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Abstract

This paper studies the welfare effects of different credit arrangements and how these effects depend on the trading mechanism and inflation. In a competitive market, a deviation from the Friedman rule is always sub-optimal. Moreover, credit arrangements can be welfare-reducing, because increased consumption by credit users will drive up the price level so that money users have to reduce consumption when facing a binding liquidity restraint. By adopting an optimal trading mechanism, however, these welfare implications can be overturned. Price discrimination under the optimal mechanism helps internalize the price effects. First, small deviations from the Friedman rule are no longer welfare-reducing. Second, increasing the access to credit becomes welfare-improving. Finally, the model is extended to study the welfare effects of credit systems when credit serves as means of payment, and endogenous credit constraint.

JEL classification: E40, E50
Bank classification: Credit and credit aggregates; Payment, clearing, and settlement systems

Résumé

Dans cette étude, les auteurs examinent les effets de différents modes de crédit sur le bien-être et la sensibilité de ces effets au mécanisme de négociation et à l'inflation. Dans un marché soumis à la concurrence, un écart par rapport à la règle de Friedman est toujours sous-optimal. De plus, le crédit peut réduire le bien-être étant donné qu'en accroissant la consommation de ceux qui y ont recours, il fait monter le niveau des prix, obligeant ainsi les utilisateurs de monnaie à réduire leur consommation s'ils sont confrontés à une contrainte de liquidité. Il est toutefois possible de neutraliser ces effets sur le bien-être en introduisant un mécanisme de négociation optimal. Dans ce cas, la discrimination par les prix aide à internaliser les effets de prix. Premièrement, un faible écart par rapport à la règle de Friedman n'a plus d'incidence réductrice sur le bien-être. Deuxièmement, un accès accru au crédit se traduit par une amélioration du bien-être. Pour terminer, les auteurs étoffent le modèle afin d'étudier les effets sur le bien-être de l'utilisation de systèmes de crédit, lorsque le crédit est un moyen de paiement, et les effets de contraintes de crédit endogènes.

Classification JEL : E40, E50
Classification de la Banque : Crédit et agrégats du crédit; Systèmes de paiement, de compensation et de règlement

1 Introduction

Recent policy debates on regulating the retail payments system are motivated by concerns about the efficiency and welfare implications of different payment instruments and their pricing schemes. Conducting policy analysis on these issues from first principles requires a general equilibrium model in which the fundamental roles of different payment arrangements are explicitly captured. Thanks to recent developments in monetary theory[1], it is now widely recognized that, in an environment with imperfect information and limited commitment, money is essential as a means of payment.[2] Moreover, the allocation in a monetary economy is typically inefficient when some agents are money constrained (for example, due to sub-optimal monetary policy or liquidity shocks). As a result, some forms of credit arrangement may help to improve efficiency by relaxing agents' liquidity constraints. What remain less well understood are the welfare effects of different credit arrangements and their interaction with monetary policy and the trading mechanism. This paper is an attempt to use modern, micro-founded monetary theory to address these issues.

Specifically, this paper investigates the following questions: Does availability of credit always improve social welfare in a competitive environment? If not, what is the source of inefficiency? What sorts of trading/pricing mechanisms are needed to mitigate this inefficiency? Do technologies of production, trading, and enforcement matter for these questions?

Let us briefly describe the model and give the basic intuition behind our findings. Owing to information frictions in the goods market, buyers need to acquire non interest-bearing money in order to trade for consumption. Money-users therefore bear a cost of holding liquidity which is particularly onerous when inflation is high. Credit-

[1] Williamson and Wright (2010) describe the most recent development in this literature.

[2] See, for example, Kocherlakota (1998).

users, however, can economize on the use of cash and (at least partially) avoid this inflation tax by acquiring credit from a bank which is in the form of outside money in the benchmark case. Therefore, inflation may generate a redistributive effect across different types, lowering money-users' consumption while increasing credit-users' consumption. Overall, however, deviation from the Friedman rule is still sub-optimal and inflation is welfare-reducing in a competitive environment.

Since using money is costly due to inflation, one may expect that increasing the use of credit can always enhance welfare because a buyer, by gaining access to credit, can now avoid the inflation tax and enjoy a higher level of consumption. We call this the "composition effect" because as the economy is composed of more credit-users, ceteris paribus, welfare tends to go up. But this is only a partial equilibrium argument. There is an additional general equilibrium "price effect": an increase in consumption of new credit-users will drive up the market price and reduce cash buyers' consumption. At first glance, this pecuniary externality should not lead to any welfare loss according to standard arguments. One need to notice, however, that the first welfare theorem can fail when there are distortions in the economy, and pecuniary externalities can have welfare consequences (Greenwald and Stiglitz,1986). In the current environment, the presence of binding liquidity constraints implies that more people using credit can tighten money-users' liquidity constraints and hence lower the aggregate welfare whenever the price effect dominates the composition effect. As a result, with competitive pricing, the introduction of credit services can be welfare reducing. These are the main findings of the benchmark model discussed in Section 3.

Since the negative effects of inflation and credit hinge on the assumption of competitive pricing, it is natural to ask whether such efficiencies can be mitigated by the optimal design of the trading mechanism and pricing protocol. Instead of focusing on one specific trading mechanism, we employ the mechanism design approach to solve for the set of optimal allocations subject to technological and incentive feasibility constraints. The welfare implications of inflation and credit arrangements are significantly different under the optimal trading mechanism. First, deviation from the Friedman rule is not necessarily sub-optimal. By appropriately splitting the trade surpluses of different parties, the first best allocation can be supported for low inflation. Second, under the optimal trading mechanism, the price effect can now be internalized and thus the provision of credit and payment services by banks become welfare-improving. Note that the implementation of these allocations relies on the availability of certain enforcement and information technologies to the mechanism. For example, it is important to set the terms of trade according to the buyers' type, something resembling price discrimination across money-users and credit-users. These results will be derived in Section 4.

We then study two extensions of our benchmark model. We first allow banks to issue credit in the form of inside money which is accepted as a means of payment in goods transactions. In this environment, banks provide an additional payment service (i.e. imagine a private payment system without the use of outside money). We show that the price effect dominates the composition effect more strongly in this case than the benchmark model resulting in the introduction of credit as a means of payment can be further welfare-reducing. Finally, we study an extension of the benchmark model in which loan repayments are only imperfectly enforceable: a defaulter is punished by being excluded from access to banking in the future. In this setting, we uncover a new channel operating through the implied endogenous credit constraint. For example, an increase in access to credit will reduce the value of default through the price effect, and hence relax credit-users' credit constraints, potentially improving the social welfare.

There is a recent literature that develops monetary models to understand the micro-foundation of money and credit. Our model is closely related to the money and banking model developed by Berentsen, Camera and Waller (2007) which in turns builds on Lagos and Wright (2005). The basic question of the literature is to examine the welfare effects of credit arrangement and inflation in an environment with explicit information and commitment frictions. Recent work building on these ideas includes Calvacanti and Wallace (1999), Chiu and Meh (2011), Monnet and Roberds (2008), Sanches and Williamson (2010), Sanches (2011), Gu, Mattesini and Wright (2012), Gu, Mattesini, Monnet and Wright (2012)

The rest of the paper proceeds as follows. Section 2 describes the basic model. Section 3 derives the welfare effects of monetary policy and credit arrangements in a competitive environment. Section 4 revisits the questions under the optimal trading arrangements applying the mechanism design approach. Section 5 explores an extension with inside-money loans, and Section 6 an extension with endogenous credit constraint. Section 7 concludes. Formal proofs of lemmas and propositions can be found in the Appendix.

2 Benchmark Model

The basic economic environment is similar to Berentsen et al. (2007) based on the framework of Rocheteau and Wright (2005). Time is discrete and runs forever. Each period is divided into two subperiods, day and night. Agents meet at a Walrasian market in both subperiods. There is a continuum of infinitely-lived agents who differ along three dimensions. First, they permanently belong to one of two groups in the day market, called *buyers* and *sellers*. We normalize the measure of each group to 1. Second, all buyers experience preference shocks during the day: with probability π, a buyer wants to consume, while with complementary probability, the buyer does not want to consume. These shocks are i.i.d. across agents and time. Third, only a fraction

α of buyers have access to a banking sector.[3] All sellers have access to banking.

In the night market all agents produce and consume, but in the day market a buyer can only consume and a seller can only produce. Goods are perishable. Moreover, all goods trades are anonymous during the day, and all histories of goods trading are private information. As a result, sellers require immediate payment and a medium of exchange is essential for trade. There exists fiat money that is perfectly divisible. The supply M grows at a constant gross rate γ. New money is injected ($\gamma > 1$) or withdrawn ($\gamma < 1$) via lump sump transfers $\tau M = (\gamma - 1) M$ to all agents at the beginning of night. We restrict attention to policies where $\gamma \geq \beta$, where $\beta \in (0, 1)$ is the discount factor, since it is easy to check that there is no equilibrium otherwise. To examine equilibrium at the Friedman rule, we take the limit of equilibria as $\gamma \to \beta$.

For a seller who produces q units of output during the day, consumes x units of output and produces y units of output at night, the instantaneous utility is

$$-c(q) + v(x) - y,$$

We assume that $v'(x) > 0$, $v''(x) < 0$ for all x, and there exists $x^* > 0$ such that $v'(x^*) = 1$. The cost of production satisfies $c(0) = 0$, $c'(q) > 0$, $c''(q) \geq 0$. Similarly, the instantaneous utility of a buyer is

$$\varepsilon u(q) + v(x) - y,$$

where q is the quantity consumed during the day and $\varepsilon \in \{0, 1\}$ is the i.i.d. preference shock, with $\Pr(\varepsilon = 1) = \pi$ and $\Pr(\varepsilon = 0) = 1 - \pi$. The assumption of $u(q)$ includes $u(0) = 0$, $u'(0) = +\infty$, $u'(q) > 0$, and $u''(q) < 0$.

In the banking sector, there are competitive banks that can make credit arrangement, as in Berentsen et al. (2007). Banks process a record keeping technology that

[3] We assume the type is permanent. Having i.i.d. type, however, will not affect the main results of the paper.

can keep track of financial histories, but not trading histories in the goods market. Since record keeping is only available for financial transactions, trade credit between buyers and sellers is not feasible. Instead, banks can make nominal loans and take deposits. These financial services are available only at the beginning of the day after the preference shock is realized and before goods trading. Finally, without loss of generality, assume that all financial contracts are one-period contracts and thus loans and deposits are not rolled over across periods. Banks can commit to repay their depositors. Banks can also perfectly enforce loan repayment by the borrowers in the benchmark model. We consider limited enforcement of loan repayment as an extension.

In Berentsen et al. (2007), banks are subject to a cash constraint when making loans, in the sense that all loans have to be backed by money deposit.[4] An alternative credit arrangement is where banks are not subject to the cash-in-advance constraint so that loans (or credit) can be used as a payment instrument. Banks provide both credit and payment services. To understand the role of credit and credit arrangement, we will examine the second type of credit arrangement in section 5.

The timing in our model is as follows (see Figure 1). At the beginning of each period, buyers observe their preference shocks and the banking sector opens where agents with access to it can borrow loans or make deposits. Then, the banking sector closes and agents trade goods in the day market. Agents receive lump-sum transfers τM, consume and produce as well as settle financial claims at night.

2.1 Night Market Problem

Let b denote a buyer who has access to credit, n denote a buyer who does not have access to credit, and s denote a seller. Let $W^j(m, \ell, d)$ be the value function of a type $j \in \{b, n, s\}$ agent who enters the night market holding m units of money, ℓ loans and d

[4]Berentsen et al. (2007) interpret this restriction as 100% reserve requirement. In this case, fiat money serves as the only means of payment in the day market.

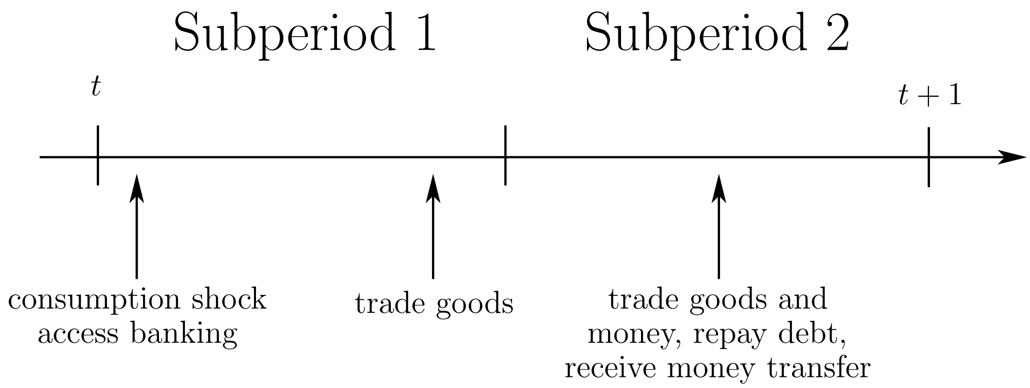

Figure 1: Timeline of Events

deposits. Denote an agent j's value function of carrying m dollars into the day market by $V^j(m)$. We normalize the price of the consumption good in the day market to 1 and denote the value of a dollar in units of consumption by ϕ. The value of the agent j in the night market is

$$W^j(m,\ell,d) = \max_{x,y,m_+}\left[v(x) - y + \beta V^j(m_+)\right] \tag{1}$$

$$\text{st. } x + \phi m_+ = y + \phi(m + \tau M) + \phi\left(1+r^d\right)d - \phi(1+r)\ell, \tag{2}$$

where r is the nominal loan rate and r^d is the nominal deposit rate. Type n agents cannot use banks and thus have $\ell = d = 0$. Substituting (2) into (1), the problem simplifies to

$$\begin{aligned} W^j(m,\ell,d) &= \phi\left[m + \tau M - (1+r)\ell + \left(1+r^d\right)d\right] \\ &\quad + \max_{x,m_+}\left[v(x) - x - \phi m_+ + \beta V^j(m_+)\right]. \end{aligned}$$

