-1} & 0 \end{bmatrix}, \ \Omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ . \\ . \\ \omega_{\bar{J}} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 & . & . & 1 \end{bmatrix}.$$

Here, the first $\bar{J} \times \bar{J}$ matrix Γ characterizes the dynamics of natural population changes, where κ_j represents the fertility rate of the age j cohort. The second matrix Ω determines the net immigration rates $\left\{\omega_j : j = 1, ..., \bar{J}\right\}$ into each age cohort. The model assumes that, in each period, the number of arriving immigrants at a particular age is proportional to the total population in the previous period. It also assumes that immigrants are identical to members of the native population, as in Waldron and Zampolli (2010). This avoids the need to separate the decisions of immigrants from those of natives, which would otherwise greatly increase the

computational cost.

In a steady-state equilibrium, the total population grows at some constant rate n_p. It is known that the number $(1+n_p)$ is the dominant eigenvalue of the population projection matrix $\Gamma + \Omega$, and the associated eigenvector $(\tilde{\mu}_1, ..., \tilde{\mu}_{\bar{J}})$ gives the stationary age distribution, where $\tilde{\mu}_j \geq 0$ denotes the share of the age j cohort over the total population.

Individuals start their adult life or become economically active at some age j_0, with $1 < j_0 < \bar{J}$. Individuals of an age less than j_0 are supported by their parents. I do not model how adults form households. Instead, it is assumed that there exists a life-cycle profile of household size $\left\{\zeta_j : j_0 \leq j \leq \bar{J}\right\}$, where ζ_j represents the effective household size with the household head of age j. There are in total $\bar{J} - j_0 + 1$ different adult-age cohorts. Denote $J = \left\{j_0, ..., \bar{J}\right\}$ as the set of adult ages. Throughout this paper, I normalize the eigenvector $(\tilde{\mu}_1, ..., \tilde{\mu}_{\bar{J}})$ such that the resulting adult distribution $\left(\mu_{j_0}, ..., \mu_{\bar{J}}\right)$, which is of model interest, has a measure one in the steady state.

2.2 Preference

A household derives utility from the consumption of non-housing goods (c) and housing services (h) according to a period utility function $u : \mathbb{R}^3_+ \to \mathbb{R}$, $(c, h, \zeta) \mapsto u(c, h, \zeta)$, which is assumed to be strictly increasing, strictly concave and to satisfy the standard properties of differentiability and Inada conditions in the first two arguments. It is assumed that there exists a linear technology that transforms one unit of housing stock at the beginning of each period into one unit of housing service during that period. The third argument ζ of the period utility function denotes the effective household size, which captures the economies of scale in household consumption as argued in Lazear and Michael (1980). The life-cycle profile of household size $\left\{\zeta_j : j \in J\right\}$ is taken as given by households.

Preferences are assumed to be time separable with a constant discount factor $\beta > 0$. A household values consumption streams $\{c_j, h_j\}_{j=j_0}^{\bar{J}}$ according to

$$E_0 \left\{ \sum_{j=j_0}^{\bar{J}} \beta^{j-j_0} u\left(c_j, h_j, \zeta_j\right) \right\}, \tag{2}$$

where E_0 denotes an expectation operator conditional on the information available at age j_0, and expectations are formed with respect to the stochastic processes governing both mortality and earning shocks to be discussed below. I assume that households do not leave voluntary bequests upon their death, and their remaining wealth will be collected and redistributed equally among the living. For simplicity, the period utility for being dead is normalized to zero.

2.2.1 Endowment

Households at age j_0 are endowed with some financial asset a_{j_0} and housing asset h_{j_0}. I assume that $a_{j_0} = h_{j_0} = 0$. Every period, each household at age j receives stochastic earnings $v_j\epsilon$, where v_j and ϵ represent an age-dependent component and stochastic component of earnings, respectively. As for the age component v_j, it is assumed that there exists an age $j^* \in J$ with $j^* < \bar{J}$ such that $v_j > 0$ for $j < j^*$ and $v_j = 0$ for $j \geq j^*$. One can interpret j^* as a mandatory retirement age, and consequently households at age $j_0 \leq j < j^*$ as workers and those at age $j \geq j^*$ as retirees. Here, the number v_j reflects the average labor productivity of an age j household, and it is simply set to 0 for $j \geq j^*$ as retirement is mandatory at age j^*. Meanwhile, the period-specific stochastic component ϵ is assumed to be independent of age and follow a finite-state first-order Markov process $\{E, \Pi\}$, where $E = \{\epsilon_1, ..., \epsilon_\ell\}$ and $\Pi = (\pi_{ik})_{1 \leq i,k \leq \ell}$ denote, respectively, the state space and its associated one-period transition probabilities with

$$\pi_{ik} = \Pr\left(\epsilon' = \epsilon_k | \epsilon = \epsilon_i\right) \text{ and } \sum_{k=1}^{\ell} \pi_{ik} = 1, \text{ for } i = 1, ..., \ell. \tag{3}$$

Here, a prime is used to denote a variable in the next period. This Markov process is identical for all households and there is no uncertainty over the aggregate earning. Denote $\Delta = (d_1, ..., d_\ell)'$ as the unique invariant probability measure associated with Π. By definition, Δ solves the linear system $\Pi'\Delta = \Delta$ with $\sum_{i=1}^{\ell} d_i = 1$. The initial realization of the stochastic efficiency shock is drawn from Δ for all households.

2.3 Housing Market

House purchases are financed with long-term mortgage loans. There exists an exogenous menu of finitely many mortgage contracts provided by a financial intermediary whose roles will be discussed in the next section. Those contracts differ in their downpayment requirements, contract lengths and mortgage interest rates in the model. Denote $M = \{1, ..., \bar{m}\}$ as the set of available mortgage contracts. For a mortgage loan of type $m \in M$, the numbers $\lambda_m \in [0, 1]$, $L_m \in \mathbb{N}$ and $r_m \in \mathbb{R}_+$ specify the associated downpayment ratio, contract length and mortgage interest rate, respectively. Since this paper does not model default and focuses on the effect of mortgage innovation on house prices, I assume that mortgage interest rates coincide with the interest rate r on deposits.

Let P denote the house price in a steady-state equilibrium. A household who chooses to finance a house purchase of size h with a mortgage of type m borrows $D_{L_m} = (1 - \lambda_m) Ph$ and pays a downpayment $H_{L_m} = \lambda_m Ph$, which equals the initial equity in the house. The mortgage contract is self-amortizing, with each mortgage payment consisting of both interest and principal

components. According to the principle that the present value of mortgage payments equals the initial mortgage debt, the periodic mortgage payment q is determined by

$$\sum_{k=1}^{L_m} \frac{q}{(1+r)^k} = D_{L_m},\ q = rD_{L_m}\left(1-(1+r)^{-L_m}\right)^{-1}. \tag{4}$$

Let $n \in \{0,1,...,L_m\}$ be the number of remaining mortgage payments and D_n be the outstanding mortgage balance. Given the above repayment structure of the mortgage contract, D_n evolves according to

$$D_{n-1} = (1+r)\,D_n - q = D_n - (q - rD_n)\,,\ 0 \le n \le L_m,$$

where rD_n and $q - rD_n$ denote the parts of payment q toward the interest and principal, respectively. As a result, the equity accumulated in the house evolves as

$$H_{n-1} = H_n + q - rD_n,\ 0 \le n \le L_m.$$

As time passes, the mortgage balance decreases, whereas home equity increases.

The housing market has a fixed housing stock $H_s > 0$. This is a similar assumption as in Nakajima (2005) and Sommer, Sullivan and Verbrugge (2011), who study a similar issue of house prices. It is important to realize that the total housing stock is only fixed relative to the population in this paper, since the adult population is assumed to be of measure one throughout the analysis.

Houses are costly to maintain and trade. The consumption of housing services depreciates the housing stock and requires maintenance expenses. These periodic expenses for a house of size h are $\delta_h Ph$, where δ_h represents the proportional maintenance cost. Meanwhile, trading houses is subject to transaction costs. Let ϕ_s and ϕ_b denote the proportional transaction cost associated with selling and buying houses, respectively. The total incurred transaction cost when changing the house size from h to h' is thus equal to $\phi_s Ph + \phi_b Ph'$.

2.4 Financial Sector

There is a competitive financial sector represented by a financial intermediary. The intermediary receives deposits from households. It also receives mortgage payments from existing homeowners, and principal payments from households who sell their houses and from households who die unexpectedly with an outstanding mortgage balance. These funds are used to pay back deposits with promised interest and offer mortgage loans to households in need of financing their house purchases.

2.5 Government

In this economy, the government is engaged in three activities: collecting income tax to finance government expenditures, running a pay-as-you-go social security program and redistributing the wealth of households who die unexpectedly.

The government balances its budget every period by choosing its spending G according to the amount of revenue collected from income taxation. Households are subject to an income tax code represented by a differentiable tax function $T(y)$, where y is the taxable income. To capture the progressivity of the U.S. tax code, it is assumed that $T'(y) > 0$ and $T''(y) < 0$.[5] Also, mortgage interest payments are tax deductible.

The government also runs a simple pay-as-you-go social security program. It collects the payroll tax from workers' earnings at a rate τ. All the revenues are distributed equally to retirees as their retirement benefits. The social security benefit at age j is denoted by b_j such that $b_j = 0$ if $j < j^*$ and $b_j = b$ if $j \geq j^*$. The retirement benefit b is assumed to equal a fraction ρ (i.e., the replacement ratio) of average labor earnings. The social security program is self-financing and independent of the budget of government expenditures.