The first order conditions are $v'(x) = 1$ and

$$\beta\frac{dV^j(m_+)}{dm_+} \leq \phi, \text{ “}=\text{” if } m_+ > 0. \tag{3}$$

It follows that the optimal choice of (x, m_+) is independent of (m,ℓ,d) for all agents. This is a natural result from assuming quasi-linear utility in the day market, as first formalized by Lagos and Wright (2005). The envelope conditions imply that

$$\begin{aligned}
\frac{\partial W^j(m,\ell,d)}{\partial m} &= \phi, \\
\frac{\partial W^j(m,\ell,d)}{\partial \ell} &= -\phi(1+r), \\
\frac{\partial W^j(m,\ell,d)}{\partial d} &= \phi(1+r^d).
\end{aligned}$$

The value function $W^j(m,\ell,d)$ is linear in (m,ℓ,d) and can be rewritten as

$$W^j(m,\ell,d) = W^j(0,0,0) + \phi m - \phi(1+r)\ell + \phi\left(1+r^d\right)d.$$

2.2 Day Market Problem

Moving back to the day market, a seller who holds m^s units of money at the beginning of the day market has

$$V^s(m^s) = \max_{q^s,\ell,d} -c(q^s) + W^s(m^s+\ell-d+pq^s,\ell,d) \text{ st. } d \le m^s,$$

where p is the competitive price of goods during the day. Let λ_d^s be the Lagrange multiplier, the first order conditions are

$$\begin{aligned}
c'(q^s) &= \phi p, \qquad (4)\\
\phi r &= 0, \\
\phi r^d &= \lambda_d^s.
\end{aligned}$$

It is immediate that a seller will not borrow unless $r=0$, and will deposit all the money holding whenever $r^d>0$. The envelope condition thus gives $dV^s(m)/dm = \phi(1+r^d)$. From (3), a seller's demand for money satisfies

$$r^d \le \frac{\gamma-\beta}{\beta}, \text{ “}=\text{” if } m^s > 0. \qquad (5)$$

That is, sellers may hold 0 or any positive amount of money depending on the deposit rate. If the deposit rate is strictly less than $\gamma/\beta - 1$, then seller strictly prefers holding 0 unit of money.

All buyers experience i.i.d. preference shocks at the beginning of the day. Consider first the fraction α buyers who have access to credit. One can show that those who want to consume will never deposit money in the bank and those who do not want to consume will never take out loans. Hence, the value of a buyer holding m^b units of money in the day market is

$$\begin{aligned} V^b\left(m^b\right) &= \max_{q^b,\ell,d} \pi\left[u\left(q^b\right)+W^b\left(m^b+\ell-pq^b,\ell,0\right)\right] && (6)\\ &\quad +(1-\pi)\,W^b\left(m^b-d,0,d\right), \\ \text{st.} &\quad pq^b \leq m^b+\ell, && (7)\\ &\quad d \leq m^b. && (8)\end{aligned}$$

Let λ_q and λ_d denote the Lagrange multipliers for (7) and (8), respectively. As $W^b(m,\ell,d)$ is linear in (m,ℓ,d), we can derive the first order conditions as

$$\pi u'\left(q^b\right) = \left(\pi\phi+\lambda_q\right)p, \tag{9}$$

$$\pi\phi r = \lambda_q, \tag{10}$$

$$(1-\pi)\,\phi r^d = \lambda_d. \tag{11}$$

If r is positive, (10) implies that the liquidity constraint (7) must be binding in a monetary equilibrium (i.e. $\phi > 0$). Similarly, if r^d is positive, the deposit constraint (8) must be binding. Combining (4), (9), and (10) yields

$$\frac{u'\left(q^b\right)}{c'\left(q^s\right)} = 1+\frac{\lambda_q}{\pi\phi} = 1+r. \tag{12}$$

The envelope condition of $V^b(m)$ gives

$$\frac{dV^b\left(m^b\right)}{dm^b} = \phi\left[\pi\frac{u'\left(q^b\right)}{c'\left(q^s\right)}+(1-\pi)\left(1+r^d\right)\right]. \tag{13}$$

Plugging (13) into (3), a type b buyer's demand for money satisfies

$$\pi\left[\frac{u'\left(q^b\right)}{c'\left(q^s\right)}-1\right] \leq \frac{\gamma-\beta}{\beta}-(1-\pi)\,r^d,\ \text{“}=\text{''}\ \text{if } m_+^b > 0. \tag{14}$$

For those buyers who cannot use banks, the value of holding m^n at the beginning of the day market is

$$V^n(m^n) = \max_{q^n} \pi \left[u(q^n) + W^n(m^n - pq^n)\right] + (1-\pi) W^n(m^n) \text{ st. } pq^n \leq m^n. \quad (15)$$

One can show that the constraint $pq^n = m^n$ must be binding. The first order condition implies a type n buyer's money demand satisfies

$$\pi \left[\frac{u'(q^n)}{c'(q^s)} - 1\right] = \frac{\gamma - \beta}{\beta}. \quad (16)$$

Comparing (14) and (16), we can see that $q^n < q^b$ for any $\gamma > \beta$ and $r^d > 0$. As long as the deposit rate is positive, type b buyers enjoy a higher q^b because they can take out loans to expand their consumption. Finally, goods market clearing condition is

$$q^s = \pi \left[\alpha q^b + (1-\alpha) q^n\right]. \quad (17)$$

2.3 Banking Problem

In the benchmark economy which is labeled as economy 1, the size of the loans are constrained by the amount of deposits that a bank has, so banks can only lend out *outside-money loans*. In the day market, only (outside) money is accepted as a means of payment. Therefore, in this economy, banks take deposits and make loans to channel money balances across agents, but they do not provide any payment services.[5] Competitive banks take as given the market rates r^d and r and choose the amount of money deposits D and money loans L to

$$\max_{L,D} \left(rL - r^d D\right) \quad (18)$$

subject to $L \leq D$.

[5]Alternatively, this can be viewed as a form of "narrow banking" with 100% reserve requirement: banks can create transferrable IOU's but these IOU's have to be fully backed by outside money.

It is straightforward to show that, in equilibrium,

$$\begin{cases} r = r^d, \\ D \geq L, \\ r^d(D - L) = 0. \end{cases} \tag{19}$$

That is, the deposit rate and the loan rate are equal. The interest rates are positive, unless there is an excess supply of deposit.

To complete the description of the banking sector, the loan market clearing condition is

$$D - L = m^s + \alpha m^b - \pi\alpha p q^b. \tag{20}$$

The amount of deposits net of loans $D - L$ equals to the supply of deposits $m^s + \alpha m^b$ net of the demand for loans $\pi\alpha p q^b$. Since banks can only make outside-money loans, they are subject to a cash-in-advance constraint, implying that $D - L \geq 0$.

2.4 Equilibrium

Having solved for the optimal decisions problems faced by buyers, sellers and banks, we combine these decisions to define a steady state equilibrium. For economy 1, using (5), (12), (14), (16), (19) and (20), we can characterize a steady state equilibrium as a list of $\left(r_1, q_1^b, q_1^n, q_1^s\right)$ satisfying (17) and

$$r_1 = i, \tag{21}$$

$$\frac{u'\left(q_1^n\right)}{c'\left(q_1^s\right)} = 1 + \frac{i}{\pi}, \tag{22}$$

$$\frac{u'\left(q_1^b\right)}{c'\left(q_1^s\right)} = 1 + i. \tag{23}$$

Here $i = \gamma/\beta - 1$ is the nominal interest rate for a loan between two consecutive night markets.

In equilibrium, sellers and type b buyers are just indifferent between bringing money to the banking sector or not (as long as some agents bring money to deposit). There is an indeterminacy with respect to their money holdings. However, their individual money holdings are payoff-equivalent and thus irrelevant for real allocations and welfare. Note that, q_1^b and q_1^n are not independent unless $c''\left(q_1^s\right) = 0$.

In the special case when $\alpha = 0$, no one has access to credit in the economy. This is equivalent to a pure monetary economy which is labeled as economy 0. The steady state equilibrium is a list of (q_0^n, q_0^s) that satisfies $q_0^s = \pi q_0^n$ and (22).

The aggregate welfare in either economy is defined as

$$\mathcal{W} = 2v(x) - 2x + \pi\alpha u(q^b) + \pi(1-\alpha)u(q^n) - c(q^s), \tag{24}$$

where x is determined independently by $v'(x) = 1$. Given this definition, the first best allocation can be found by maximizing (24) subject to (17). The solution of (q^b, q^n, q^s) satisfies

$$u'(q^b) = u'(q^n) = c'(q^s) \text{ and } q^s = \pi q^b = \pi q^n.$$

Denote $q^* = q^b = q^n$.

Remark 1 *All the results derived in this paper are based on the centralized trading of goods. One may question whether the results still hold if the trading of goods is decentralized and bilateral. We argue that they will not change as long as some factors of production are traded in a multilateral fashion.*

To see that, suppose instead that meetings in the day market are bilateral, and the terms of trade are determined by take-it-or-leave-it offers from the buyers. Each buyer makes an offer (D, q) to buy q goods by paying D dollars. Suppose a seller produces output q by employing labor hours h according to a linear production function $q = F(h) = h$.[6] *Labor hours are traded in a centralized market at a (real) wage rate w. Each seller chooses how much labor to supply H to the market and how much labor to be hired h.*

[6] Assuming a concave production function $F(h)$ will not eliminate the price effect. Instead, it will introduce an additional effect due to diminishing marginal productivity. A price-making buyer will internalize this effect, while a price-taking buyer will not.

The disutility of supplying labor is $\psi(H)$ with $\psi' > 0, \psi'' > 0$. The payoff of a seller with an offer (D, q) is thus

$$V^s(q, D) = \phi D - wq + \max_H [wH - \psi(H)].$$

Obviously, $w = \psi'(H)$ in a competitive labor market. Moreover, any offer from the buyer to the seller must satisfy $V^s(q, D) = V^s(0, 0)$. It is then straightforward to show that, the same set of equilibrium conditions can be derived, by appropriately relabeling H and $\psi(H)$ as q^s and $c(q)$.

3 Welfare in competitive equilibrium

In this section, we examine different factors that affect the economy's aggregate welfare. In particular, the factors include money growth rate and access to credit.

3.1 Inflation and welfare

We begin with the effect of inflation on welfare. Inflation is a policy parameter that is controlled by the monetary authority. In the monetary theory literature, monetary policy is usually neutral, but not super-neutral. The results summarized by Lemma 1 also share this feature.[7]

Lemma 1 *Effects of inflation: $dq_1^n/di < 0$, $dq_1^s/di < 0$, $dq_1^b/di \gtrless 0$, $d\mathcal{W}/di < 0$.*

Lemma 1 summarizes the effects of inflation on consumption and welfare. Note that inflation has heterogenous effects on different types of buyers, working through two different channels. First, higher inflation raises the liquidity costs for both types of buyers, inducing them to reduce consumption and thus aggregate output q_1^s drops. The

[7] Since the (gross) inflation rate is $p_{t+1}/p_t = \gamma = (1+i)\beta$ in the steady state, we can study the effects of inflation by examining i. Also recall that subscripts "1" denotes economy 1 in which banks lends out outside-money loans.

second channel is that when q_1^s falls, the marginal cost of production goes down too, which partially offsets the inflation cost. The net effect is the sum of the two. Lemma 1 shows that inflation always reduces money-user's consumption, while it may increase or decrease the credit-user's consumption.[8] In general, inflation has a smaller effect on credit-users than on money-users because the access to credit arrangement allows agents to partially avoid the inflation tax. Knowing that inflation reduces (q_1^n, q_1^s) and possibly q_1^b, it is not surprising that inflation reduces welfare. One can also show that the Friedman rule is the optimal monetary policy.

3.2 Access to credit and welfare

Given the fact that inflation is welfare-reducing, and that inflation typically is less costly to credit-users than to money-users, it is natural to expect that increasing the access to credit can be welfare-improving. The following two Lemmas establish that this is not true in general.

Lemma 2 *Effects of access to credit on allocation:* $dq_1^n/d\alpha \leq 0$, $dq_1^b/d\alpha \leq 0$ *and* $dq_1^s/d\alpha > 0$, *with strict inequalities iff* $c'' = 0$.

Lemma 2 shows that as α rises, q_1^s increases, but both q_1^n and q_1^b decrease. This is because a rise in α has the following two effects on the economy. The first effect is that a higher α increases the composition of credit-users. Since $q_1^b > q_1^n$, q_1^s tends to increase. We can label it as the *composition effect*. The second effect is a general equilibrium *price effect*: a higher q_1^s drives up $c'(q_1^s)$ whenever $c'' > 0$. It follows that the price level in the day market becomes higher and hence both types of buyers tend to consume less, which leads to lower q_1^n and q_1^b. Clearly, if $c'' = 0$, the price effect is

[8] The proof of Lemma 1 in the appendix shows that one can impose additional assumptions to have $dq_1^b/di < 0$.

absent and (q_1^n, q_1^b) are not affected by α, with q_1^s going up through the composition effect.

Away from the Friedman's rule ($i > 0$), we have the following lemma for any $\alpha \in (0, 1)$:

Lemma 3 *Effects of access to credit on welfare: $d\mathcal{W}/d\alpha < 0$ if $c'' > 0$, $c' > 0$, and π is small.*

One can show that $d\mathcal{W}/d\alpha > 0$ when $c''(q) = 0$. It implies that the composition effect alone tends to improve the welfare in the economy. As trades involving credit-users generate larger trading surplus, a higher fraction of credit-users raises social welfare. In the case where $c''(q) > 0$, the price effect associated with a higher α makes the economy worse because of lower q^n and q^b. In the presence of two opposing effects of α on welfare, Proposition 2 provides a sufficient condition that makes the price effect as the dominant effect on welfare.

It seems counterintuitive that an increase in α can reduce the social welfare. After all, the use of credit is completely voluntary and can only expand a credit-user's feasibility set. Moreover, while the rest of the economy are impacted by an increase in α through the price effect, it is only a pecuniary externality. Standard argument suggests that pecuniary externalities by themselves are not a source of inefficiency in a competitive equilibrium. One needs to notice, however, that pecuniary externalities can have welfare consequences when there are distortions in the economy.[9] In the current setting, the first welfare theorem fails in the presence of binding liquidity constraints. A rise in α leads to a higher market price which then tightens other agents' liquidity constraints, and potentially create inefficiencies. The intuition behind this result can be illustrated by the graphs in Figure 2.

[9] In general, pecuniary externality can lead to inefficiencies in economies with incomplete markets (Greenwald and Stiglitz,1986).

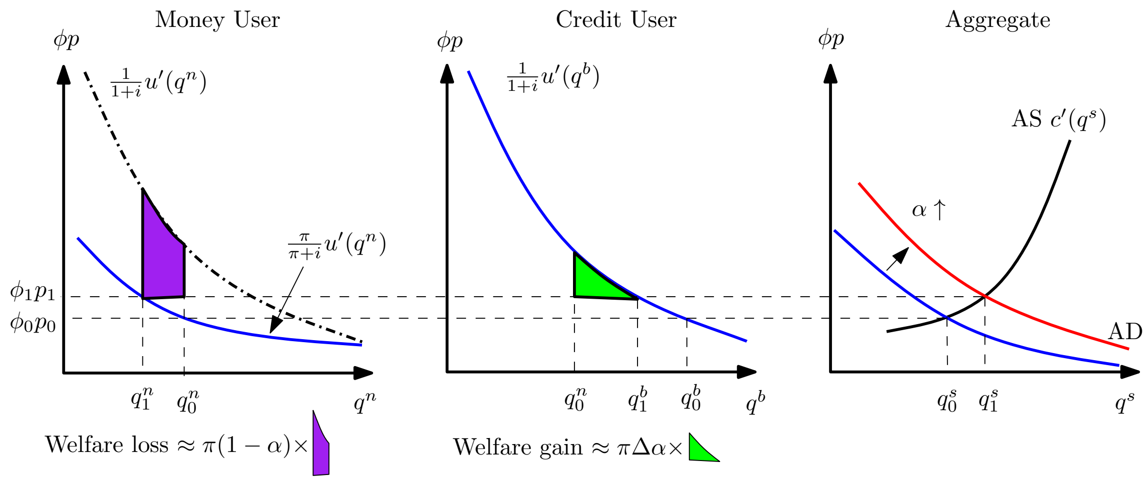

Figure 2: Welfare Effects of Increasing α

The left diagram plots the individual demand curve (blue curve) for a money-user who is facing a binding liquidity constraint, while the middle diagram shows the demand for a credit-user. These curves are derived from the first order conditions (22) and (23). Notice that the left diagram puts the money-user's demand and credit-user's demand (dash-dot curve) together. The gap between the two curves at q_1^n captures the wedge between the marginal utilities of a money-user and a credit-user due to the former's inability to access credit arrangement. The bigger the wedge, the more inefficient is the allocation that the market achieves. The inefficiency wedge is determined by two parameters: i and π. The higher the inflation rate and the higher the trading friction, the more costly it is for a money-user to carry liquidity to finance the trade.

The right diagram plots the aggregate demand and the aggregate supply which corresponds to (4). When there is an increase in α, the aggregate demand curve shifts out as the red curve in the right diagram, driving up the market price. In the middle diagram, because of the increase in α, some money-users become credit-users and consume q^b rather than q^n, there is welfare gain from the composition effect reflected by the green area. Meanwhile, the higher price level decreases consumption from those money-users since it tightens their liquidity constraints. The welfare loss from the price effect is depicted by the purple area. Whenever the purple area is larger than the green area, an increase in α leads to lower aggregate welfare.[10]

Under the assumptions made above, we can summarize the results in this section:

Proposition 1 *In a competitive equilibrium, inflation lowers aggregate consumption and welfare, and has bigger impact on money-users than on credit-users. An increase in the access to credit reduces consumption of both money-users and credit-users, and lowers welfare when π is small.*

4 Welfare under optimal trading mechanism

The previous section considers the benchmark case in which day markets are competitive, and illustrates how inflation and credit arrangements can lead to inefficient outcome, partly due to a general equilibrium price effect. It is then natural to ask whether such inefficiencies can be mitigated by adopting an optimal pricing arrangement. To address this question, we follow the mechanism design approach developed by Hu, Kennan and Wallace (2009) and Rocheteau (2011) to solve for the efficient allocation and find out the optimal pricing mechanism.

Here, we will briefly discuss the mechanism design problem, with details formally presented in the appendix. First, suppose that the buyer/seller status, the realization of the consumption shock and whether an agent has access to credit or not are public information. During the day, agents play a two-stage game specified by a mechanism. In the first stage of the game, everyone announces his real money balance. A mechanism

[10]Notice that the sufficient condition requires a small π. This is because a small π represents a small fraction of buyers that want to consume, which leads to a smaller gain from the composition effect. The conditions stated in Lemma 2 are essentially conditions for the price effect to dominate the composition effect. Simulation exercise suggests that the price effect tends to dominate the composition effect when π is small, i is small or α is small. This is intuitive because any of these parameters small reflects a small marginal gain from the composition effect.

maps the announced real money holdings of a type $j \in \{b, n, s\}$ agent to a proposed allocation $(q, z) \in R_+ \times Z$, where q is the quantity consumed by a type $j \in \{b, n\}$ agent or the quantity produced by a type s agent, and z is a transfer of real money balances from the agent.[11] In the second stage of the game, everyone says "yes" or "no" simultaneously. Anyone saying "no" receives $(0, 0)$. Anyone saying "yes" receives (q, z) according to the rule specified by the mechanism.

Following similar arguments as in Rocheteau (2011), the allocation (q, z) should not depend on a seller's money balance for truthful announcement of the seller's money balance. The mechanism will ensure that the allocation satisfies a buyer's truthful announcement of his money balance. Rocheteau (2011) considers decentralized, pairwise trading and implements allocations that are in the pairwise core. Here, we consider centralized, multilateral trading and the mechanism implements allocation that is immune to individual deviation (Nash implementable).[12]

The optimal mechanism maximizes the aggregate welfare subject to the relevant technological constraints and incentive constraints by choosing (q^j, z^j) for $j \in \{b, n, s\}$. The technological constraints are the feasibility constraints with respect to goods and money holdings. The incentive constraints include the participation constraints that ensure all agents participate in the mechanism, and the truth-telling constraints that no one wants to misreport his money holdings.[13]

Notice that the trading mechanism is more flexible than a competitive market.

[11] Note that, unlike Rocheteau (2011), there are two types of buyers b and n with different payment technology.

[12] In that sense, the mechanism is more powerful than a competitive market because it can rule out "side trades". If side trades were allowed, the allocation prescribed by the mechanism may not be immuned to group deviation. In our environment, if the mechanism were allowed only to implement allocations that are immuned to group deviation, one would expect that the set of feasible allocation will shrink to competitive equilibrium allocations.

[13] Actually, the truth-telling constraints coincide with the participation constraints in this model since the mechanism itself ensures that agents have no incentives to over or under report their money balances. Please see the proof of Lemma 4 for a detail description of the mechanism.

First, the mechanism is not restricted to linear pricing. Second, the mechanism has an option to contingent (q, z) on the agent' type and on the (self-reported) money holding. Third, agents not accepting the mechanism's proposal are not allowed to conduct side-trades. These flexibilities allow the mechanism to achieve better allocations than a competitive market.

The timing of events is the following. At the beginning of the day, agents can choose to deposit money or take out loans taking the market interest rates of deposits and loans as given. Afterwards, they play the game specified by the mechanism. Trading takes place according to the allocation generated by agents' actions. Activities in the night market remain the same as before.

4.1 Inflation and welfare

We begin with the effect of inflation on welfare.

Lemma 4 *Effects of inflation on welfare: there exists a unique $i_1 > 0$ such that the first best allocation can be implemented if and only if $i \leq i_1$. Moreover, $d\mathcal{W}/di = 0$ for $i \leq i_1$, and $d\mathcal{W}/di < 0$ for $i > i_1$.*

Under the optimal trading mechanism, when $i \leq i_1$, both money-users and credit-users consume the first-best quantity q^*, and thus inflation does not affect welfare. This finding is in sharp contrast with that reported in Lemma 1 for a competitive market in which inflation is always welfare-reducing.

When $i > i_1$, the allocation (q_1^b, q_1^n, q_1^s) is characterized by

$$\frac{u'(q_1^b) - c'(q_1^s)}{u'(q_1^n) - c'(q_1^s)} = \frac{c'(q_1^s) - \frac{1}{1+i}u'(q_1^b)}{c'(q_1^s) - \frac{\pi}{i+\pi}u'(q_1^n)}, \tag{25}$$

$$\frac{\pi\alpha}{1+i}u(q_1^b) + \frac{\pi^2(1-\alpha)}{i+\pi}u(q_1^n) = c(q_1^s), \tag{26}$$

and (17). The allocation features $\frac{\pi}{\pi+i}u'(q_1^n) < c'(q_1^s) < u'(q_1^n)$ and $\frac{1}{1+i}u'(q_1^b) < c'(q_1^s) < u'(q_1^b)$. Recall that in a competitive market, equilibrium conditions imply that $\frac{\pi}{\pi+i}u'(q_1^n) =$

$c'(q_1^s)$ and $\frac{1}{1+i}u'(q_1^b) = c'(q_1^s)$. Intuitively, when the mechanism cannot implement the first best allocation, the constrained allocation from the mechanism lies between the first best allocation and the allocation from a competitive market.

To understand the difference between a competitive market and an optimal trading mechanism, note that a competitive market does not distinguish between the two different types of buyers and type b buyers do not internalize the effect of their consumption on type n buyers. Therefore, pecuniary externality can hurt the economy when the liquidity constraint is binding. The optimal mechanism instead can distinguish between the two types of buyers and assign proper allocations and payment schemes to ensure that both types participate in the mechanism. Compared with the market equilibrium, the mechanism can redistribute from type b buyers to type n buyers to mitigate any externality generated by the price effect. This redistribution can be done via an appropriate price discrimination which can be highlighted by comparing the prices paid by the two types. Denote $p_1^b \equiv z_1^b/q_1^b$ and $p_1^n \equiv z_1^n/q_1^n$ as the prices paid by type b and type n respectively, we have the following lemma.

Lemma 5 *Price discrimination: There exists $\underline{i} < i_1$ and $\bar{i} > i_1$ such that $p_1^b \gtreqless p_1^n$ for $i < \underline{i}$, and $p_1^b > p_1^n$ for $i \in (\underline{i}, \bar{i})$. Moreover, if the utility function has a constant elasticity of scale (i.e., $u'(q)\, q/u(q)$ is constant), then $p_1^b > p_1^n$ for all $i > \bar{i}$.*

Figure 3 illustrates how the trading mechanism supports the (constrained) optimal allocation by adjusting the terms of trade. The dash-dot curve represents the optimal price range for credit-users while the solid curve represents the range for money-users. When i is lower than i_1, incentive constraints are not binding. There are multiple (z^b, z^n) pairs consistent with the first best allocation $q^b = q^n = q^*$. Therefore, there is a range of prices that allows the trade surplus be optimally split between the three parties (i.e. money-users, credit-users and sellers) to satisfy all the incentive constraints given the first best allocation. That is why $p_1^b \gtreqless p_1^n$ for $i < \underline{i}$. As i increases, the liquidity cost for buyers goes up and it becomes harder to support the first best. As a result, the upper bounds for z^b and z^n drop while the lower bounds for z^b and z^n go up, until getting to the threshold i_1 beyond which the first best is not implementable and the optimal terms of trade are uniquely determined. Because credit-users can access credit to partially offset the effect of inflation, their participation constraints are less binding than money-users, the maximum prices for credit-users are higher than those for money-users. Given this, for any $i \in (\underline{i}, \bar{i})$, in order to implement the (constrained) optimal allocation, it is necessary to charge credit-users a higher price and cross subsidize money-users. We can further characterize the optimal prices when $i > \bar{i}$ when the utility function exhibits constant elasticity. In that case, $p_1^b > p_1^n$ for all $i > \bar{i}$. The general idea is that credit-user generates a negative price externality on money-users. In order to internalize this externality, the pricing mechanism needs to price-discriminate between different types by charging credit-users a higher price as indicated in the graph. Note that this type of welfare-improving price discrimination is infeasible in a centralized, competitive market because it will induce side-trades to exploit arbitrage opportunities. These arbitrage activities, however, are prohibited under the current trading mechanism.

4.2 Access to credit and welfare

Under the optimal trading mechanism, the effect of credit is quite different. In contrast to Lemma 3, the following lemma confirms that the adoption of price discrimination makes the usage of credit welfare-improving. Moreover, an increase in the access to credit makes it easier to support the first best allocation by raising the threshold inflation rate i_1.

Lemma 6 *Effects of access to credit on welfare: $d\mathcal{W}/d\alpha \geq 0$ for i close to i_1 and*

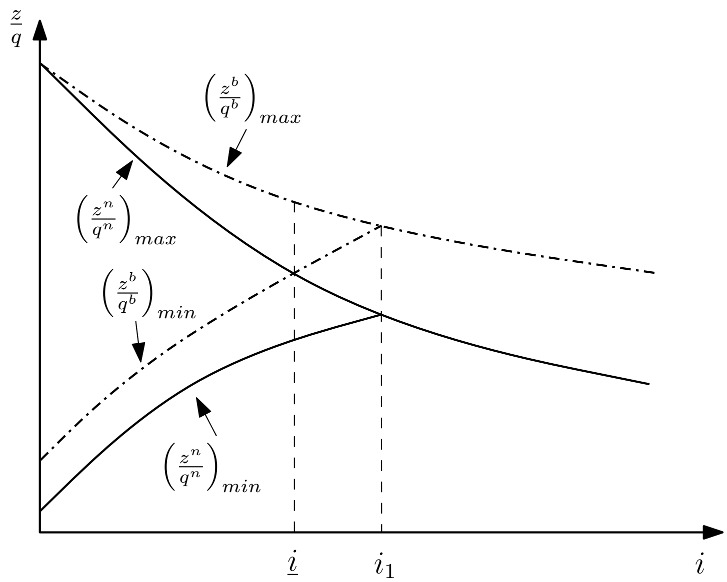

Figure 3: Optimal pricing mechanism

$di_1/d\alpha > 0$.