Finally, the government redistributes the wealth of households who die unexpectedly. Both housing and financial assets are sold and any outstanding mortgage debt is paid off. The remaining value of these assets is distributed uniformly among the living households as a lump-sum payment tr, which is determined such that the total amount of accidental bequests agrees with that of lump-sum transfers.

2.6 Household's Problem

A household's decision depends on that household's current state variables. The relevant information at the start of each period includes financial asset a, housing asset h, mortgage type m, the number of remaining payments n on the current mortgage, idiosyncratic earning shock ϵ and age j. These individual state variables are summarized as a state vector $s = (a, h, m, n, \epsilon, j)$. Denote the space of individual state vectors as $S = A \times H \times M \times N \times E \times J$, where $A \subseteq \mathbb{R}_+$, $H \subseteq \mathbb{R}_+$ and $N \subseteq \mathbb{N}$. If there is no remaining balance on the current mortgage, then n is simply set equal to zero. This paper imposes non-negativity constraints on both financial and housing assets, i.e., $a \geq 0$, $h \geq 0$, and thus short-selling in both markets is prohibited. These two constraints also prevent households from dying with a negative wealth.

Before stating the household's optimization problem, I first analyze the household's tax obligation given a state $s \in S$. The associated gross taxable income $\tilde{y}(s)$ consists of earnings $v_j\epsilon$, accrued interests on deposits ra, social security benefit b_j minus the mortgage interest

[5] Including a progressive income tax code is important for understanding the household's portfolio allocation between financial assets and housing assets, which is likely to be affected by the tax code.

payment rD_n, which is tax deductible. Thus,

$$\tilde{y}(s) = v_j \epsilon + b_j + ra - rD_n. \tag{5}$$

Here, the lump-sum transfer is not subject to income tax and social security tax payment is not tax deductible. The tax obligation of the household in state s is then defined by

$$t(s) = T(\tilde{y}(s)), \tag{6}$$

where $T(\cdot)$ denotes the progressive tax function. The after-tax income excluding deposit interests is given by

$$y(s) = (1-\tau) v_j \epsilon + tr + b_j - t(s). \tag{7}$$

On the right-hand side of equation (7), the first three terms represent the total income before adjusting for income taxation: the first term denotes earnings net of social security tax; the second term represents the lump-sum transfer received from the government; the third term stands for social security benefit. Recall that $b_j = 0$ for $j < j^*$, i.e., the social security benefit is zero for workers.

The household's optimization problem can be formulated recursively. A household in state $s = (a, h, m, n, \epsilon, j) \in S$ has two options: keep the same house and continue with the existing mortgage ($h' = h$, $m' = m$), or sell the current house and acquire a different house ($h' \neq h$, $m' \in M$). Given the current information in s, the household chooses the option with the higher value, i.e., the value function $v : S \to \mathbb{R}$ satisfies

$$v(s) = \max\{v_k(s), v_c(s)\}, \tag{8}$$

where $v_k : S \to \mathbb{R}$ and $v_c : S \to \mathbb{R}$ represent the value function associated with the option of keeping and changing existing houses, respectively. To simplify the notation, I define an indicator function associated with the two options $I : S \to \{0, 1\}$,

$$I(s) = \mathbf{1}(h' \neq h),$$

where $\mathbf{1}(A)$ is an indicator that takes value one when event A occurs and zero otherwise. Thus, $I(s)$ equals one when a housing position is changed and zero otherwise.

If the household decides to stay in the current house and continue with the existing mortgage, then the household uses periodic after-tax earning $y(s)$ and gross saving from the last period $(1+r)a$ to buy non-housing consumption c, pay periodic mortgage payment q, maintain the

current house $\delta_h Ph$ and save for the next period a'. The associated Bellman equation is

$$v_k(s) = \max_{c,a'} \left\{ u\left(c, h, \zeta_j\right) + \beta\psi_j \sum_{\epsilon' \in E} \pi\left(\epsilon, \epsilon'\right) v\left(s'\right) \right\} \tag{9}$$

$$st.\ c + a' + q + \delta_h Ph = y(s) + (1+r)\, a \tag{10}$$

$$c \geq 0,\ a' \geq 0,$$

where the state vector next period is

$$s' = \left(a', h, m, \max\left\{n-1, 0\right\}, \epsilon', j+1\right). \tag{11}$$

Note that $q = 0$ when the household has paid off all the debt on the existing mortgage.

If the household chooses to acquire a new house, then the current property must first be sold and any remaining balance on the existing mortgage loan be paid off. The sale of the current house generates a net revenue $(1-\phi_s)\, Ph - D$, where ϕ_s is the proportional transaction cost associated with selling a house. The household then chooses a new mortgage to finance the new house purchase. The associated Bellman equation is

$$v_c(s) = \max_{c,h',a',m'} \left\{ u\left(c, h', \zeta_j\right) + \beta\psi_j \sum_{\epsilon' \in E} \pi\left(\epsilon, \epsilon'\right) v\left(s'\right) \right\}$$

$$st.\ c + a' + q + \left(\lambda_{m'} + \phi_b\right) Ph' + \delta_h Ph = y(s) + (1+r)\, a + (1-\phi_s)\, Ph - D \tag{12}$$

$$c \geq 0,\ a' \geq 0,\ h' \geq 0,\ h' \neq h,\ m' \in M,$$

where the state vector next period is

$$s' = \left(a', h', m', L_{m'}, \epsilon', j+1\right). \tag{13}$$

On the left-hand side of the budget equation (12), the term $\left(\lambda_{m'} + \phi_b\right) Ph'$ represents the total initial expenditure arising from the newly obtained house of size h'. It consists of the down-payment $\lambda_{m'} Ph'$ associated with the newly chosen mortgage contract m' and the transaction cost $\phi_b Ph'$ associated with purchasing this new house.

Denote $I : S \to \{0,1\}$, $c : S \to \mathbb{R}_+$, $a' : S \to A$, $h' : S \to H$ and $m' : S \to M$ as the optimal policy functions of the house adjustment decision, non-housing consumption, financial position, housing position and mortgage selection, respectively. By assumption, $h'(s) = h$ and $m'(s) = m$ if $I(s) = 0$. Let $\mathcal{A}$, $\mathcal{H}$, $\mathcal{M}$, $\mathcal{N}$, $\mathcal{E}$ and $\mathcal{J}$ be the σ-algebra generated by sets A, H, M, N, E and J, respectively. Denote $\mathcal{S} = \mathcal{A} \times \mathcal{H} \times \mathcal{M} \times \mathcal{N} \times \mathcal{E} \times \mathcal{J}$. To study the distribution of households over the state space S, consider a probability space $(S, \mathcal{S}, \Phi)$, where Φ is the probability measure defined over $\mathcal{S}$. For any Borel set $S' \in \mathcal{S}$, the number $\Phi(S')$ measures the

mass of households whose individual state vectors lie in S'.

Finally, define the transition function consistent with individual policy functions as $Q : S \times \mathcal{S} \to [0,1], Q(s,S') = \Pr(s' \in S'|s)$ for any $s = (a,h,m,n,\epsilon,j) \in S$ and $S' = A' \times H' \times M' \times N' \times E' \times J' \in \mathcal{S}$. Thus, it holds that

$$Q(s,S') = \sum_{\epsilon' \in E'} \psi_j \pi(\epsilon'|\epsilon) \mathbf{1}((a',h',m',n',\epsilon',j+1) \in S'), \tag{14}$$

where

$$n' = I(s) \times L_{m'} + (1 - I(s)) \times \max\{n-1, 0\}. \tag{15}$$

The number $Q(s,S')$ gives the probability that a household in a current state s will move to some state within the set S' next period.

2.7 Equilibrium

I next define the steady-state equilibrium of this model economy.

Definition 1 *Given a set of time-invariant fiscal policies $\{T(\cdot), \rho\}$ and initial conditions, a steady-state equilibrium is a collection of value function $v : S \to \mathbb{R}$; optimal policy functions $I : S \to \{0,1\}$, $c : S \to \mathbb{R}_+$, $a' : S \to A$, $h' : S \to H$ and $m' : S \to M$; prices $\{r, P\}$; government policies $\{\tau, b, tr\}$; and distribution Φ such that*

1. *Given prices $\{r,P\}$, the value function $v(\cdot)$ and policy functions $I(\cdot)$, $c(\cdot)$, $a'(\cdot)$, $h'(\cdot)$ and $m'(\cdot)$ solve the household's optimization problem.*

2. *The government expenditure budget is balanced:*

$$G = \int_S t(s) \Phi(ds), \tag{16}$$

   where $t(\cdot)$ is the tax obligation function given in (6).

3. *The social security program is self-financing:*

$$b \sum_{j=j^*}^{\bar{J}} \mu_j = \tau \sum_{j=j_0}^{j^*-1} \sum_{i=1}^{\ell} \mu_j v_j d_i \epsilon_i. \tag{17}$$

4. *The lump-sum transfers equal the accidental bequests:*

$$tr = \int_S \left(1 - \psi_j\right) \left(a'(s) + \left(\left(1 - \phi_s\right) P h'(s) - D_{n'}\right)\right) \Phi(ds). \tag{18}$$

5. *Markets clear:*

   (a) *Housing market clears:*

$$\int_S h'(s)\,\Phi(ds) = H_s. \tag{19}$$

   (b) *Financial market clears:*

$$\begin{aligned} 0 \;=\; & \int_S a'(s)\,\Phi(ds) + \int_S q(s)\,\Phi(ds) \\ & + \int_{\{s\in S:\ I(s)=1\}} D_n \Phi(ds) + \int_S \left(1-\psi_j\right) D_{n'}\Phi(ds) \\ & - \int_{\{s\in S:\ I(s)=1\}} (1-\lambda_{m'})\,Ph'(s)\,\Phi(ds) - \int_S (1+r)\,a\Phi(ds). \end{aligned} \tag{20}$$

6. *For the transition function Q consistent with individual decisions as defined in (14), the distribution function Φ is time invariant, i.e., for any Borel set $S' \in \mathcal{S}$,*

$$\Phi(S') = \int_S Q(s, S')\,\Phi(ds).$$

The above definition is standard. For the social security program, the payroll tax rate τ is determined as follows. Since the retirement benefit b is assumed to equal a fraction ρ of average labor earnings, it then follows that

$$b = \rho \frac{\sum_{j=j_0}^{j^*-1}\sum_{i=1}^{\ell}\mu_j v_j d_i \epsilon_i}{\sum_{j=j_0}^{j^*-1}\mu_j}. \tag{21}$$

This, along with equation (17), implies the following payroll tax rate:

$$\tau = \rho \frac{\sum_{j=j^*}^{\bar{J}}\mu_j}{\sum_{j=j_0}^{j^*-1}\mu_j}. \tag{22}$$

The right-hand side of the balance equation (18) of lump-sum transfers represents the total wealth left over by households who are alive today but die unexpectedly next period: the integral associated with integrand $a'(s)$ represents the total financial wealth of those households, and that with integrand $(1-\phi_s)\,Ph'(s) - D_{n'}$ represents the net housing value after paying off any outstanding mortgage debt. This wealth is collected by the government and redistributed uniformly among the living households as a lump-sum payment tr.

For the market-clearing equation (20), the four terms on the first two lines on the right-hand side of the equation capture the total amount of funds available at the intermediary: on the first line, the first term is the total amount of new deposits by households at the intermediary,

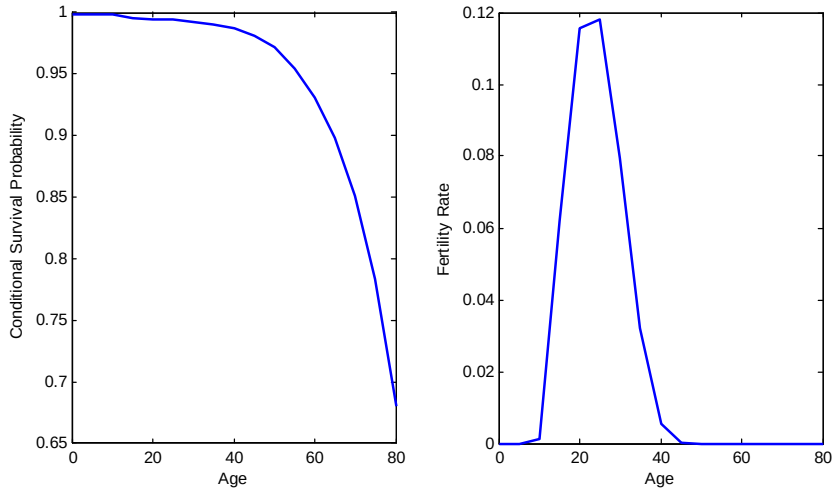

Figure 3: Conditional Survival Probabilities and Fertility Rates

and the second term accounts for mortgage payments received from households; on the second line, the first term measures repayments of outstanding mortgage balances from sales of existing houses by those households adjusting their houses, and the second term measures repayments of outstanding debt of households who die unexpectedly. The two terms on the third line represent the total amount of funds made available to households: the first term represents new mortgage loans and the second term captures gross payments on old deposits.

3 Calibration

This section describes the calibration of the benchmark model to the U.S. data. The set of model parameters is split into two subsets. The first subset consists of those parameters whose values have been estimated in previous studies or estimated directly from the data without the need to solve the model. The second subset includes parameters estimated by method of moments such that selected moments of the U.S. data are matched.

3.1 Demographics

One period in the model equals five years. Individuals start their adult life at age 20 ($j_0 = 5$) and can live up to age 80 ($\bar{J} = 17$). Thus, there are 13 adult generations who are economically active. Retirement is mandatory at age 65 ($j^* = 14$). The conditional survival probabilities $\{\psi_j\}_{j=1}^{\bar{J}}$ and fertility rates $\{\kappa_j\}_{j=1}^{\bar{J}}$ are taken, respectively, from the U.S. Life Tables in 1991 and U.S. Vital Statistics on Natality in 1991, reported by the National Center for Health Statistics. I plot the survival probabilities and fertility rates in Figure 3.

Immigrants are defined as individuals who were foreign-born and whose parents were not U.S. citizens.[6] I do not use direct evidence to estimate the net immigration parameters $\{\omega_j\}_{j=1}^{\bar{J}}$, because there is a scarcity of official emigration data relative to immigration data. For instance, the U.S. Census Bureau does not collect data on the number of people, either citizens or non-citizens, who emigrate from the United States. Furthermore, many emigrants from the United States do not plan to become permanent emigrants, but to be expatriates for a limited amount of time. The high dynamics of the emigration-prone groups make emigration from the United States indiscernible from temporary country leave. For this consideration, I use an indirect method to estimate these parameters. More precisely, I calibrate $\{\omega_j\}_{j=1}^{\bar{J}}$ such that the implied steady-state population growth rate of the population dynamics system (1), which is the dominant eigenvalue of the population projection matrix $\Gamma + \Omega$, matches the population growth rate observed in the data.[7] Also, I impose the same assumption as in Waldron and Zampolli (2010) that $\omega_j = 0$ for any $j \neq j_0$, i.e., all immigrants arrive at their youngest adult age 20.[8] This assumption is made to simplify the estimation but it can also be defended on the ground that many immigrants arrive at an early working age. Kandel (2011) finds that six of every 10 foreign-born persons are between age 25 to 54, whereas only four of every 10 native-born persons fall within this age group; meanwhile, children under the age of 18 make up 24.3 percent of the native-born population but only 7.3 percent of the foreign-born population. In the benchmark model, I calibrate the immigration rate to match an annual population growth rate of 0.99 percent in 1994, which results in a parameter value ω_{j_0} of 0.0767.

To calibrate the effective household size $\left\{\zeta_j\right\}_{j=j_0}^{\bar{J}}$, I first compute the life-cycle profile of average household size from the 1995 Survey of Consumer Finances (SCF), a triennial survey sponsored by the Federal Reserve Board in cooperation with the Department of Treasury. The age of a household corresponds to the age of the head of the household in the survey. The average household size for each age group is calculated as the average size of those households whose heads' ages fall into this age group. The estimated series of household sizes are then converted into series of effective household sizes using household equivalence scales, which capture the economies of scale in household consumption. The household equivalence scale measures the change in consumption expenditures needed to keep the welfare of a household constant when its size varies. The specific scale sizes vary across different studies, whose estimates have both advantages and drawbacks. In this paper, I follow the strategy of Fernandez-Villaverde and Krueger (2007) and use the mean of eight different reported estimates in the benchmark

[6] Similar to Storesletten (2000), I do not model the impact of illegal immigration.

[7] In the scenario with no immigration, i.e., $\omega_1 = ... = \omega_{\bar{J}} = 0$, the implied population growth rate given the calibrated survival and fertility rates is -14.1 percent per period (-2.98 percent per year). But with a moderate immigration flow, the implied steady-state growth rate is small and positive. This supports the common view that immigration is a crucial source of U.S. population growth.

[8] They calibrate the immigration parameter ω_{j_0} in a different way by choosing it to match the total net immigration into the United Kingdom.

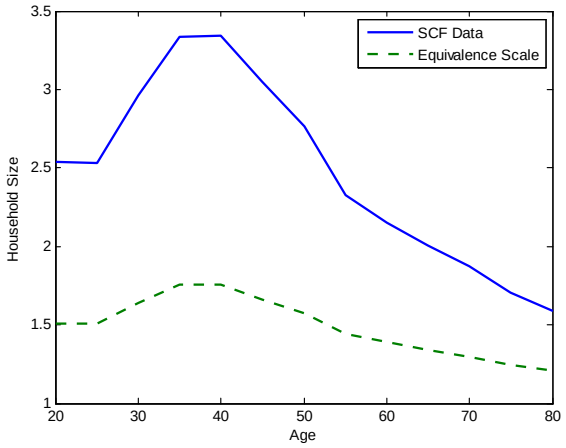

Figure 4: Life-Cycle Profile of Average Household Size

calibration. The results are reported in Table 1. The scale series are then fitted by a fourth-order polynomial, which is used to convert the estimated average household size into the effective household size for each age group.

Table 1: Household Equivalence Scales

Household Size	1	2	3	4	5
Equivalence Scale	1	1.34	1.65	1.97	2.27

Figure 4 plots the life-cycle profiles of the average household size based on the SCF data and equivalence scales. Both profiles exhibit a hump shape over the life cycle, where the average household size peaks at age 40 and then declines afterward.