Finally, we can summarize the results in this section:

Proposition 2 *Under the optimal trading mechanism, moderate inflation does not reduce aggregate consumption or welfare. Optimal trading mechanism typically involves price discrimination between money-users and credit-users. An increase in the access to credit makes it easier to support the first best allocation, and can improve welfare when inflation is not too high.*

5 Welfare with inside-money loans

In the previous sections, we consider the case in which banks offer only *outside-money* loans. As such, banks have to hold outside-money deposit in order to make loans. In this section, we examine the robustness of the finding by considering an alternative credit arrangement where in contrast to the benchmark economy, banks can lend out *inside-money loans*which could be in the form of bank IOUs. We label this economy as economy 2. The benchmark economy is labeled as economy 1, and a monetary economy without credit is labeled as economy 0.

In the day market, bank IOUs are accepted as a means of payment. Therefore, in this economy, banks provide an additional payment service. Competitive banks can take money as deposits and promise to pay a rate r^d. Banks also issue IOUs to buyers who want to borrow to trade in the goods market. Without loss of generality, assume that these IOUs are nominal, so the bank promises to pay 1 unit of money to redeem each IOU in the second subperiod. Each bank takes as given the market rates r^d and r and chooses the amount of deposit D and loans L to solve (18) *without* any constraint. Obviously, in equilibrium,

$$\begin{cases} r = 0, \\ r^d \cdot D = 0. \end{cases} \tag{27}$$

Here, since banks can make loans costlessly, free entry ensures that banks make no profit from offering loans, and thus the loan rate is zero. Also, banks will not take deposit unless the deposit rate is zero.

In economy 2, the agent's decision problem is similar to the benchmark economy, thus we can define a steady state equilibrium as a list of $\left(r_2, q_2^b, q_2^n, q_2^s\right)$ satisfying (17) and

$$r_2 = 0, \tag{28}$$

$$\frac{u'\left(q_2^n\right)}{c'\left(q_2^s\right)} = 1 + \frac{i}{\pi}, \tag{29}$$

$$\frac{u'\left(q_2^b\right)}{c'\left(q_2^s\right)} = 1. \tag{30}$$

In equilibrium, $r^d = 0$ and thus those who can use banks do not have any incentive to deposit.

Comparing the above equilibrium conditions to similar conditions (21) to (23) in economy 1, we can see that the deposit rate and the loan rate are positive in economy 1 while they are 0 in economy 2. Banks in economy 1 channel money balances from those who have additional liquidity to those who needs liquidity. Since any loan taken in the day market was from deposits that were accumulated in the previous night market and the repayment of loans is in the current night market, the nominal interest rate on loans is i. In contrast, bank IOUs can be used as a means of payment so that there is no need for banks to accept deposits. Given that the repayment of the bank IOUs is within the same period, the equilibrium interest rate charged by banks is 0. The difference in the loan rate leads to the result that type b buyers in economy 1 are directly affected by i (and monetary policy) while those in economy 2 are only indirectly affected through $c'(q_2^s)$.

The basic results still hold in economy 2 where the formal proof can be found in the appendix: In a competitive equilibrium, inflation lowers aggregate consumption and welfare. Allowing more people to use credit is not necessarily welfare-improving due to the negative price effect similar in the benchmark case. However, the relative strength of this price effect is different in both economies. Lemma 7 provides a welfare comparison among the pure monetary economy, economy 1, and economy 2.

Lemma 7 *Credit systems and welfare in a competitive equilibrium: the welfare is ranked as the following:* $\mathcal{W}_0 > \mathcal{W}_1 > \mathcal{W}_2$*, when* π *and* α *are small, and* $i > 0$*.*

Here, $\mathcal{W}_0$, $\mathcal{W}_1$, and $\mathcal{W}_2$ denote respectively the welfare in an economy without credit, with outside-money loans, and with inside-money loans. The finding of this lemma may be a bit surprising. One may expect that economy 2 would entail higher welfare than economy 1 because banks are subject to less constraint and banks can issue inside money as a means of payment. However, Lemma 7 overturns this thinking and the intuition can be illustrated in Figure 4. For any given price, the credit-user's demand in economy 1 is smaller than the one in economy 2. The reason is that everyone in economy 1 has to use money as a payment that is subject to inflation distortion

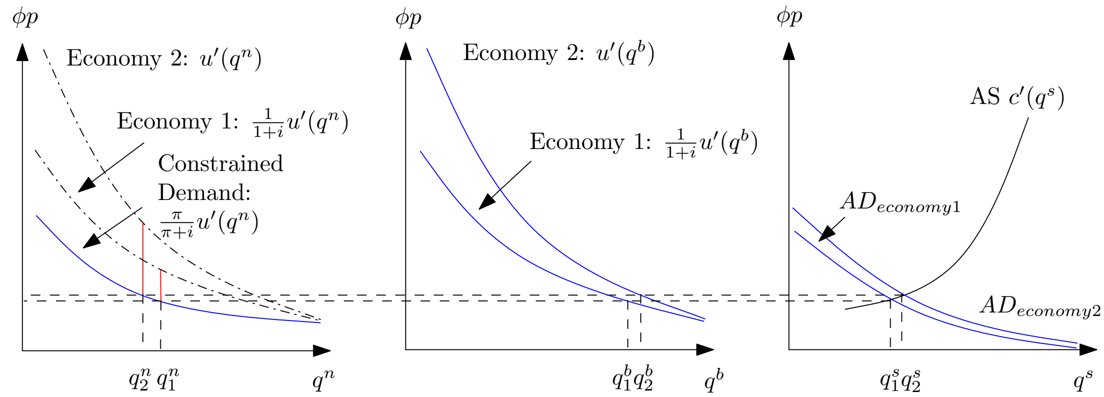

Figure 4: Economy 1 Dominates Economy 2

while some people in economy 2 can pay with credit to avoid such distortion. Given that money-user's demands are the same in both economies, the inefficiency wedge (measured by the vertical distance between the demand curves of credit-users and money-users) is bigger in economy 2. Hence, other things being equal, the price effect generates smaller welfare loss in economy 1. As mentioned before, there is also a composition effect that can positively affect welfare. Lemma 7 shows that when both π and α are small, the price effect dominates the composition effect. Since the negative price effect is strongest in economy 2 and then followed by economy 1, the ranking of welfare is $\mathcal{W}_0 > \mathcal{W}_1 > \mathcal{W}_2$.

As before, we now move to examine the case when the optimal trading mechanism is employed. In contrast to the competitive market, under the optimal pricing mechanism, the welfare ranking is reversed:

Lemma 8 *Credit systems and welfare under optimal trading mechanisms: the welfare in economy 2 weakly dominates the welfare in economy 1.*

The intuition behind this lemma is that credit-users in economy 2 are not directly subject to inflation distortion comparing to credit-users in economy 1. As a result, the participation constraints in economy 2 is less binding than in economy 1, and thus enlarges the set of allocations that the optimal mechanism can implement. Translate this into welfare, we have $\mathcal{W}_2 \geq \mathcal{W}_1$. Finally, we can summarize the results in this section:

Proposition 3 *The welfare effects of credit arrangements with inside-money loans and with outside-money loans are qualitatively similar. Their relative welfare ranking however depends on the trading mechanism. In a competitive market, welfare is lower when banks are allowed to lend out inside-money loans. Under the optimal trading mechanism, welfare is higher when banks are allowed to lend out inside-money loans.*

6 Welfare with endogenous credit constraint

So far we have been assuming that borrowers fully commit to repaying their loans. In this section, we consider the case with limited commitment: the only punishment available is that a borrower who fails to repay his loan is excluded from the banking sector in all future periods. This will generate an endogenous credit limit restricting the maximum size of a loan that a borrower can obtain.

6.1 Equilibrium

The decision problems for sellers and type n buyers are unaffected. Only the decision problem for type b buyers needs to be modified. In the day market, a type b buyer now has to solve (6) subject to an additional borrowing constraint:

$$\ell \leq \bar{\ell}, \tag{31}$$

with $\bar{\ell}$ being the endogenous credit limit to ensure no default in equilibrium. Let λ_ℓ denote the Lagrange multiplier for (31). The only change is the first order condition with respect to ℓ which now becomes

$$\pi\phi r = \lambda_q - \lambda_\ell.$$

In the night market, we need first to examine the incentive for a type b buyer to default. If a borrower defaults on his loans, he cannot access the banking sector for all future periods. Hence, the continuation value is the same as the value of a type n buyer. The no default condition requires that

$$W^b\left(m+\ell-pq^b,\ell,0\right)\geq W^n\left(m+\ell-pq^b,0,0\right).$$

Therefore the credit limit $\bar{\ell}_1$ can be obtained by solving for $\bar{\ell}$ which satisfies

$$W^b\left(m+\bar{\ell}-pq^b,\ell,0\right)=W^n\left(m+\bar{\ell}-pq^b,0,0\right).$$

In the appendix, it is shown that this borrowing limit is given by

$$\bar{\ell}_1=\frac{\beta\left\{\pi\left[\left(u\left(q^b\right)-c'\left(q^s\right)q^b\right)-\left(u\left(q^n\right)-c'\left(q^s\right)q^n\right)\right]-ic'\left(q^s\right)\left(\pi q^b-q^n\right)\right\}}{\phi\left(1+r\right)\left(1-\beta\right)}. \tag{32}$$

Notice that, the first term on the RHS of (32) captures the increase in trading surplus by having access to credit. Other things being equal, a drop in q^n increases this term and thus relaxes the borrowing constraint.

Depending on whether (31) is binding and whether credit is used, there are three types of equilibrium in the steady state.

1. Equilibrium with unconstrained credit consists of $\left(r_1,q_1^b,q_1^n,q_1^s\right)$ satisfying (17), (21)-(23), as well as the condition that the borrowing constraint is not binding:

$$\left(1-\pi\right)c'\left(q_1^s\right)q_1^b<\phi\bar{\ell}_1.$$

2. Equilibrium with constrained credit consists of $\left(r_1,q_1^b,q_1^n,q_1^s,\bar{\ell}_1\right)$ satisfying (17), (22), (32) and

$$\frac{u'\left(q_1^b\right)}{c'\left(q_1^s\right)}=1+\frac{i}{\pi}-\frac{\left(1-\pi\right)}{\pi}r_1, \tag{33}$$

$$\left(1-\pi\right)c'\left(q_1^s\right)q_1^b=\phi\bar{\ell}_1. \tag{34}$$

In this case, one can show that $r_1 < i$. Sellers do not bring any money to the day market. Type b buyers are still willing to bring money to the day market since they are credit constrained and thus derive higher marginal value from higher money balances..

3. Equilibrium without credit which is equivalent to a pure monetary equilibrium is self-fulfilling, under the expectation that borrowers will not repay their loans in future.

6.2 Findings

When the repayment of loans cannot be perfectly enforced, the economy can be in different types of steady state equilibrium depending on parameter values. The unconstrained credit equilibrium behaves exactly like the equilibrium in the economy with perfect enforcement. A more interesting equilibrium is the constrained credit equilibrium because the endogenous credit limit will be affected by monetary policy, access to credit and credit arrangement. This is a new channel that does not exist in the perfect enforcement economy. There also exists an equilibrium without credit. This equilibrium always exists owing to the self-fulfilling nature of the equilibrium. That is, a zero credit limit is consistent with an equilibrium where banks expect borrowers to default and borrowers indeed will have incentives to default if they obtain any credit from banks.

Berentsen et al. (2007) find conditions that characterize these three types of equilibrium only near the neighborhood $\gamma \to \beta$ and $\beta \to 1$. Here we prove that with some restrictions on the utility function and the production function, we can characterize the existence of different types of equilibrium more generally. In particular, there exists an

$\bar{\gamma}_1 > 0$ such that

$$\begin{cases} \gamma \leq \min\{1, \bar{\gamma}_1\}: & \text{only equilibrium without credit exists} \\ \gamma \in (1, \bar{\gamma}_1]: & \text{constrained credit equilibrium exists} \\ \gamma > \bar{\gamma}_1: & \text{unconstrained credit equilibrium exists} \end{cases}$$

See Proposition (4) in the appendix for formal proofs of this result. When the inflation rate is very low, credit is not viable and the equilibrium credit limit is 0. This is because for borrowers, the gain from using credit is not high enough so that they do not have incentive to repay. As inflation increases, the welfare of type n buyers drops and thus the borrowing constraint is relaxed. When the inflation rate is moderate, credit is used, but the borrowing constraint is still binding. When the inflation is sufficiently high, an unconstrained credit equilibrium exists.

In a constrained credit equilibrium, the presence of the credit limit brings an additional link between q_1^b and q_1^n. Compared to Lemma 2, the effects of credit on allocations and welfare are different. We find that in the constrained credit equilibrium,

$$\frac{dq_1^b}{d\alpha} \geq 0, \frac{dq_1^n}{d\alpha} \leq 0, \frac{dq_1^s}{d\alpha} \geq 0, \text{ and } \frac{d\mathcal{W}_1}{d\alpha} > 0.$$

whenever π is sufficiently small. See Proposition (5) in the appendix for formal proofs. When more buyers can use credit, q_1^s increases because of the composition effect. It follows that q_1^n drops, which lowers the value of default in this economy with limited enforcement. This may then relax type b's borrowing constraint, and thus increase q_1^b and aggregate welfare. Notice that this credit limit channel does not exist in the benchmark model where we have $dq_1^b/d\alpha < 0$ and $d\mathcal{W}_1/d\alpha > 0$ for sufficiently small π.

7 Conclusion

This paper uses modern monetary theory to study the welfare effects of inflation and different credit arrangements. We show that, in a monetary economy, credit and payment arrangements are not necessarily welfare-improving because agents may fail to internalize the effects of their actions on others' liquidity constraints. Moreover, the welfare implications of different payment/credit arrangements depend critically on the design of the trading mechanism as well as on the fundamental technologies (e.g. production, trading and enforcement). The optimal trading mechanism typically exhibits nonlinear pricing and price discrimination across different types. Trade surplus has to be optimally split among different parties so as to satisfy agents' incentive constraints and to induce agents to internalize price externalities. The ability to prohibit side-trades is also important for achieving the constrained optimal allocation. While our exercise establishes a welfare benchmark for policy analysis, future research should study whether and how these desirable allocations can be implemented in a decentralized environment.

This paper illustrates the above ideas by developing a simple, stylized model. We expect that the main forces behind the welfare and policy implications remain relevant in more general setting. For example, one might introduce and examine other realistic features such as adoption costs of credit/payment technology by consumers/merchants. Unless the trading/pricing arrangement is appropriately designed, the equilibrium allocation will typically be inefficient. Applying the mechanism design approach may help us understand the essentiality of certain payment structure (e.g. surcharging, card rewards, card membership fees) for correcting inefficiencies. We leave this for future research.

Appendix

A Proof of Lemma 1

Proof. Totally differentiating the system (17), (22), (23), we get

$$\Phi_1 \begin{pmatrix} dq_1^n \\ dq_1^b \\ dq_1^s \end{pmatrix} = \begin{pmatrix} \frac{1}{\pi} c'\left(q_1^s\right) & 0 \\ c'(q_1^s) & 0 \\ 0 & \pi(q_1^b - q_1^n) \end{pmatrix} \begin{pmatrix} di \\ d\alpha \end{pmatrix},$$

where

$$\Phi_1 = \begin{pmatrix} u''\left(q_1^n\right) & 0 & -(1+\frac{i}{\pi})c''\left(q_1^s\right) \\ 0 & u''\left(q_1^b\right) & -(1+i)c''\left(q_1^s\right) \\ -\pi\left(1-\alpha\right) & -\pi\alpha & 1 \end{pmatrix}.$$

Thus,

$$|\Phi_1| = u''\left(q_1^n\right)\left[u''\left(q_1^b\right) - \pi\alpha\left(1+i\right)c''\left(q_1^s\right)\right] - \left(1-\alpha\right)\left(\pi+i\right)u''\left(q_1^b\right)c''\left(q_1^s\right) > 0.$$

The comparative statics with respect to i in economy 1 is given by

$$\begin{aligned} dq_1^n/di &= c'\left(q_1^s\right)\left[\frac{1}{\pi}u''\left(q_1^b\right) - \alpha\left(1-\pi\right)c''\left(q_1^s\right)\right]/|\Phi_1| < 0, \\ dq_1^b/di &= c'\left(q_1^s\right)\left[u''(q_1^n) + \left(1-\pi\right)\left(1-\alpha\right)c''\left(q_1^s\right)\right]/|\Phi_1| \lesseqgtr 0, \\ dq_1^s/di &= c'\left(q_1^s\right)\left[\left(1-\alpha\right)u''\left(q_1^b\right) + \pi\alpha u''(q_1^n)\right]/|\Phi_1| < 0. \end{aligned}$$

Assuming that $u'''(q) > 0$, $c'''(q) = 0$, and $u''(q_1^*) + \left(1-\pi\right)\left(1-\alpha\right)c''\left(q_1^*\right) < 0$, the sign of dq_1^b/di is negative.

Totally differentiating the total welfare with respect to i,

$$\begin{aligned} \frac{d\mathcal{W}_1}{di} &= \pi\alpha u'(q_1^b)\frac{dq_1^b}{di} + \pi\left(1-\alpha\right)u'\left(q_1^n\right)\frac{dq_1^n}{di} - c'(q_1^s)\frac{dq_1^s}{di} \\ &= \pi\alpha\frac{dq_1^b}{di}\left[u'(q_1^b) - c'(q_1^s)\right] + \pi\left(1-\alpha\right)\frac{dq_1^n}{di}\left[u'\left(q_1^n\right) - c'(q_1^s)\right] \\ &= \pi\alpha i c'(q_1^s)\frac{dq_1^b}{di} + \left(1-\alpha\right)i c'(q_1^s)\frac{dq_1^n}{di} < 0, \end{aligned}$$

using (22) and (23). ■

B Proof of Lemma 2

Proof. Following the same steps as in the proof of Lemma 1, the comparative statics with respect to α for economy 1 is given by

$$\begin{aligned}
dq_1^n/d\alpha &= (\pi + i)\,(q_1^b - q_1^n)u''\left(q_1^b\right)c''\left(q_1^s\right)/|\Phi_1| \leq 0, \\
dq_1^b/d\alpha &= \pi\,(1+i)\,(q_1^b - q_1^n)u''\left(q_1^n\right)c''\left(q_1^s\right)/|\Phi_1| \leq 0, \\
dq_1^s/d\alpha &= \pi(q_1^b - q_1^n)u''\left(q_1^n\right)u''\left(q_1^b\right)/|\Phi_1| > 0.
\end{aligned}$$

■

C Proof of Lemma 3

Proof. Following the same steps as in the proof of Lemma 2, totally differentiating the total welfare with respect to α,

$$\begin{aligned}
\frac{d\mathcal{W}_1}{d\alpha} &= \pi\left[u(q_1^b)-u(q_1^n)\right]+\pi\alpha u'(q_1^b)\frac{dq_1^b}{d\alpha}+\pi\left(1-\alpha\right)u'(q_1^n)\frac{dq_1^n}{d\alpha}-c'(q_1^s)\frac{dq_1^s}{d\alpha} \\
&= \pi\left[u(q_1^b)-u(q_1^n)\right]+\pi\alpha u'(q_1^b)\frac{dq_1^b}{d\alpha}+\pi\left(1-\alpha\right)u'(q_1^n)\frac{dq_1^n}{d\alpha} \\
&\quad -\pi\alpha c'(q_1^s)\frac{dq_1^b}{d\alpha}-\pi\left(1-\alpha\right)c'(q_1^s)\frac{dq_1^n}{d\alpha}-\pi c'(q_1^s)\left(q_1^b-q_1^n\right) \\
&= \pi\left\{\left[u(q_1^b)-q_1^b c'(q_1^s)\right]-\left[u(q_1^n)-q_1^n c'(q_1^s)\right]\right\}+\pi\alpha i c'(q_1^s)\frac{dq_1^b}{d\alpha}+\left(1-\alpha\right)ic'(q_1^s)\frac{dq_1^n}{d\alpha}.
\end{aligned}$$

Notice that the difference in trade surplus in the first term is strictly positive but has an upper bound under the assumption that $c'(0)>0$. Given that $dq_1^b/d\alpha<0$ and $dq_1^n/d\alpha<0$, the second term and the third term are strictly negative. Therefore, when π is small, $0<\alpha<1$ and $i>0$, $d\mathcal{W}/d\alpha<0$. ■

D Maximization Problem of the Mechanism

After agents make deposit/loan decision, they could hold different amount of real money balances depending on whether they have access to credit or not. We follow Rocheteau (2011) to design the mechanism so that agents will truthfully report their money balances. Recall that a mechanism maps an agent's type $j \in \{b, n, s\}$ and his announced money balance to an allocation (q^j, z^j). To support the desired allocation (q^j, z^j) for a type j buyer, the mechanism will propose (q^j, z^j) if the announce money balance is no less than z^j, and will propose $(0, 0)$ otherwise. Under this mechanism, a buyer carrying less than z^j has no (strict) incentive to misreport because any report below z^j gives him zero payoff, while any report above z^j is infeasible. Similarly, a buyer carrying more than z^j has no incentive to misreport because reporting below z^j generates zero payoff while all reports above z^j are payoff equivalent. So a type j agent will carry z^j to the mechanism stage. The participation constraint (PC) that we specify below will guarantee that buyers do not gain from getting $(0, 0)$. The PC for an agent who does not have access to credit is

$$-\gamma z_1^n + \beta z_1^n + \beta\pi \left[u(q_1^n) - z_1^n\right] \geq 0.$$

Loan market clearing condition implies that $\pi\phi\ell = (1-\pi)\phi k = (1-\pi)\phi\hat{m}_1^b$. The amount of money available for an agent with access to credit is thus $z_1^b = \phi\hat{m}_1^b + \phi\ell = \frac{1}{\pi}\phi\hat{m}_1^b$. The PC for an agent who has access to credit is

$$-\pi\gamma z_1^b + \beta\pi z_1^b + \beta\pi \left[u(q_1^b) - z_1^b\right] \geq 0.$$

For a seller, the PC is

$$-c(q_1^s) + z_1^s \geq 0.$$

As we do not restrict to pairwise trading, the market clearing conditions imply (17) and

$$z_1^s = \pi\alpha z_1^b + \pi(1-\alpha)z_1^n.$$

The PCs can be rearranged as

$$-iz_1^n + \pi \left[u(q_1^n) - z_1^n\right] \geq 0, \tag{D.1}$$

$$-i\pi z_1^b + \pi \left[u(q_1^b) - z_1^b\right] \geq 0, \tag{D.2}$$

$$-c(q_1^s) + z_1^s \geq 0. \tag{D.3}$$

Having specified the trading mechanism and the PCs, we focus on the optimal mechanism that maximizes (24) such that (D.1) to (D.3) are satisfied.

E Proof of Lemma 4

Proof. The Lagrange is

$$\begin{aligned}
\mathcal{L} \;=\; & \max_{q_1^n, q_1^b, z_1^n, z_1^b} \pi\alpha u(q_1^b) + \pi(1-\alpha)u(q_1^n) - c\left[\pi\alpha q_1^b + \pi(1-\alpha)q_1^n\right] \\
& + \lambda_1 \left\{-iz_1^n + \pi\left[u(q_1^n) - z_1^n\right]\right\} \\
& + \lambda_2 \left\{-i\pi z_1^b + \pi\left[u(q_1^b) - z_1^b\right]\right\} \\
& + \lambda_3 \left\{\ \pi\alpha z_1^b + \pi(1-\alpha)z_1^n - c\left[\pi\alpha q_1^b + \pi(1-\alpha)q_1^n\right]\right\}.
\end{aligned}$$

The FOCs are

$$\begin{aligned}
\pi\left(1-\alpha+\lambda_1\right)u'(q_1^n) &= (1+\lambda_3)\pi(1-\alpha)c'(q_1^s), \\
\pi\left(\alpha+\lambda_2\right)u'(q_1^b) &= (1+\lambda_3)\,\pi\alpha c'(q_1^s), \\
\lambda_1(i+\pi) &= \lambda_3\pi(1-\alpha), \\
\lambda_2(1+i) &= \lambda_3\alpha.
\end{aligned}$$

Notice that if $\lambda_3 = 0$, then $\lambda_1 = \lambda_2 = 0$ and if $\lambda_3 > 0$, then $\lambda_1, \lambda_2 > 0$. We analyze these two cases in the following.

When $\lambda_1 = \lambda_2 = \lambda_3 = 0$, it is straightforward that $q_1^n = q_1^b = q^*$. The necessary and sufficient condition to implement this allocation is

$$c(\pi q^*) \leq \pi\left[\frac{\alpha}{1+i} + \frac{\pi(1-\alpha)}{i+\pi}\right]u(q^*). \tag{E.1}$$

Notice that at the Friedman rule ($i = 0$), the condition is satisfied because $c(\pi q^*) \leq \pi c(q^*) \leq \pi u(q^*)$. The LHS of (E.1) does not depend on i, while the RHS of (E.1) is decreasing in i. One can show that there exists a unique i_1 such that $i \geq i_1$, the allocation $q_1^n = q_1^b = q^*$ and $q_1^s = \pi q^*$ can be implemented.

When $\lambda_1, \lambda_2, \lambda_3 > 0$, all PCs are binding. The allocation (q_1^n, q_1^b, q_1^s) is characterized by (25), (26) and (17). ■

F Proof of Lemma 5

Proof. For $i < i_1$, the mechanism can implement the first best allocation that $q_1^b = q_1^n = q^*$ and $q_1^s = \pi q^*$. However, there is a set of $\left(z_1^b, z_1^n\right)$ to achieve the first best as long as they satisfy the participation constraints

$$\begin{aligned} u\left(q^*\right) &\geq (1+i)\, z_1^b, &\text{(F.1)}\\ \frac{\pi}{\pi+i} u\left(q^*\right) &\geq z_1^n, &\text{(F.2)}\\ \pi\alpha z_1^b + \pi\left(1-\alpha\right) z_1^n &\geq c\left(\pi q^*\right). &\text{(F.3)} \end{aligned}$$

From (F.1) and (F.2), the maximum payments for different buyers are

$$\begin{aligned} z_{1max}^b &= \frac{u\left(q^*\right)}{1+i}, &\text{(F.4)}\\ z_{1max}^n &= \frac{\pi u\left(q^*\right)}{\pi+i}, &\text{(F.5)} \end{aligned}$$

which are decreasing in i. Given z_{1max}^b, constraint (F.3) gives us

$$z_1^n \geq \frac{c\left(\pi q^*\right)}{\pi\left(1-\alpha\right)} - \frac{\alpha z_1^b}{1-\alpha} \geq \frac{c\left(\pi q^*\right)}{\pi\left(1-\alpha\right)} - \frac{\alpha z_{1max}^b}{1-\alpha}.$$

Therefore, the minimum payment for a money-user is

$$z_{1min}^n = \frac{c\left(\pi q^*\right)}{\pi\left(1-\alpha\right)} - \frac{\alpha u\left(q^*\right)}{\left(1-\alpha\right)\left(1+i\right)},$$

which is increasing in i. Similarly, given z_{1max}^n, constraint (F.3) yields the minimum payment for a credit-user as

$$z_{1min}^b = \frac{c\left(\pi q^*\right)}{\pi\alpha} - \frac{\left(1-\alpha\right)\pi u\left(q^*\right)}{\alpha\left(\pi+i\right)},$$

which is also increasing in i. Given the properties of maximum and minimum payments with i, it is easy to see that $z_{1max}^b = z_{1min}^b$ and $z_{1max}^n = z_{1min}^n$ when $i = i_1$. The set of optimal payments is then characterized by

$$\left\{\left(z_1^b, z_1^n\right) : z_{1min}^b \leq z_1^b \leq z_{1max}^b, z_{1min}^n \leq z_1^n \leq z_{1max}^n\right\}$$

which is depicted in Figure 3.