3.2 Preference

The period household utility function is assumed to be of the constant relative risk-aversion type given by

$$u(c,h,\zeta)=\zeta\frac{\left(\left(\frac{c}{\zeta}\right)^{\theta}\left(\frac{h+\eta}{\zeta}\right)^{1-\theta}\right)^{1-\sigma}}{1-\sigma}=\zeta^{\sigma}\frac{\left(c^{\theta}\left(h+\eta\right)^{1-\theta}\right)^{1-\sigma}}{1-\sigma}, \tag{23}$$

where η is an arbitrarily small positive number and it makes the utility function finite when $h=0$, i.e., one can survive without a house but one cannot survive without food. The parameter σ denotes the coefficient of relative risk aversion and is set to 2, a value that lies in the middle of the range commonly used in the literature. Meanwhile, the aggregation between non-housing goods and housing services is assumed to be Cobb-Douglas, which is widely used in the housing literature. Indeed, Fernandez-Villaverde and Krueger (2005) argue that the Cobb-Douglas

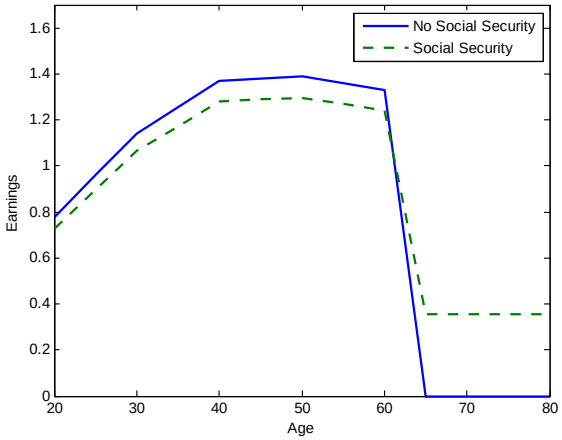

Figure 5: Life-Cycle Profile of Average Earnings

aggregation function, which implies a unit elasticity between housing and non-housing goods, is an empirically reasonable choice.[9] I estimate the aggregation parameter θ and time discount factor β endogenously by method of moments as described in section 3.6.

3.3 Endowment

In each period, an age j household with an earning shock ϵ receives an earning $v_j\epsilon$. The deterministic life-cycle average earning profile $\{v_j : j \in J\}$ is taken from Hansen (1993). Figure 5 plots the life-cycle profiles of earnings before and after adjustment with social security. Clearly, both earning profiles exhibit a hump shape over the life cycle, with a peak at around age 50. Also note that $v_j = 0$ for $j \geq j^*$ as retirement is mandatory at age 65.

For the parameterization of the stochastic component of earning process ($E = \{\epsilon_1, ..., \epsilon_\ell\}$, $\Pi = (\pi_{ik})_{1\leq i,k\leq \ell}$), I first estimate the continuous-state stochastic earning process $\{\tilde{\epsilon}\}$ from the data. Its logarithm counterpart $\{u = \log \tilde{\epsilon}\}$ is assumed to follow an AR(1) process:

$$u' = \rho_u u + \varepsilon', \tag{24}$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$ is the driving innovation process. The unconditional standard deviation of u is $\sigma_u = \sqrt{\sigma_\varepsilon^2/(1-\rho_u^2)}$. Using the Panel Study of Income Dynamics (PSID) data aggregated into a five-year period, Yang (2009) estimates that $\rho_u = 0.85$, $\sigma_\epsilon^2 = 0.30$ and thus $\sigma_u = 1.04$. I use these parameter estimates in the benchmark model. Note that compared to the one-year-period estimation as in Storesletton, Telmer and Yaron (2004), the persistence is lower and the

[9] For instance, using aggregate data and the constant elasticity of substitution aggregation function $g(c,h) = (\theta c^\eta + (1-\theta)h^\eta)^{1/\eta}$ between housing and non-housing goods, Ogaki and Reinhart (1998) obtain an estimate of $\eta = 0.143$, which is statistically not different from zero.

variance is higher due to the five-year setting.

The continuous-state process (24) is then discretized into a three-state Markov chain ($E = \{\epsilon_1, \epsilon_2, \epsilon_3\}, \Pi = (\pi_{ik})_{1\leq i,k\leq 3}$) with $\epsilon_1 < \epsilon_2 < \epsilon_3$. I choose three discrete states as a compromise between computation feasibility and approximation quality.[10] The discretization relies on the method proposed by Tauchen (1986). As a result, I obtain the set of idiosyncratic earning shocks[11]

$$E = \{0.263, 0.745, 2.106\}, \tag{25}$$

the associated transition matrix

$$\Pi = \begin{bmatrix} 0.747 & 0.248 & 0.005 \\ 0.171 & 0.658 & 0.171 \\ 0.005 & 0.248 & 0.747 \end{bmatrix} \tag{26}$$

and its implied invariant probability distribution

$$\Delta = \begin{bmatrix} 0.290 & 0.420 & 0.290 \end{bmatrix}. \tag{27}$$

3.4 Housing Market

For parameters that capture institutional features of housing markets, I assume in the benchmark model that the only available mortgage is a traditional FRM with a 30-year term and a 20 percent downpayment requirement.[12] Both purchasing and selling houses incur transaction costs: purchasing costs arise from buyers' search cost and mortgage initiation cost, while selling costs involve ongoing brokerage fees. Gruber and Martin (2003) document from the Consumer Expenditure Survey that the buying cost is about 2.5 percent of the house value and the selling cost is, on average, 7 percent. I adopt their estimates and set $(\phi_b, \phi_s) = (0.025, 0.07)$. For home maintenance cost, according to Harding, Rosenthal and Sirmans (2007), the depreciation rate for housing units used as shelter is estimated to be between 2.5 percent and 3 percent. I set δ_h at 0.03 in the benchmark model.

The aggregate housing stock H_s is estimated as follows. First, I use data from the U.S. Bureau of Economic Analysis to calibrate the housing stock-output ratio observed in 1994. From the 1994 National Income and Product Accounts (NIPA), I compute the output Y_{1994} by

[10] Although a larger number of states would be more desirable, the current three-state choice is reasonable considering the low persistence of the stochastic earning process.

[11] The mean of idiosyncratic earning shocks is normalized to one using the invariant distribution Δ.

[12] This is a typical downpayment ratio of primary mortgage loans used in the housing literature. Indeed, employing the data on the value of homes purchased and the amount borrowed on the first mortgage from the 1995 American Housing Survey, Chambers, Garriga and Schlagenhauf (2009a) find that the downpayment fraction for the first-time home purchase is 0.1979, whereas the fraction for households who previously owned a home is 0.2462.

subtracting the item of housing and utilities from GDP. This exclusion is due to the fact that housing service is not a part of output in the model. From the Fixed Assets Tables, I compute the housing stock H_{1994} as the residential fixed stock, including both private and government residential stock. This gives an estimated ratio H_{1994}/Y_{1994} equal to 1.233. Second, assuming the standard long-run labor share 0.7 of the U.S. national income, I calibrate the housing stock H_s as

$$H_s = \frac{1}{5} \times \frac{H_{1994}}{Y_{1994}} \times \frac{1}{0.7} \times \bar{y}_{\text{model}}, \tag{28}$$

where $\bar{y}_{\text{model}}$ represents the average household income in the benchmark model and the factor $1/5$ makes the stock-income ratio consistent with the five-year model period.

3.5 Government

To capture the progressivity of federal income tax rates in the current U.S. tax system, I use the estimated tax function from Gouveia and Strauss (1994) to represent the income tax code. They model the progressive tax schedule of the U.S. federal income tax with the following tax functional form:

$$T(y) = \tau_0 \left(y - \left(y^{-\tau_1} + \tau_2 \right)^{-\frac{1}{\tau_1}} \right), \tag{29}$$

where $T(y)$ is the tax bill associated with a taxable income y. The tax function is characterized by three policy parameters: τ_0 is a scaling factor; τ_1 is a curvature factor of the tax function; and τ_2 is a unit factor determining the unit used to measure income and the size of income deduction. Gouveia and Strauss (1994) estimate this tax function for the U.S. data in the 1980s and obtain that, in 1989, $\tau_0 = 0.258$, $\tau_1 = 0.768$ and $\tau_2 = 0.031$. The benchmark model uses the same parameter estimates for τ_0 and τ_1. However, since tax function (29) is estimated for incomes in 1990 U.S. dollars and is not unit free, I follow Erosa and Koreshkova (2007) to normalize τ_2 to $\tilde{\tau}_2$ such that the average tax rate is equalized in the U.S. economy and in the benchmark model, which implies that

$$\tilde{\tau}_2 = \tau_2 \left(\frac{\bar{y}_{\text{model}}}{\bar{y}_{1990}} \right)^{-\tau_1}, \tag{30}$$

where $\bar{y}_{\text{model}}$ is the average household income in the benchmark model, and $\bar{y}_{1990}$ is the average household income in the 1990 U.S. economy.

For the social security program, retirees receive a fixed social security benefit as a fraction of their average earnings before retirement. The replacement rate ρ is set at 30 percent, which then implies a payroll tax rate τ of 6.68 percent according to (22).

3.6 Estimation of (β, θ)

There are still two parameters, namely the discount factor β and the aggregation parameter θ, that remain to be estimated. Instead of using direct evidence to pin down these two parameters, I use the method of moments by choosing their values so that selected moments are matched. Let $\Theta = (\beta, \theta)$ be the vector of structural parameters to be estimated. Let $\{F_k\}_{k=1}^2$ and $\{F_k(\Theta)\}_{k=1}^2$ represent the two specified aggregate statistics observed in the data and computed from the model economy parameterized with Θ, respectively. The aim is to find some vector Θ^* such that $F_k(\Theta^*) = F_k$, $k = 1, 2$. Here, evaluating $F_k(\Theta)$ for a given Θ requires numerically solving the household's optimization problem and finding the equilibrium prices. The computational algorithm to find an equilibrium is described in the appendix.