Notice that for any $\pi < 1$, $z^b_{1max} > z^n_{1max}$ for any $i < i_1$. Hence if $z^b_{1min} > z^n_{1max}$, the set of optimal payments must have the property that $z^b_1 > z^n_1$ so that the implied prices satisfy $p^b_1 = z^b_1/q^* > z^n_1/q^* = p^n_1$. This happens when $i > \underline{i}$ where $\underline{i}$ is the solution for $z^b_{1min} = z^n_{1max}$ which is

$$\underline{i} = \pi \left(\frac{\pi u\left(q^*\right)}{c\left(\pi q^*\right)} - 1 \right).$$

Since i_1 solves

$$c\left(\pi q^*\right) = \left[\frac{\pi\alpha}{1+i} + \frac{\pi^2\left(1-\alpha\right)}{\pi+i} \right] u\left(q^*\right),$$

it is straightforward to verify that $\underline{i} < i_1$.

For $i \geq i_1$, the allocation is no longer the first best. The participation constraints in (F.1), (F.2) and (F.3) must be binding. Hence the payments are unique (see Figure 3). In this case, $z^b_1 = u\left(q^b_1\right)/\left(1+i\right)$ and $z^n_1 = \pi u\left(q^n_1\right)/\left(\pi+i\right)$ and $\left(q^b_1, q^n_1, q^s_1\right)$ satisfies equations (25), (26) and (17). Because $p^b_1\left(i_1\right) > p^n_1\left(i_1\right)$, p^b_1 and p^n_1 are continuous in i as $\left(q^b_1, q^n_1, q^s_1, z^b_1, z^n_1\right)$ are continuous in i, we can conclude that there is a $\bar{i} > i_1$ such that $p^b_1 > p^n_1$. Combining with the previous case, it must be true that $p^b_1 > p^n_1$ for any $i \in (\underline{i}, \bar{i})$.

Next we prove that $q^b_1 > q^n_1$ for any $\alpha, \pi \in (0,1)$ and $i_1 < i < +\infty$. Suppose it is not true, then $u'\left(q^b_1\right) \geq u'\left(q^n_1\right)$ for all $i > i_1$. From equilibrium condition (25), we have $u'\left(q^b_1\right) - c'\left(q^s_1\right) \geq u'\left(q^n_1\right) - c'\left(q^s_1\right)$ implies

$$c'\left(q^s_1\right) - \frac{1}{1+i} u'\left(q^b_1\right) \geq c'\left(q^s_1\right) - \frac{\pi}{\pi+i} u'\left(q^n_1\right),$$

which turns out that

$$\frac{\pi}{\pi+i} u'\left(q^n_1\right) \geq \frac{1}{1+i} u'\left(q^b_1\right).$$

Because $\pi/\left(\pi+i\right) < 1/\left(1+i\right)$ for any $\pi < 1$, it must be true that $u'\left(q^n_1\right) > u'\left(q^b_1\right)$ which leads to a contradiction.

Now we turn to the prices. Recall that $p_1^b = \frac{u\left(q_1^b\right)}{(1+i)q_1^b}$ and $p_1^n = \frac{\pi u\left(q_1^n\right)}{(\pi+i)q_1^n}$ for $i > i_1$, if u has a constant elasticity, then $p_1^b = \frac{u'\left(q_1^b\right)}{\rho(1+i)}$ and $p_1^n = \frac{u'\left(q_1^n\right)\pi}{\rho(\pi+i)}$ where $\rho \equiv u'(q)\,q/u(q)$. $p_1^b > p_1^n$ if and only if $u'\left(q_1^b\right)/(1+i) > \pi u'\left(q_1^n\right)/(\pi+i)$. Suppose the last inequality is not true, then

$$c'\left(q_1^s\right) - \frac{u'\left(q_1^b\right)}{1+i} \geq c'\left(q_1^s\right) - \frac{\pi u'\left(q_1^n\right)}{\pi+i}.$$

From the equilibrium condition (25), we must have $u'\left(q_1^b\right) - c'\left(q_1^s\right) \geq u'\left(q_1^n\right) - c'\left(q_1^s\right)$ which contradicts to $u'\left(q_1^n\right) > u'\left(q_1^b\right)$. Therefore, $p_1^b > p_1^n$ for $i > \underline{i}$ by combining with previous results. ■

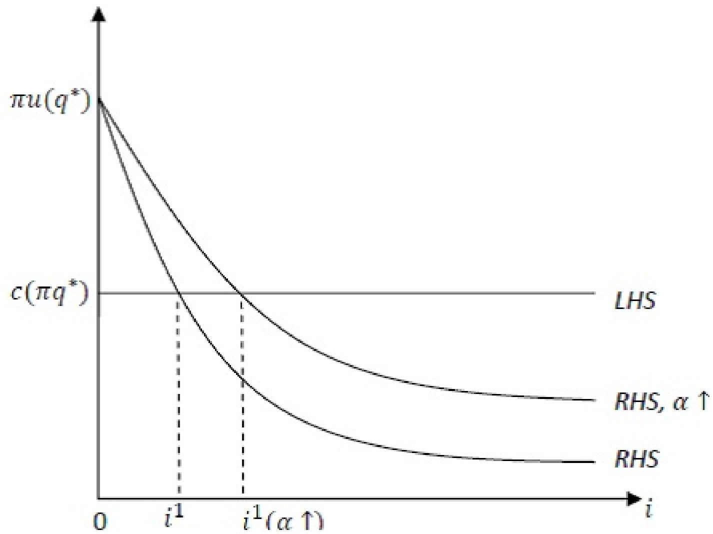

Figure 5: Unconstrained Allocation in Economy 1

G Proof of Lemma 6

Proof. From Lemma 4, it is obvious that $d\mathcal{W}/d\alpha = 0$ for any $i < i_1$ since the economy has already achieved the first best. When $i = i_1$, recall that the necessary and sufficient condition to implement the first best allocation is (E.1). Because the RHS of (E.1) is increasing in α while the LHS is constant, it follows that an increase in α raises i_1. Hence, the first best allocation is more likely to be achieved when α increases, i.e., $d\mathcal{W}/d\alpha|_{i=i_1} > 0$. Then by continuity, $d\mathcal{W}/d\alpha > 0$ for i close to i_1. ■

H Deriving the basic results for economy 2

Totally differentiating the system (17), (29) and (30), we get

$$\Phi_2 \begin{pmatrix} dq_2^n \\ dq_2^b \\ dq_2^s \end{pmatrix} = \begin{pmatrix} \frac{1}{\pi}c'\left(q_2^s\right) & 0 \\ 0 & 0 \\ 0 & \pi(q_2^b - q_2^n) \end{pmatrix} \begin{pmatrix} di \\ d\alpha \end{pmatrix},$$

where

$$\Phi_2 = \begin{pmatrix} u''\left(q_2^n\right) & 0 & -(1+\frac{i}{\pi})c''\left(q_2^s\right) \\ 0 & u''\left(q_2^b\right) & -c''\left(q_2^s\right) \\ -\pi\left(1-\alpha\right) & -\pi\alpha & 1 \end{pmatrix}.$$

Thus,

$$|\Phi_2| = u''\left(q_2^n\right)\left[u''\left(q_2^b\right) - \pi\alpha c''\left(q_2^s\right)\right] - \left(1-\alpha\right)\left(\pi+i\right)u''\left(q_2^b\right)c''\left(q_2^s\right) > 0.$$

The comparative statics with respect to i is given by

$$\begin{aligned} dq_2^n/di &= \frac{1}{\pi}c'\left(q_2^s\right)\left[u''\left(q_2^b\right) - \pi\alpha c''\left(q_2^s\right)\right]/|\Phi_2| < 0, \\ dq_2^b/di &= \left(1-\alpha\right)c'\left(q_2^s\right)c''\left(q_2^s\right)/|\Phi_2| \geq 0, \\ dq_2^s/di &= \left(1-\alpha\right)c'\left(q_2^s\right)u''\left(q_2^b\right)/|\Phi_2| < 0. \end{aligned}$$

In terms of changes in i on welfare, we get

$$\begin{aligned} \frac{d\mathcal{W}_2}{di} &= \pi\alpha u'(q_2^b)\frac{dq_2^b}{di} + \pi\left(1-\alpha\right)u'\left(q_2^n\right)\frac{dq_2^n}{di} - c'(q_2^s)\frac{dq_2^s}{di} \\ &= \pi\alpha\frac{dq_2^b}{di}\left[u'(q_2^b) - c'(q_2^s)\right] + \pi\left(1-\alpha\right)\frac{dq_2^n}{di}\left[u'\left(q_2^n\right) - c'(q_2^s)\right] \\ &= \left(1-\alpha\right)ic'(q_2^s)\frac{dq_2^n}{di} < 0, \end{aligned}$$

using (29) and (30).

The comparative statics with respect to α for economy 2 is given by

$$\begin{aligned} dq_2^n/d\alpha &= (\pi+i)(q_2^b - q_2^n)u''\left(q_2^b\right)c''\left(q_2^s\right)/|\Phi_2| \leq 0, \\ dq_2^b/d\alpha &= \pi(q_2^b - q_2^n)u''\left(q_2^n\right)c''\left(q_2^s\right)/|\Phi_2| \leq 0, \\ dq_2^s/d\alpha &= \pi(q_2^b - q_2^m)u''\left(q_2^n\right)u''\left(q_2^b\right)/|\Phi_2| > 0. \end{aligned}$$

In terms of welfare, we have

$$\begin{aligned}\frac{d\mathcal{W}_2}{d\alpha} &= \pi\left[u(q_2^b)-u(q_2^n)\right]+\pi\alpha u'(q_2^b)\frac{dq_2^b}{d\alpha}+\pi\left(1-\alpha\right)u'(q_2^n)\frac{dq_2^n}{d\alpha}-c'(q_2^s)\frac{dq_2^s}{d\alpha}\\ &= \pi\left\{\left[u(q_2^b)-q_1^b c'(q_2^s)\right]-\left[u(q_2^n)-q_1^n c'(q_2^s)\right]\right\}+\left(1-\alpha\right)ic'(q_2^s)\frac{dq_2^n}{d\alpha}.\end{aligned}$$

It follows that $d\mathcal{W}/d\alpha<0$ when π is small, $0<\alpha<1$ and $i>0$.

I Proof of Lemma 7

Proof. First observe that when $\alpha \to 0$, $q_1^n = q_2^n = q_0$, $q_1^s = q_2^s$, and $q_1^b < q_2^b$ by the first order conditions (22) and (29). Also notice that $\mathcal{W}_0 = \mathcal{W}_1 = \mathcal{W}_2$ when $\alpha \to 0$. The derivatives of $\mathcal{W}$ with respect to α and evaluating at $\alpha = 0$ and $\pi = 0$ are

$$\begin{aligned}
\frac{d\mathcal{W}_0}{d\alpha}|_{\alpha=0,\pi=0} &= 0, \\
\frac{d\mathcal{W}_1}{d\alpha}|_{\alpha=0,\pi=0} &= ic'(0)\,\frac{dq_1^b}{d\alpha}|_{\alpha=0,\pi=0}, \\
\frac{d\mathcal{W}_2}{d\alpha}|_{\alpha=0,\pi=0} &= ic'(0)\,\frac{dq_2^b}{d\alpha}|_{\alpha=0,\pi=0}.
\end{aligned}$$

Because

$$\begin{aligned}
\frac{dq_1^b}{d\alpha}|_{\alpha=0,\pi=0} &= \frac{q_1^b - q_1^n}{\frac{u''\left(q_1^n\right)}{ic''\left(q_1^s\right)} - 1} < 0, \\
\frac{dq_2^b}{d\alpha}|_{\alpha=0,\pi=0} &= \frac{q_2^b - q_2^n}{\frac{u''\left(q_2^n\right)}{ic''\left(q_2^s\right)} - 1} < 0,
\end{aligned}$$

and $\frac{dq_1^b}{d\alpha}|_{\alpha=0,\pi=0} > \frac{dq_2^b}{d\alpha}|_{\alpha=0,\pi=0}$, we must have $\frac{d\mathcal{W}_0}{d\alpha}|_{\alpha=0,\pi=0} > \frac{d\mathcal{W}_1}{d\alpha}|_{\alpha=0,\pi=0} > \frac{d\mathcal{W}_2}{d\alpha}|_{\alpha=0,\pi=0}$. By the continuity of $\mathcal{W}$, we can conclude that $\mathcal{W}_0 > \mathcal{W}_1 > \mathcal{W}_2$ in the neighborhood of $(\alpha, \pi) = (0, 0)$. ■

J Proof of Lemma 8

In order to prove Lemma 8, we first show the following lemma to characterize the allocation in economy 2.

Lemma 9 *In economy 2, if $\alpha \geq \frac{\pi u(q^*)}{c(\pi q^*)}$, the first best allocation can always be implemented. If $\alpha < \frac{\pi u(q^*)}{c(\pi q^*)}$, then there exists a unique i_2 such that the first best allocation can be implemented if and only if $i \leq i_2$.*

Proof. Consider the optimal mechanism in economy 2. Let the amount of inside-money loans in real terms allocated to type b buyers be a. Following notations in the previous economy except that allocation related to credit-users has subscript 2, the relevant PCs are

$$-iz_2^n + \pi\left[u(q_2^n) - z_2^n\right] \geq 0, \tag{J.1}$$

$$-iz_2^b + \pi\left[u(q_2^b) - z_2^b - a\right] \geq 0, \tag{J.2}$$

$$-c(q_2^s) + z_2^s \geq 0. \tag{J.3}$$

An optimal mechanism maximizes (24) such that (J.1) to (J.3) are satisfied.