Table 2: Benchmark Model Parameters

Parameter		Value
Demographics		
$\bar{J}$	maximum life expectancy	17
j^*	mandatory retirement age	14
j_0	youngest adult age	5
ω_5	immigration rate	0.0767
Preference		
σ	relative risk-aversion coefficient	2.0
β	discount factor	0.925
θ	non-housing preference	0.90
Housing		
L	mortgage length	6
λ	downpayment requirement	0.200
δ_h	housing maintenance cost	0.030
ϕ_b	transaction cost of buying a house	0.025
ϕ_s	transaction cost of selling a house	0.070
Government		
τ_0	tax scaling factor	0.258
τ_1	tax curvature factor	0.768
τ_2	tax unit factor	0.031
ρ	replacement ratio	0.300
τ	payroll tax rate	0.0668

The first targeted moment is the equilibrium interest rate. Using the Federal Reserve Economic Data, I compute an average annual real interest rate $r = 3.2\%$ during the early 1990s. The choice of this target is motivated by the fact that the discount factor β directly

impacts households' willingness to borrow and lend, and thus the interest rate. The second targeted moment is the ratio of non-housing consumption over output, which is estimated to be 0.54 from NIPA in 1994. The aggregation factor θ, which measures the relative preference between non-housing goods and housing services, affects the allocation of earnings among these two goods. The calibration procedure gives $\beta = 0.925$ and $\theta = 0.90$. Table 2 summarizes the benchmark model parameterization.

3.7 Life-Cycle Profiles of Consumption and Asset Holdings

This section investigates the life-cycle profiles of consumption and asset holdings generated from the parameterized benchmark model. They are important for understanding the mechanisms of how changes in economic fundamentals affect asset prices in the next section. The averages are obtained by integrating the variable of interest with respect to the stationary distribution of households holding age fixed. For instance, the life-cycle profile of non-housing consumption $\{C_j : j \in J\}$ is computed as $C_j = \frac{1}{\mu_j} \int_S c(s)\, \Phi(da \times dh \times dm \times dn \times d\epsilon \times \{j\})$, $j \in J$, which is the average non-housing consumption among households within the age j cohort.

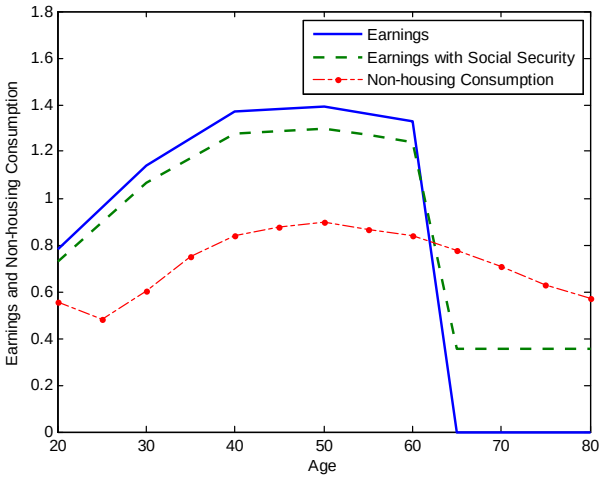

Figure 6: Life-Cycle Profile of Earnings and Non-Housing Consumption

Figure 6 plots the life-cycle profiles of non-housing consumption, earnings and earnings adjusted with social security. As seen previously in Figure 5, the life-cycle profiles of earnings exhibit a hump shape. This hump-shaped pattern also occurs in the life-cycle profile of non-housing consumption, peaking at around age 50.[13] The result is consistent with previous studies on household consumption, such as Fernandez-Villaverde and Krueger (2005) and Yang (2009). This pattern is attributed to the presence of housing and mortality risk: in their early years,

[13]The relatively high non-housing consumption in the first period results from the fact that households start with zero housing assets and thus choose a high non-housing consumption to avoid too low utility.

households find it optimal to compromise their non-housing consumption in order to build up their housing stock, which generates housing service and provides collateralizable insurance against idiosyncratic earning shocks; in their later years, households reduce their non-housing consumption as they discount their future consumption at a higher rate when facing increasing mortality rates. Meanwhile, due to consumption-smoothing behavior, the life-cycle profile of non-housing consumption is less pronounced than that of earnings.

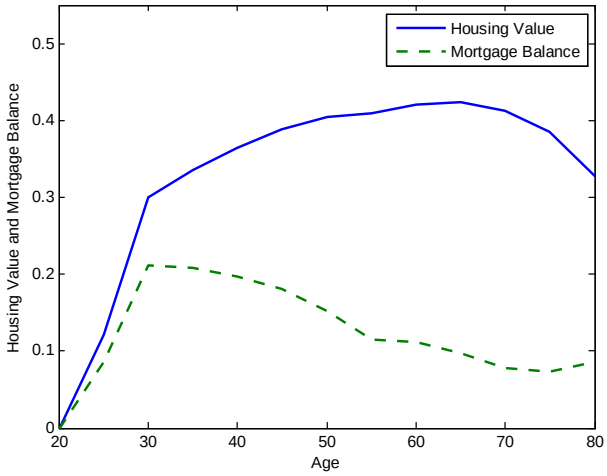

Figure 7: Life-Cycle Profile of Housing Value and Mortgage Balance

Figure 7 plots the life-cycle profiles of housing value and mortgage balance. Housing consumption increases sharply in the first two periods and then increases gradually with a peak at around age 65, and decreases slowly afterward. By forgoing some non-housing consumption, households build up their housing stock quickly early in life to reap its benefits of housing service and collateral insurance. These two benefits of housing, along with the transaction cost associated with housing adjustment, prevent households from downsizing their houses quickly late in life. The relatively flat profile of housing asset holdings compared to non-housing consumption is consistent with empirical findings in Feinstein and McFadden (1989), Venti and Wise (2000) and Yang (2009). These studies find that elderly households do not substantially decrease their housing consumption. Meanwhile, as expected from the model setting, the life-cycle profile of outstanding mortgage balance lies below that of housing value. Note that the positive average mortgage debt late in life lies in the assumption that when senior households trade down their houses, they will get new mortgages and thus evoke new mortgage debt. Furthermore, households hold their largest mortgage balance around age 30. After that, they gradually pay off their mortgage debt and save in financial assets. It is also interesting to note a slight rise in the mortgage debt of households late in their life. This small increase is due to mortality risk. Before the last period, many senior households typically have paid off all or most of their

mortgage debt. However, foreseeing their death after the last period, they trade down their houses a period before, which evokes a slight rise in the average mortgage balance of the last age cohort.

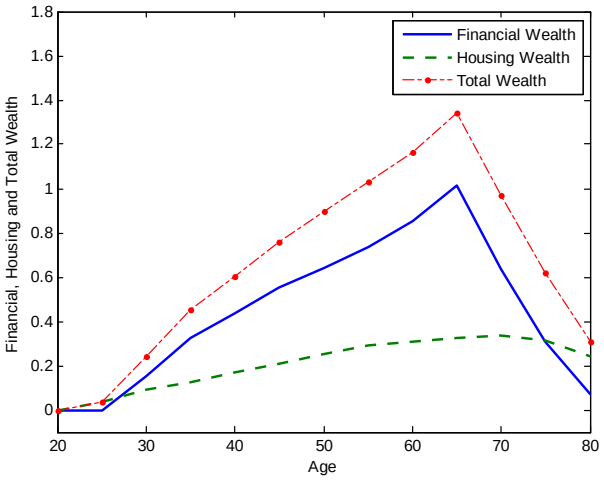

Figure 8: Life-Cycle Profile of Wealth Holdings

Figure 8 plots the life-cycle profiles of financial wealth, housing wealth and total wealth. Here, housing wealth is defined as the difference between housing value and outstanding mortgage balance. As the figure shows, a household virtually owns no financial assets even at the beginning of the second period, because of the downpayment requirement for purchasing a house in the first period. This agrees with the important fact that young households do not save in financial assets but rather in housing assets. After the housing stock is built up, they start saving in financial assets to prepare for retirement, with their financial wealth peaking at age 65. After retirement, households dissave their financial assets to sustain their non-housing consumption, but do not significantly decrease their housing stock. The life-cycle profile of housing wealth is less pronounced than that of financial wealth, since households build up their home equity gradually. As seen in Figure 7, households start with relatively little home equity at a young age. As time passes, they pay off their mortgage debt and accumulate home equity. It is evident that households in their 20s have the lowest demand for either asset in the life cycle, whereas 35-65 are their prime asset accumulating years. This implies that a population with a smaller share of age cohorts in their 20s tends to have a larger demand for both assets, putting upward pressure on their prices.

4 Changes in House Prices and Mortgage Debt

This section employs the parameterized model to analyze the observed changes in U.S. house prices and mortgage debt between 1994 and 2005. Three experiments are performed to disentangle the individual contribution of population aging, mortgage innovation and historically low interest rates to the run-up in house prices and mortgage debt. Each experiment varies one factor at a time, holding others constant relative to the benchmark scenario. The corresponding contribution of this varying factor is then estimated by comparing the resulting new equilibrium to the benchmark equilibrium. More precisely, the first experiment replaces the population distribution in 1994 with its counterpart in 2005. The second experiment introduces an additional mortgage product into the benchmark model. The third experiment considers the effects of declining interest rates. I also measure the combined effects of these three factors by allowing their changes to occur simultaneously.

4.1 Effects of Population Aging

The U.S. population has gotten progressively older in the past few decades. Factors contributing to this demographic change include declining fertility rates, falling mortality rates at older ages and slowing immigration. As studied in the existing literature, demography matters for asset prices. As seen in Figure 7, the average housing demand of households at older ages is higher than at younger ages. Thus, given a fixed population size, aggregate housing demand is larger in a population with a smaller share of age cohorts in their 20s.