Formally, the Lagrange is

$$\begin{aligned}
\mathcal{L} = & \max_{q_2^n, q_2^b, z_2^n, z_2^b, a} \pi\alpha u(q_2^b) + \pi(1-\alpha)u(q_2^n) - c\left[\pi\alpha q_2^b + \pi(1-\alpha)q_2^n\right] \\
& + \lambda_1\left\{-iz_2^n + \pi\left[u(q_2^n) - z_2^n\right]\right\} \\
& + \lambda_2\left\{-iz_2^b + \pi\left[u(q_2^b) - z_2^b - a\right]\right\} \\
& + \lambda_3\left\{\pi\alpha\left(z_2^b + a\right) + \pi(1-\alpha)z_2^n - c\left[\pi\alpha q_2^b + \pi(1-\alpha)q_2^n\right]\right\}.
\end{aligned}$$

The FOCs are

$$
\begin{aligned}
\pi(1-\alpha+\lambda_1)u'(q_2^n) &= (1+\lambda_3)\,\pi(1-\alpha)c'(q_2^s), \\
\pi(\alpha+\lambda_2)u'(q_2^b) &= (1+\lambda_3)\,\pi\alpha c'(q_2^s), \\
\lambda_1(i+\pi) &= \lambda_3\pi(1-\alpha), \\
\lambda_2(i+\pi) &= \lambda_3\pi\alpha, \\
\lambda_2 &= \lambda_3\alpha.
\end{aligned}
$$

If $i > 0$, the optimal choice of z_2^b is 0. For an agent that can use credit, it is not optimal to hold any money. Depending on whether $\lambda_3 = 0$, there are two types of solution.

When $\lambda_1 = \lambda_2 = \lambda_3 = 0$, the solution of (q_2^n, q_2^b, q_2^s) is characterized by $q_2^n = q_2^b = q^*$ and $q_2^s = \pi q^*$. The necessary and sufficient condition to implement this allocation is

$$
c(\pi q^*) \leq \pi\left[\alpha + \frac{\pi\,(1-\alpha)}{i+\pi}\right]u(q^*). \tag{J.4}
$$

Again, one can show that the LHS of (J.4) does not depend on i while the RHS of (J.4) is decreasing in i. Note that if $c(\pi q^*) \leq \pi\alpha u(q^*)$ or $\alpha \geq \frac{\pi u(q^*)}{c(\pi q^*)}$, (J.4) holds for any i. That is, the first best allocation can be implemented for any i when α is big enough. If $\alpha < \frac{\pi u(q^*)}{c(\pi q^*)}$, then there exists a critical i_2 such that (E.1) holds if and only if $i \leq i_2$. An increase in α leads to an increase in i_2. In addition, $i_2 > i_1$ because the RHS of (J.4) always lies above the RHS of (E.1). See Figures 6 and 7 for a graphical illustration of the two cases.

When $\lambda_1, \lambda_2, \lambda_3 > 0$, all PCs are binding. The solution of (q_2^n, q_2^b, q_2^s) is from

$$
u'(q_2^b) = c'(q_2^s), \tag{J.5}
$$

$$
\pi\alpha u(q_2^b) + \frac{\pi^2\,(1-\alpha)}{i+\pi}u(q_2^n) = c(q_2^s), \tag{J.6}
$$

and (17). ■

Proof of Lemma 8. As the optimal mechanism maximizes the same objective function, one way to see the welfare comparison is to examine the PCs imposed by

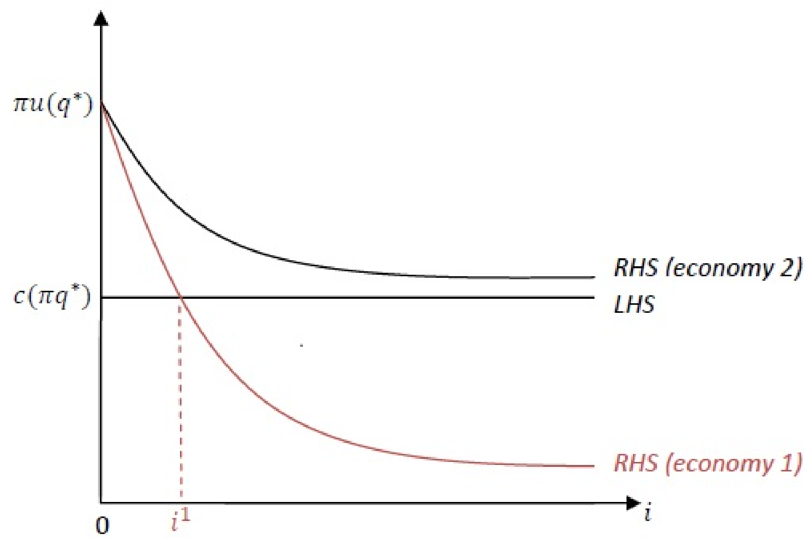

Figure 6: Unconstrained Allocation in Economy 2 – Case 1

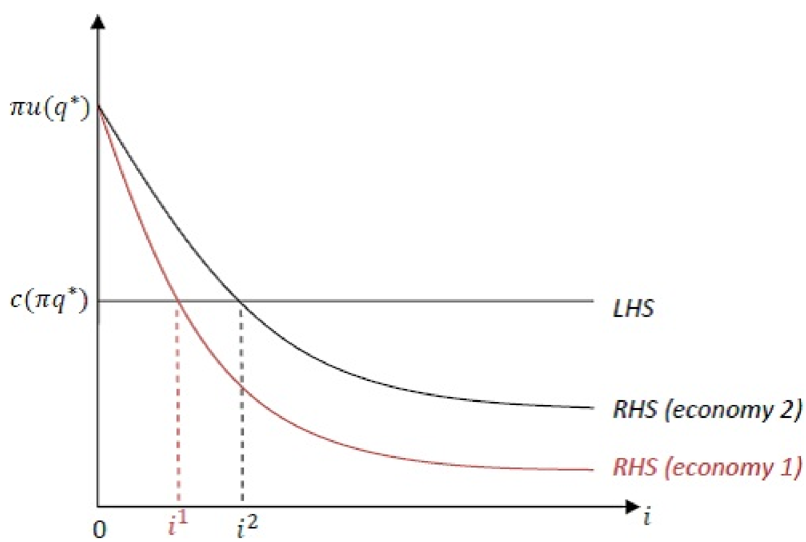

Figure 7: Unconstrained Allocation in Economy 2 – Case 2

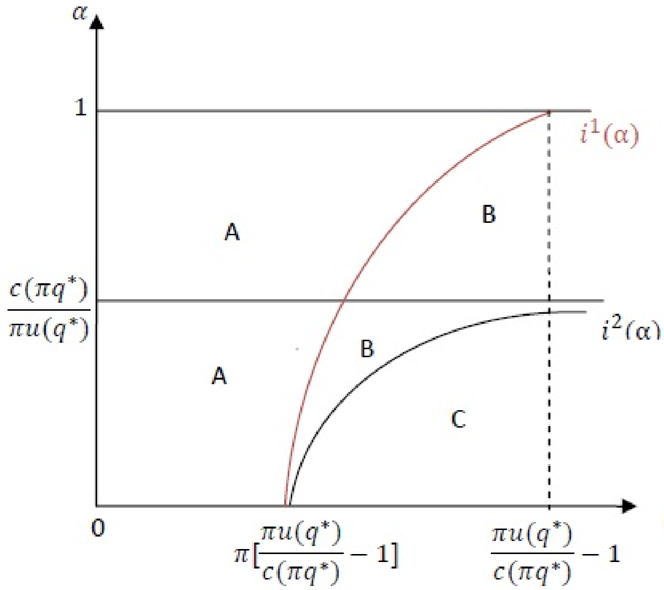

Figure 8: Welfare Comparison

these two different economies. Notice that if we let $z_2^b = \pi z_1^b$ and $a = (1-\pi)z_1^b$ in (J.2) and (J.3), then (J.2) and (J.3) become equivalent to (D.2) and (D.3). Given that (J.1) the same as (D.1), any allocation that is implementable in economy 1 should be feasible in economy 2. It implies that welfare in economy 2 cannot be worse than the welfare in economy 1. So from a mechanism design perspective, welfare in economy 2 always weakly dominates welfare in economy 1.

Figure 8 illustrates the welfare comparison between two economies. In area A, both economy can implement the first best allocation. Only economy 2 can implement the first best allocation in area B. In area C, neither economy can implement the first best allocation, however, any allocation that is implementable in economy 1 is also feasible in economy 2. ■

K Economy with Endogenous Credit Constraint

Deriving the credit constraint. Let m_{+}^{b} be the optimal money balance that a buyer will carry to the next day market when he chooses not to default. Similarly m_{+}^{n} denotes the optimal money balance when he chooses to default in the next day. The difference between the two values is

$$W^{b}\left(m+\ell-pq^{b},\ell,0\right)-W^{n}\left(m+\ell-pq^{b},0,0\right) \tag{K.1}$$

$$=\beta\left[V_{+}^{n}\left(m_{+}^{b}\right)-V_{+}^{n}\left(m_{+}^{n}\right)\right]-\phi\left(m_{+}^{b}-m_{+}^{n}\right)-\phi\left(1+r\right)\ell=0 \tag{K.2}$$

The values in the night market can be rewritten as

$$\begin{aligned}
V_{+}^{n}\left(m_{+}^{n}\right) &= \pi u\left(q_{+}^{n}\right)+\pi\left[W_{+}^{n}(0)+\phi_{+}\left(m_{+}^{n}-p_{+}q_{+}^{n}\right)\right] \\
&\quad +(1-\pi)\left[W_{+}^{n}(0)+\phi_{+}m_{+}^{n}\right], \\
&= \pi u\left(q_{+}^{n}\right)+\bar{W}_{+}^{n}(0)+\phi_{+}m_{+}^{n}-\pi\phi_{+}p_{+}q_{+}^{n}, \\
&= \pi u\left(q^{n}\right)+\phi_{+}\tau M_{+}+v\left(x^{*}\right)-x^{*}+\beta V_{++}^{n}\left(m_{++}^{n}\right)-\phi_{+}m_{++}^{n} \\
&\quad +\phi_{+}m_{+}^{n}-\pi\phi_{+}p_{+}q_{+}^{n},
\end{aligned}$$

and

$$\begin{aligned}
V_{+}^{b}\left(m_{+}^{b}\right) &= \pi u\left(q_{+}^{b}\right)+\pi\left[W_{+}^{b}(0,0,0)+\phi_{+}\left(m_{+}^{b}+\ell_{+}-p_{+}q_{+}^{b}\right)-\phi_{+}\left(1+r_{+}\right)\ell_{+}\right] \\
&\quad +(1-\pi)\left[W_{+}^{b}(0,0,0)+\phi_{+}\left(m_{+}^{b}-d_{+}\right)+\phi_{+}\left(1+r_{+}^{d}\right)d_{+}\right], \\
&= \pi u\left(q_{+}^{b}\right)+\phi_{+}\tau M_{+}+v\left(x^{*}\right)-x^{*}+\beta V_{++}^{b}\left(m_{++}^{b}\right)-\phi_{+}m_{++}^{b} \\
&\quad +\phi_{+}m_{++}^{b}-\pi\phi_{+}p_{+}q_{+}^{b},
\end{aligned}$$

where the last equality is derived from the banking sector clearing and free entry conditions. In a stationary equilibrium, all real variables are constant, hence, $V^{j}\left(m^{j}\right)=$

$V_+^j\left(m_+^j\right)$ and $\phi m^j = \phi_+ m_+^j$. We have

$$\begin{aligned}
V_+^n\left(m_+^n\right) &= \frac{1}{1-\beta}\left[\pi u\left(q^n\right) + \phi_+ \tau M_+ + v\left(x^*\right) - x^* + \phi_+\left(1-\gamma\right) m_+^n - \pi\phi_+ p_+ q^n\right], \\
V_+^b\left(m_+^b\right) &= \frac{1}{1-\beta}\left[\pi u\left(q^b\right) + \phi_+ \tau M_+ + v\left(x^*\right) - x^* + \phi_+\left(1-\gamma\right) m_+^b - \pi\phi_+ p_+ q^b\right].
\end{aligned}$$

Therefore

$$V_+^b\left(m_{+1}^b\right) - V_+^n\left(m^n\right) = \frac{\pi\left[u\left(q^b\right) - u\left(q^n\right)\right] + \phi_+\left(1-\gamma\right)\left(m_+^b - m_+^n\right) - \pi\phi_+ p_+\left(q^b - q^n\right)}{1-\beta}. \tag{K.3}$$

Since $i > 0$, $m_+^b = p_+ q^b - \ell_+ = p_+ q^b - \left(1-\pi\right) m_+^b/\pi \Rightarrow m_+^b = \pi p_+ q^b$, and $m_+^n = p_+ q^n$ for $\gamma > \beta$. Plugging (K.3) into (K.1) and replacing m_+^j with q^j gives

$$\phi\ell = \frac{\beta\left\{\pi\left[u\left(q^b\right) - u\left(q^n\right) - c'\left(q^s\right)\left(q^b - q^n\right)\right] - \frac{\gamma-\beta}{\beta} c'\left(q^s\right)\left(\pi q^b - q^n\right)\right\}}{\left(1+r\right)\left(1-\beta\right)}. \tag{K.4}$$

■

Proposition 4 *Suppose the utility function is CRRA with the coefficient of relative risk aversion $\sigma < 1$. Let q^* be the solution of $u'\left(q^*\right) = c'\left(\pi q^*\right)$ and assume that $u''\left(q^*\right) < -\left(1-\pi\right)\left(1-\alpha\right) c''\left(\pi q^*\right)$ and $c'''\left(q\right) = 0$.*

There exists a $\hat{\beta}$ close to 1 such that for any $\beta \in \left(\hat{\beta}, 1\right)$ and any $\hat{\gamma} > \beta$ where $\hat{\imath} = \hat{\gamma}/\beta - 1 \in \left(0, \pi\left(\frac{\beta}{1-\beta} - 1\right)\right)$, if the following condition holds

$$\frac{\ln \frac{\pi + \hat{\imath}}{\pi\left(1+\hat{\imath}\right)}}{\ln \frac{\beta}{\left(1-\beta\right)\left(1-\pi\right) + \beta\pi}} \leq \sigma < 1,$$

then the following is true for all $\gamma \in \left[\beta, \hat{\gamma}\right]$:

There is an $\bar{\gamma}_1 > 0$ such that

(i) if $\gamma > \bar{\gamma}_1$, a unique unconstrained credit equilibrium exists;

(ii) if $1 < \gamma \leq \bar{\gamma}_1$, a unique constrained credit equilibrium exists;

(iii) if $\gamma \leq \min\left\{1, \bar{\gamma}_1\right\}$, only an equilibrium without credit exists.