This section quantifies the impact of population aging on the U.S. housing market and examines whether demographics may have been accountable for the surge in house prices and mortgage debt during the period between 1994 and 2005. The steady-state population distribution in 2005 is determined from the population model (1) using the observed population growth rate in 2005 rather than that in 1994. This section also estimates the quantitative importance of immigration on house prices via its effect on the age distribution of the population.

Table 3: Effects of Demographics on Macroeconomic Aggregates

	Benchmark	2005 Demographics	Change	Actual Change
Interest Rate	0.159	0.153	−0.60%	
House Price	0.978	0.997	+1.94%	+45.0%
Mortgage Debt	0.128	0.130	+1.56%	+56.0%

Notes:

1. The interest rate is measured in a five-year period.
2. The size of the change in the interest rate is given in percentage points.

Table 3 summarizes the effects of population aging on macroeconomic aggregates if only the demographic structure were changed during this period. The first column shows equilibrium aggregates in the benchmark economy, and the second column reports equilibrium aggregates in the economy, with demographics in the benchmark year 1994 being replaced by those in 2005. The third column shows the change in the 2005 levels relative to the benchmark. Demographic changes during this period increase house prices by 1.94 percent and lower interest rates by 0.6 percent. Population aging, as seen in Figure 2, decreases the share of young households but increases the share of households in their prime asset accumulation years. This in turn increases the demand for both housing and financial assets, and pushes up their prices.

Meanwhile, aggregate mortgage debt increases by 1.56 percent, a number smaller than the increase in house prices. Figure 7 shows that households owe their largest mortgage debt at their younger ages, when they have the smallest home equity in their houses. As time goes by, they gradually pay off their mortgage debt and save in financial assets. Thus, when keeping house prices constant, a population with more age cohorts over 30 has a smaller aggregate mortgage debt. This explains why mortgage debt increases less than house prices in this experiment.

The above experiment shows that demographics generate changes in house prices and mortgage debt that are both qualitatively consistent with those observed in the data. However, the model-generated changes do not match in magnitude their data counterparts. As the fourth column of Table 3 shows, house prices and mortgage debt increase, respectively, by 45 percent and 56 percent between 1994 and 2005, both far above their corresponding model predictions. This suggests that demographics are not the major factor in the actual changes in the U.S. housing market during this period. Additional factors need to be explored.

My model also allows a study of the relationship between immigration and house prices via immigrants' effects on the age distribution of the population. Immigrants have been a major source of U.S. population growth in recent decades. Declining arrivals of young immigrants since the early 1990s are one of the major drivers of ongoing population aging. To measure the quantitative importance of immigration on house prices, I perform another experiment by artificially raising the immigration rate by 1 percent over its 1994 level but keeping both mortality and fertility rates constant. The new immigration rate (0.0774) results in a steady-state annual population growth rate of 1.02 percent and an associated stationary population distribution. I then recompute the equilibrium with this distribution and report the results in Table 4.

The table shows that, holding mortality and fertility rates constant, a 1 percent increase in the immigration rate would lower house prices by 0.82 percent, with an immigration-price elasticity of 0.82. As seen in Figure 7, the age 20-24 cohort has the smallest demand for housing assets in the life cycle. As immigrants arrive within this age cohort in the model, increasing immigration decreases the aggregate housing demand, which then lowers house prices. Also, as

Figure 8 shows, households hold virtually no financial assets until their 30s, so that they can save for a downpayment. Rising arrivals of young immigrants decrease the share of age cohorts at the prime saving ages. This in turn lowers the aggregate demand for financial assets and drives down their prices.

Table 4: Effects of Immigration on Macroeconomic Aggregates

	Benchmark	+1% Immigration	Change
Immigration Rate	0.0767	0.0774	+1.00%
Interest Rate	0.159	0.161	+0.20%
House Price	0.978	0.970	−0.82%
Mortgage Debt	0.128	0.127	−0.78%

The above prediction of a negative effect of immigration on house prices might seem controversial at first glance. Relevant immigration literature typically predicts a positive effect (e.g., Ottaviano and Peri, 2007; Gonzalez and Ortega, 2010). Their underlying logic is that rising immigration increases aggregate demand for houses and imposes upward pressure on house prices. This is indeed the scale effect of immigration on aggregate housing demand. However, it is useful to recall that population size is fixed throughout the analysis in this paper. Thus, the above experiment focuses on the distribution effect of immigration on housing demand through its impact on the long-run age distribution. Increasing immigration lowers the average age of the population and decreases aggregate housing demand. Therefore, the above prediction does not contradict the previous studies, but rather captures a different effect of immigration on house prices. The exercise suggests that a comprehensive account of the impact of immigration on housing markets might need to take both effects into consideration.

4.2 Effects of Mortgage Innovation

Since the early 1990s, U.S. housing markets have undergone considerable changes with respect to housing finance. One particularly important change was the introduction of a number of new mortgage products known as combo loans in the late 1990s. These products are popular, especially among those who want to avoid large downpayments and private mortgage insurance. For instance, an "80-15-5" combo implies a primary loan for 80 percent of the house value, a secondary loan for 15 percent, and a 5 percent downpayment. Similarly, an "80-20" combo loan, or so-called no-downpayment loan, corresponds to a traditional loan with a loan-to-value (LTV) ratio of 80 percent accompanied by a second loan for a 20 percent downpayment. More formally, consider a combo loan "$(1-\lambda)$-$(1-\varkappa)\lambda$-$\varkappa\lambda$," which consists of a primary loan of maturity N_1 and interest rate r_1 for a $(1-\lambda)$ portion of the house value, and a secondary loan of maturity N_2 and interest rate r_2 for a $(1-\varkappa)\lambda$ portion of the house value, along with a

downpayment $\varkappa\lambda$. Here, the parameter $\varkappa \in [0,1]$ represents whether a downpayment is needed: when $\varkappa = 1$, it simply corresponds to a standard FRM without a secondary loan; when $\varkappa = 0$, it corresponds to a loan without downpayment. Compared to the primary loan, the secondary loan usually requires a faster repayment $N_2 \leq N_1$ and a higher interest rate $r_2 = r_1 + \iota$, where $\iota \geq 0$ denotes an interest premium. The total payment on this combo loan consists of amortized payments from two individual loans, and can be expressed as

$$q = \begin{cases} q_1 + q_2 & \text{if } N_1 - N_2 < n \leq N_1 \\ q_1 & \text{if } 0 \leq n \leq N_1 - N_2 \end{cases}, \tag{31}$$

where $n \in \{0, 1, ..., N_1\}$ denotes the number of remaining mortgage payments and q_i represents the amortized payment on each individual loan i, $i = 1, 2$, computed from (4). The periodic interest payments and outstanding balance on a combo loan is simply the sum of that on the two individual loans.

To identify the contribution of mortgage innovation to rising house prices and mortgage debt, I fix demographics at the benchmark scenario but expand the menu of available mortgage contracts for financing house purchases. In particular, the following experiment considers a mortgage set in 2005 consisting of one standard 30-year FRM with an 80 percent LTV ratio and one "80-20" combo loan, and that in 1994 has only the 30-year FRM with an 80 percent LTV ratio. For the combo loan, both its primary and secondary loans are assumed to have a 30-year maturity, with an interest premium of $\iota = 2\%$ on the latter. Given the expanded menu of mortgages, each time a household decides to finance a house purchase, the household chooses from the two alternative mortgage contracts, which trade off between lower downpayments and higher interest rates.

Table 5: Effects of Mortgage Innovation on Macroeconomic Aggregates

	Benchmark	2005 (FRM+Combo)	Change	Actual Change
Interest Rate	0.159	0.174	+1.50%	
House Price	0.978	0.970	−0.82%	+45.0%
Mortgage Debt	0.128	0.152	+18.8%	+56.0%

Table 5 shows that introducing this additional combo loan significantly increases mortgage debt by nearly 19 percent. This prediction is promising, since mortgage innovation alone is capable of explaining nearly 34 percent of the observed change in the mortgage debt. However, the model also shows that the availability of this combo loan lowers house prices by 0.82 percent and raises interest rates by 1.50 percent. This prediction over asset prices is quite surprising, since conventional views suggest that mortgage innovation should have been responsible for driving up house prices during this housing boom. Many households who would otherwise be excluded from the housing market due to downpayment constraints could now finance their

house purchases with the newly introduced combo loan. This should have created upward pressure on house prices. However, the model shows that, keeping other factors constant, mortgage innovation indeed has a nearly trivial effect on house prices.

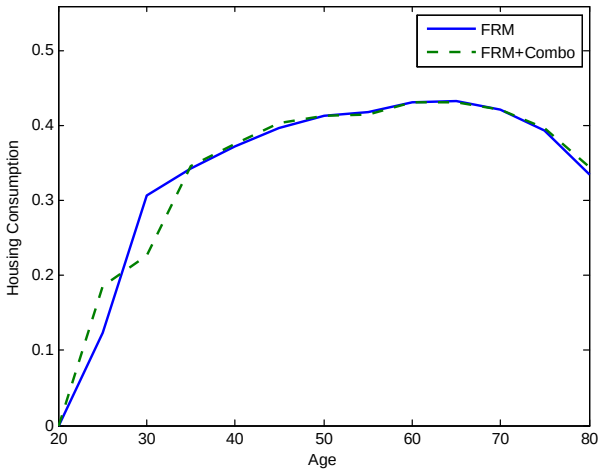

Figure 9: Effects of Mortgage Innovation on Housing Consumption

To explain these patterns of macroeconomic changes, I investigate how households adjust their consumption-saving behavior in response to mortgage innovation. Figure 9 shows the effects of introducing this combo loan on the life-cycle profile of housing consumption. Obviously, the availability of this new mortgage product mostly affects the housing consumption of young households in their 20s, but has very little effect on that of households above this age group. With the combo loan's more lax downpayment requirement, households in the age cohort of 20-24 are able to finance a larger house as expected, and their average house size increases by nearly 50 percent. However, having bought the larger houses with combo loans, those young households bear a larger burden of interest payments. This, along with the higher selling cost of existing bigger houses, prevents them from quickly upsizing their houses at age 25-29, which they would do in the benchmark scenario. Our computation shows that the average house size of this age cohort drops by nearly 26 percent.[14]

Figure 10 shows the effects of introducing this combo loan on the life-cycle profile of mortgage balance. With an expanded mortgage set, mortgage debt increases across various age groups. Causes of rising mortgage debt, however, differ over the life cycle. Before age 25, households hold more mortgage debt because many of those young households purchase bigger houses with

[14]This result does not contradict the findings in Chambers, Garriga and Schlagenhauf (2009a). They find that mortgage innovation accounts for between 56 percent and 70 percent of the increase in homeownership during this boom period. If one considers homeownership as having a housing stock over a certain minimum level, then the result can be interpreted as showing that mortgage innovation increases the average house size of young age cohorts and thus increases the homeownership among young households. Indeed, the authors show that the increase in total homeownership is mainly driven by rising homeownership among young households.

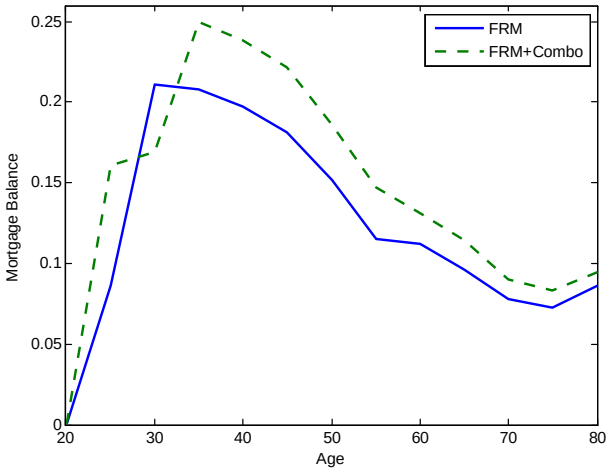

Figure 10: Effects of Mortgage Innovation on Mortgage Balance

lower downpayments using the combo loan. Nevertheless, households hold a larger mortgage balance for a different reason late in life. As seen in Figure 9, the availability of the combo loan has barely any effect on the housing consumption of households beyond age 35. This suggests that mortgage innovation does not induce those households to purchase larger houses. In fact, some of those households who happen to adjust their houses choose to finance with the combo loan and thus increase their debt holdings for the following two reasons: first, although they might have accumulated enough financial wealth to meet the downpayment requirement of the traditional FRM, the combo loan can help them increase their savings in financial assets for retirement; second, since mortgage interest payment is tax deductible, larger interest payments on the combo loan make it more tax attractive than FRM. This explains why the mortgage debt of those households rises, on average, though their housing consumption remains roughly unchanged. Finally, it is straightforward that falling mortgage debt at age 30 mainly results from the decrease in the average house size of households at this age as seen in Figure 9.

But why does mortgage innovation not drive up house prices? The answer is that mortgage innovation indeed has two offsetting effects on aggregate housing demand in a general equilibrium setting. On the one hand, it relaxes financial constraints and makes the housing market more accessible to more households, which thus spurs housing demand. On the other hand, it increases the demand for mortgage loans, which pushes up the interest rate as shown in Table 5. Consequently, both the returns on financial assets and the cost of mortgage borrowing rise. The substitution effect induces middle-aged households to shift their wealth from housing assets to financial assets. Figure 11 shows the effects of introducing the combo loan on the life-cycle profile of financial wealth. Rising interest rates induce households over age 30 to invest more in financial assets than in the benchmark model. The difference is particularly pronounced for

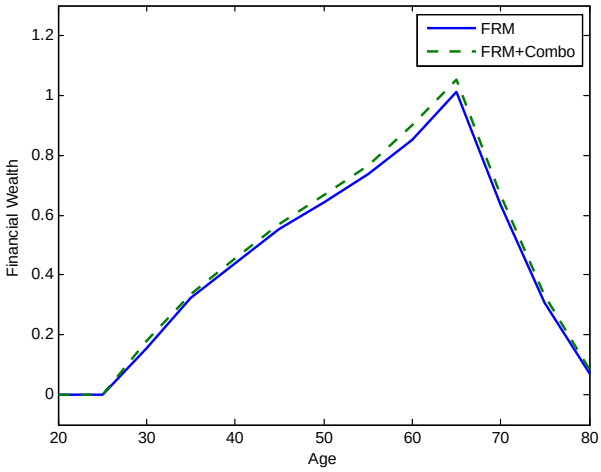

Figure 11: Effects of Mortgage Innovation on Financial Wealth

households at the prime saving ages of 35-65. The results show that though mortgage innovation makes borrowing easier, middle-aged households do not significantly upsize their houses, as seen in Figure 9, but would rather increase their savings in financial assets. As a result, the positive impact of relaxed borrowing constraints on housing demand is offset by households' reallocating their wealth toward financial assets due to rising interest rates. This explains why mortgage innovation alone has barely any effect on house prices.

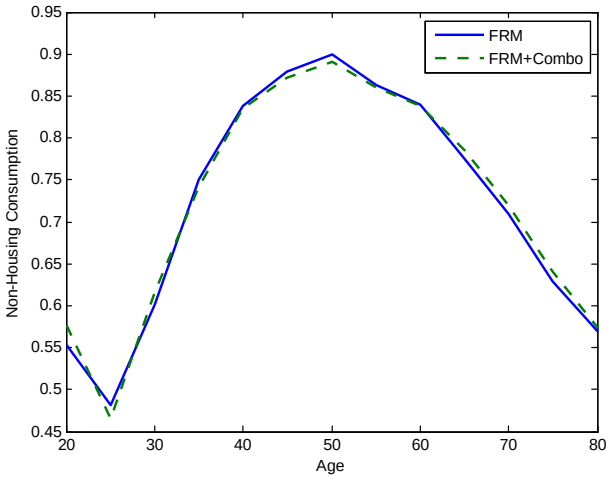

Figure 12: Effects of Mortgage Innovation on Non-Housing Consumption

It is also interesting to investigate how mortgage innovation affects a household's non-housing consumption. Figure 12 shows the effects of introducing the combo loan on the life-cycle profile of non-housing consumption. Mortgage innovation clearly has a non-uniform effect on the non-housing consumption in the life cycle. First, because of the relaxed downpayment

requirement with the combo loan, households at age 20-24 consume more than in the benchmark scenario. However, they consume less at age 25-29, due to higher interest payments arising from their increased holdings of mortgage debt. Meanwhile, as households' demand for financial assets rises due to rising interest rates, they sacrifice some of their non-housing consumption between age 35 and 60. The larger amount of savings in financial assets, however, allows them to support a higher level of non-housing consumption after retirement.

4.3 Effects of Historically Low Interest Rates

The previous experiment suggests that if mortgage innovation were the only change occurring during this period, then it would significantly increase mortgage borrowing but leave house prices nearly unaffected as a result of rising interest rates. However, U.S. interest rates reached a historically low level in the early 2000s. It is believed that the latest U.S. housing boom was caused in part by record low interest rates. A lower interest rate reduces the cost of mortgage borrowing and encourages purchases of housing assets.

To measure the quantitative importance of declining interest rates in explaining rising house prices and mortgage debt, I extend the model into an open-economy setting where interest rates are determined globally, and then lower the real interest rate in the benchmark scenario from 3.2 percent to 2 percent, a decrease broadly consistent with the actual change between 1994 and 2005. This can be justified by a global saving glut during this period. The results are reported in Table 6. Declining interest rates have a measurable impact on both house prices and mortgage debt. In response to this change in the interest rate, house prices and mortgage debt increase, respectively, by 21.3 percent and 24.2 percent, which comprise 47.3 percent and 43.2 percent of their observed changes. The model-generated changes are sizable given that the only cause is a decline in the interest rate.

Table 6: Effects of Declining Interest Rates on Macroeconomic Aggregates

	Benchmark	2005 Interest Rate	Change	Actual Change
Interest Rate	0.159	0.100	−0.59%	
House Price	0.978	1.186	+21.3%	+45.0%
Mortgage Debt	0.128	0.159	+24.2%	+56.0%

The significant impact of declining interest rates on house prices lies in households' portfolio reallocation from financial assets to housing assets. Figure 13 shows the effects of declining interest rates on the life-cycle profile of households' investment in financial assets. Households decrease their holdings of financial assets as the interest rate falls. This differs from the previous scenario, where mortgage innovation pushes up the interest rate and induces households to save more in financial assets. The decline in the interest rate, however, makes housing assets more

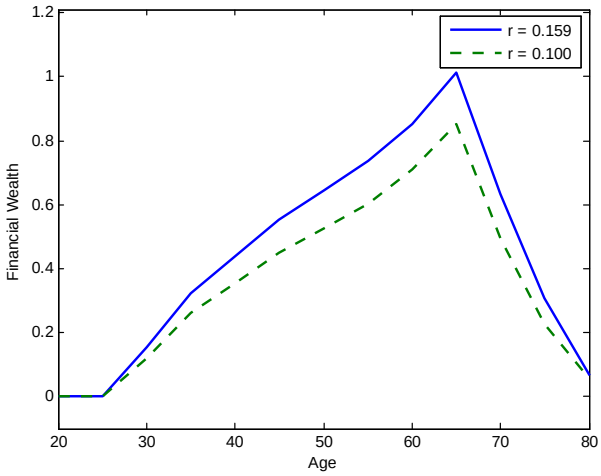

Figure 13: Effects of Declining Interest Rates on Financial Wealth

attractive. Figure 14 shows the effects of declining interest rates on the life-cycle profile of housing consumption. Households at the prime saving ages of 35-65 significantly increase their holdings of housing assets. Falling interest rates make housing assets a better saving alternative for retirement. As the figure shows, after retirement, households liquidate their home equity by quickly downsizing their housing stock to finance their non-housing consumption. Such a rising entry of middle-aged households into the housing market increases aggregate housing demand enormously and pushes up house prices.

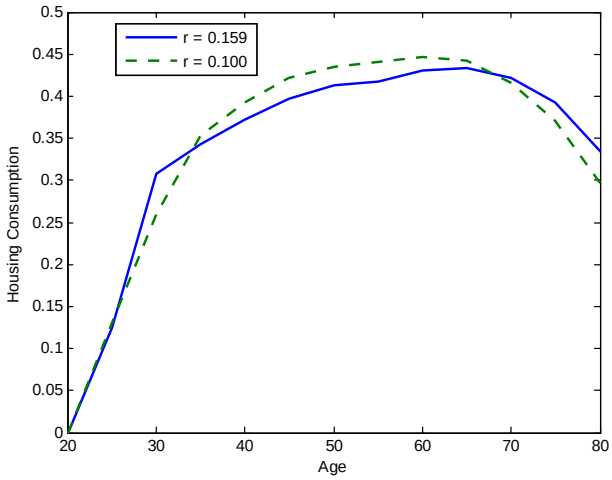

Figure 14: Effects of Declining Interest Rates on Housing Consumption

Note that housing demand is further encouraged by declining mortgage borrowing cost. Figure 15 shows the effects of declining interest rates on the life-cycle profile of mortgage balance. Apparently, the increase in mortgage debt mostly comes from households at age 35-

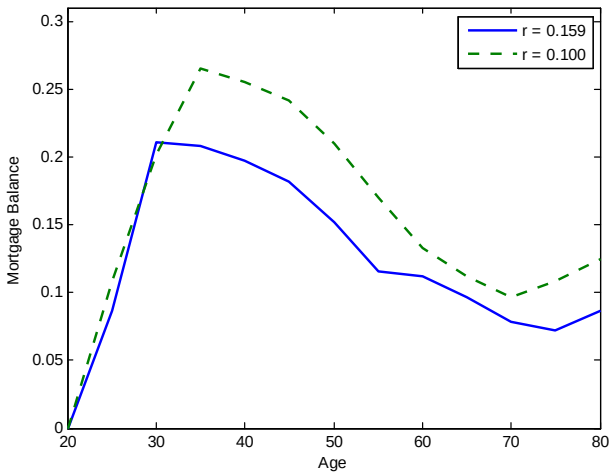

Figure 15: Effects of Declining Interest Rates on Mortgage Balance

65. It is interesting to note that lower interest rates do not significantly increase the housing consumption and mortgage borrowing of young households, as one might have expected. This is because, for young households, the benefit of a decreasing cost of mortgage borrowing is neutralized by rising house prices.

4.4 Combined Effects

In the previous three experiments, I vary only one factor each time. In this section, I measure the combined contributions of population aging, mortgage innovation and historically low interest rates to the observed changes in house prices and mortgage debt. This joint analysis is important because these factors might interact in different directions and mutually strengthen or weaken each other's effect. For instance, population aging increases the share of middle-aged cohorts, which would thus amplify the contribution of declining interest rates to rising house prices.

Table 7 reports how these three factors jointly affect house prices and mortgage debt. The table shows that, together, they generate a 27.7 percent increase in house prices and a 53.1 percent increase in mortgage debt, which account for 61.6 percent and 94.8 percent of their observed changes, respectively.

Table 7: Combined Effects on Macroeconomic Aggregates

	Benchmark	2005 Economy	Change	Actual Change
Interest Rate	0.159	0.100	−0.59%	
House Price	0.978	1.249	+27.7%	+45.0%
Mortgage Debt	0.128	0.196	+53.1%	+56.0%

To summarize the individual and combined contributions of these three factors, I also per-

form a decomposition analysis, with results reported in Table 8. The contribution of each factor to changes in house prices or mortgage debt is measured by dividing the corresponding model-generated change to its data counterpart, which provides the percentage change that can be explained by this factor. First, a declining interest rate is found to play a major role in boosting house prices and mortgage debt. It explains 47.3 percent and 43.2 percent of their observed changes, respectively. Meanwhile, by relaxing borrowing constraints, mortgage innovation accounts for 33.6 percent of the rise in mortgage debt but barely affects house prices. Furthermore, population aging explains only 4.31 percent and 2.79 percent of the observed changes in house prices and mortgage debt during this period. Finally, the interaction effects among these three factors are sizable, representing 11.7 percent and 15.3 percent of the observed changes in house prices and mortgage debt, respectively.

Table 8: Summary of Decomposition Analysis

	House Price		Mortgage Debt	
	Change	% Data	Change	% Data
Data	+45.0%		+56.0%	
Model	+27.7%	61.6	+53.1%	94.8
Population Aging	+1.94%	4.31	+1.56%	2.79
Mortgage Innovation	−0.82%	−1.82	+18.8%	33.6
Lower Interest Rate	+21.3%	47.3	+24.2%	43.2
Interaction Effect	+5.28%	11.7	+8.54%	15.3

5 Conclusion

This paper constructs an OLG general equilibrium model to examine the contributions of population aging, mortgage innovation and historically low interest rates to the sharp rise in U.S. house prices and mortgage debt between 1994 and 2005. Population aging contributes to rising house prices and mortgage debt, but it accounts for only a small portion of their observed changes. Meanwhile, mortgage innovation significantly increases the mortgage borrowing of various age cohorts, but it has a trivial effect on house prices, because interest rates rise due to rising demand for mortgage loans. This increases households' savings in financial assets and leaves their housing assets nearly unchanged. The observed run-up in house prices can, however, be justified in an open-economy setting where interest rates fall due to a global saving glut. Declining interest rates force households at prime saving ages to reallocate their wealth from financial assets to housing assets, which dramatically drives up house prices.

There are several directions for future research. A valuable extension would be to study the transitional dynamics between steady states. Taking into account the transitional path is important since it affects a forward-looking household's portfolio allocation between financial

assets and housing assets. Meanwhile, though demographics play a small role in driving this housing boom, it would be interesting to evaluate the potential impact of the ongoing retirement of baby boomers on future house prices. Furthermore, it would be useful to examine housing market dynamics by considering the effects of uncertainty shocks. Increasing uncertainty slows the housing investment of risk-averse agents. This might explain why housing markets are usually slow to recover during recessions which are characterized by high levels of uncertainty.

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Appendix

This appendix describes how to compute a steady-state equilibrium of the model. Recall that each individual state vector $s = (a, h, m, n, \epsilon, j) \in S = A \times H \times M \times N \times E \times J$ consists of six state variables: financial asset $a \in A$, housing asset $h \in H$, mortgage type $m \in M$, mortgage payment counter $n \in N$, idiosyncratic earning shock $\epsilon \in E$, and age $j \in J$. Here, the state spaces of financial asset (A) and housing asset (H) are continuous, whereas those of the rest are discrete. To find the equilibrium, I first discretize the state space $A \times H$ into $A_d \times H_d = \{a_1, ..., a_{n_a}\} \times \{h_1, ..., h_{n_h}\}$, where $n_a, n_h \in \mathbb{N}$. The upper bounds a_{n_a} and h_{n_h} are chosen to be large enough so that they do not constitute a constraint on the optimization problem. By this discretization process, I construct a six-dimensional grid $S_d = A_d \times H_d \times M \times N \times E \times J$ with $n_a \times n_h \times \bar{m} \times \bar{n} \times \ell \times \left(\bar{J} - j_0 + 1\right)$ grid points in total, where $\bar{m}$ and $\bar{n}$ denote the number of elements in the finite sets M and N, respectively. This grid is used to store the value functions, policy functions and distribution function as finite-dimensional arrays.

The steady-state equilibrium is then solved as follows:

1. Guess a house price P and an interest rate r.
2. Guess a lump-sum transfer θ from accidental bequests.
3. Solve the household's optimization problem to obtain the value function and optimal policy functions as follows:
   (a) Since the value function after the last period equals zero, i.e., $V\left(\cdot, \cdot, \cdot, \cdot, \cdot, \bar{J} + 1\right) = 0$, solve the value function for the last period of life for each grid point. This also yields policy functions for the last period.
   (b) By backward induction, repeat the above step until the first period in life.

4. Compute the associated stationary distribution Φ of households by forward induction with previously obtained policy functions, starting from the known distribution at the beginning of the life cycle.

5. Check whether the implied lump-sum transfer is consistent with the initial guess. If not, go back to step 2 and update the guess.

6. Given the stationary distribution and prices, compute aggregate demand and supply in both the housing and financial markets. If both markets clear, an equilibrium is found. If not, go back to step 1 and update prices P and r.