Proof. First consider an unconstrained credit equilibrium. In an unconstrained equilibrium. $\phi\ell < \phi\bar{\ell}$ requires that

$$\begin{aligned}(1+i)(1-\beta)(1-\pi)c'(q_1^s)q_1^b &< \beta\pi\left[u(q_1^b)-u(q_1^n)-c'(q_1^s)(q_1^b-q_1^n)\right].\\ &\quad -\beta i c'(q_1^s)(\pi q_1^b-q_1^n)\end{aligned}$$

Let $g(i,\beta)$ and $h(i,\beta)$ be the LHS and RHS of above inequality respectively. Denote $\Delta(i,\beta) \equiv g(i,\beta) - h(i,\beta)$. Taking the derivative of Δ with respect to i, we have

$$\begin{aligned}\Delta_i(i,\beta) &= c'(q_1^s)\left[(1-\beta)(1-\pi)q_1^b+\beta(\pi q_1^b-q_1^n)\right]\\ &\quad +(1-\beta)(1-\pi)(1+i)c'(q_1^s)\frac{\partial q_1^b}{\partial i}\\ &\quad +\frac{\partial q^s}{\partial i}c''(q_1^s)\left[(1-\beta)(1-\pi)(1+i)q_1^b+\beta\pi(q_1^b-q_1^n)+\beta i(\pi q_1^b-q_1^n)\right].\end{aligned}$$

We want to establish that $\Delta_i(i,\beta) < 0$. First look at the sign of derivatives $\partial q_1^j/\partial i$. From Lemma 1 and assuming that $u''' > 0$, $c''' = 0$ and $u''(q^*)+(1-\pi)(1-\alpha)c''(q^*) < 0$, we have $\partial q^j/\partial i < 0$ for $j \in \{b,n,s\}$.

Once we determine that $\partial q^j/\partial i < 0$ for $j \in \{b,n,s\}$, it suffices to show that

$$(1-\beta)(1-\pi)q_1^b+\beta(\pi q_1^b-q_1^n) \le 0, \tag{K.5}$$

and

$$(1-\beta)(1-\pi)(1+i)q_1^b+\beta\pi(q_1^b-q_1^n)+\beta i(\pi q_1^b-q_1^n) \ge 0, \tag{K.6}$$

so that $\Delta_i(i,\beta) < 0$. Condition (K.5) implies that

$$q_1^b \le \frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}q_1^n. \tag{K.7}$$

Because $q_1^n \le q_1^b$, it is necessary that $\beta/\left[(1-\beta)(1-\pi)+\beta\pi\right] \ge 1$ which implies $\beta \ge 1/2$. Condition (K.6) implies that

$$\frac{\pi+i}{1+i}\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}q_1^n \le q_1^b. \tag{K.8}$$

Hence, the sufficient condition requires that for any q_1^b and q_1^n the following inequalities hold:

$$\frac{\pi+i}{1+i}\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi} \leq \frac{q_1^b}{q_1^n} \leq \frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}. \tag{K.9}$$

From (22) and (23), plus CRRA utility, we have

$$\frac{q_1^b}{q_1^n} = \left[\frac{(1+i)\,\pi}{\pi+i}\right]^{-\frac{1}{\sigma}}.$$

Hence condition (K.9) implies

$$\frac{\ln\frac{\pi+i}{\pi(1+i)}}{\ln\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}} \leq \sigma \leq \frac{\ln\frac{\pi+i}{\pi(1+i)}}{\ln\frac{\pi+i}{1+i}\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}}.$$

Because $\beta/\left[(1-\beta)(1-\pi)+\beta\pi\right] < 1/\pi$,

$$\frac{\ln\frac{\pi+i}{\pi(1+i)}}{\ln\frac{\pi+i}{1+i}\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}} > 1.$$

The assumption on $\sigma < 1$ requires that

$$\frac{\ln\frac{\pi+i}{\pi(1+i)}}{\ln\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}} \leq \sigma < 1.$$

It is necessary that $i < \pi\left(\frac{\beta}{1-\beta}-1\right)$. Hence, given arbitrary $\hat{\imath} \in \left(0, \pi\left(\frac{\beta}{1-\beta}-1\right)\right)$, for any $i \leq \hat{\imath}$, if

$$\frac{\ln\frac{\pi+\hat{\imath}}{\pi(1+\hat{\imath})}}{\ln\frac{\beta}{(1-\beta)(1-\pi)+\beta\pi}} \leq \sigma < 1$$

and $u''(q^*) + (1-\pi)(1-\alpha)\,c''(q^*) < 0$ holds then $\Delta_i < 0$.

For any $i \leq \hat{\imath}$, $g(i,1) = 0$ and $h(i,1) > 0$ since

$$\frac{q_1^b}{q_1^n} \leq \frac{\beta}{(1-\beta)(1-\pi)+\beta\pi} < \frac{1}{\pi},$$

we must have $\Delta(i,1) < 0$. By the continuity of Δ, we can find a $\hat{\beta}$ sufficiently close to 1 such that $\Delta(i,\beta) < 0$ for all $\beta \in \left(\hat{\beta},1\right)$. Since $\Delta(0,\beta) > 0$ for all β, by the intermediate value theorem, there exists unique $\bar{\imath}$, i.e. $\bar{\gamma}$, such that $\Delta(\bar{\imath},\beta) = 0$ for any

$\beta \in \left(\hat{\beta}, 1\right)$. Because $\Delta_i(i,\beta) < 0$, if $i > \bar{\imath}$, $g(i,\beta) < h(i,\beta)$, a unique unconstrained credit equilibrium exits while if $i \leq \bar{\imath}$, $g(i,\beta) \geq h(i,\beta)$, so if a credit equilibrium exits it must be constrained.

Next consider the constrained equilibrium. The equilibrium conditions are summarized by equations (17), (22), (33) and (34) with r_1 and $\bar{\ell}_1$ replaced using (21) and (32). Let $\left(\tilde{q}^b, q^n, \tilde{q}^s, \tilde{\imath}\right)$ be the solution of the system. First from (12) and (14), we observe that $\tilde{\imath} \leq i \equiv \gamma/\beta - 1$ for all γ. Given $\tilde{\imath} \in [0, i]$, we can determine the bounds for $\tilde{q}^b$ as the following: if $\tilde{\imath} = 0$, then $\tilde{q}^b = q^n$, while $\tilde{\imath} = i$, $\tilde{q}^b = \bar{q}^b \equiv u'^{-1}\left[(1+i)\, c'(q^s)\right]$. Combining (33) and (34) and eliminating $\tilde{\imath}$ yields

$$\begin{aligned}
&\left[1 - \pi + \frac{\gamma - \beta}{\beta} - \pi\left(\frac{u'\left(\tilde{q}^b\right)}{c'\left(\tilde{q}^s\right)} - 1\right)\right](1-\beta)\, c'\left(\tilde{q}^s\right)\tilde{q}^b \\
= \;& \beta\pi\left[u\left(\tilde{q}^b\right) - u\left(q^n\right) - c'\left(\tilde{q}^s\right)\left(\tilde{q}^b - q^n\right)\right] - (\gamma - \beta)\, c'\left(\tilde{q}^s\right)\left(\pi\tilde{q}^b - q^n\right). \quad \text{(K.10)}
\end{aligned}$$

We want to prove that when $1 < \gamma \leq \bar{\gamma}$, for any q^n, there exists a $\tilde{q}^b \in \left[q^n, \bar{q}^b\right]$ solves equation (K.10). Denote LHS (K.10) of by $g\left(\tilde{q}^b\right)$ and RHS of (K.10) by $h\left(\tilde{q}^b\right)$. First evaluate g and h at q^n using (22): $g(q^n) = (1-\pi)(1-\beta)\, c'(\tilde{q}^s)\, q^n$ and $h(q^n) = (1-\pi)(\gamma-\beta)\, c'(\tilde{q}^s)\, q^n$. If $\gamma > 1$, then $g(q^n) < h(q^n)$. Observe that when $\tilde{q}^b = \bar{q}^b$,

$$g\left(\bar{q}^b\right) = (1+i)(1-\beta)(1-\pi)\, c'\left(\tilde{q}^s\right)\bar{q}^b$$

and

$$h\left(\bar{q}^b\right) = \beta\pi\left[u\left(\bar{q}^b\right) - u\left(q^n\right) - c'\left(\tilde{q}^s\right)\left(\bar{q}^b - q^n\right)\right] - \beta i c'\left(\tilde{q}^s\right)\left(\pi\bar{q}^b - q^n\right).$$

Hence $g\left(\bar{q}^b\right) - h\left(\bar{q}^b\right)$ coincides with $\Delta(i,\beta)$. Since $\Delta_i < 0$ and $i < \bar{\imath}$, it must be true that $\Delta(i,\beta) > \Delta(\bar{\imath},\beta) = 0$. Therefore $g\left(\bar{q}^b\right) > h\left(\bar{q}^b\right)$. Solution for (K.10) exists and is unique. Because q^n is arbitrary, we can conclude that the unique constrained credit equilibrium exits when $\gamma \in (1, \bar{\gamma}]$.

When $\gamma \leq \bar{\gamma} < 1$, $g(q^n) > h(q^n)$ and $g\left(\bar{q}^b\right) > h\left(\bar{q}^b\right)$. Taking derivative of g and h and evaluating them at q^n generates

$$\begin{aligned}
\frac{dg}{d\tilde{q}^b}|_{\tilde{q}^b=q^n} &= \frac{-u''(q^n)\, c'\left(\pi\tilde{q}^d\right) + \pi\alpha u'(q^n)\, c''\left(\pi\tilde{q}^d\right)}{\left[c'\left(\pi\tilde{q}^d\right)\right]^2}\pi(1-\beta)\, c'\left(\pi\tilde{q}^d\right) q^n \\
&\quad + (1-\pi)(1-\beta)\left[c'\left(\pi\tilde{q}^d\right) + \pi\alpha c''\left(\pi\tilde{q}^d\right) q^n\right], \\
\frac{dh}{d\tilde{q}^b}|_{\tilde{q}^b=q^n} &= (1-\pi)(\gamma-\beta)\left[c'\left(\pi\tilde{q}^d\right) + \pi\alpha c''\left(\pi\tilde{q}^d\right) q^n\right].
\end{aligned}$$

Hence $g'(q^n) > h'(q^n)$. Assume $c''' = 0$, one can verify that g is convex and h is concave. These observations imply that there is no solution for (K.10). Therefore credit equilibrium does not exist. ■

Proposition 5 *In a constrained credit equilibrium, we have*

$$\frac{dq_1^n}{d\alpha} \leq 0, \frac{dq_1^b}{d\alpha} \geq 0, \text{ and } \frac{dq_1^s}{d\alpha} \geq 0$$

whenever π is sufficiently small. The equality holds if and only if $\pi = 0$. Also $\mathcal{W}'(\alpha) > 0$ for small π.

Proof. The equilibrium conditions are

$$\frac{\gamma-\beta}{\beta} = \pi\left(\frac{u'\left(q_1^b\right)}{c'(q^s)} - 1\right) + (1-\pi)\, r, \tag{K.11}$$

$$\frac{\gamma-\beta}{\beta} = \pi\left(\frac{u'(q^n)}{c'(q^s)} - 1\right), \tag{K.12}$$

$$\begin{aligned}
(1+r)(1-\beta)(1-\pi)\, c'(q^s)\, q_1^b &= \beta\pi\left[u\left(q_1^b\right) - u(q^n) - c'(q^s)\left(q_1^b - q^n\right)\right] \\
&\quad - (\gamma-\beta)\, c'(q^s)\left(\pi q^1 - q^n\right).
\end{aligned} \tag{K.13}$$

and 17). Totally differentiating the system of equations generates

$$\frac{dq_1^s}{d\alpha} = K_1\frac{dq_1^n}{d\alpha}, \tag{K.14}$$

$$\frac{dq_1^b}{d\alpha} = \left(\frac{K_1}{\pi\alpha} - \frac{1-\alpha}{\alpha}\right)\frac{dq_1^n}{d\alpha} - \frac{q_1^b - q_1^n}{\alpha}, \tag{K.15}$$

$$\frac{dr}{d\alpha} = \frac{\pi u''\left(q_1^b\right)}{(1-\pi) K_2 c'\left(q_1^s\right)}\left[(1-K_2) K_1 + \frac{1-\alpha}{\alpha}K_2\right]\frac{dq_1^n}{d\alpha} + \frac{\pi u''\left(q_1^b\right)\left(q_1^b - q_1^n\right)}{(1-\pi)\, c'\left(q_1^s\right)\alpha} \tag{K.16}$$

and

$$
\begin{aligned}
(1-\beta)(1-\pi)c'(q_1^s)q_1^b\frac{dr}{d\alpha} &= \left[\begin{array}{c}\beta\pi\left(u'\left(q_1^b\right)-c'(q_1^s)\right)-\pi(\gamma-\beta)c'(q_1^s)\\ -(1+r)(1-\beta)(1-\pi)c'(q_1^s)\end{array}\right]\frac{dq_1^b}{d\alpha}\\
&-\left[\begin{array}{c}(1+r)(1-\beta)(1-\pi)c''(q_1^s)q_1^b\\ +(\gamma-\beta)c''(q_1^s)\left(\pi q_1^b-q_1^n\right)+\beta\pi c''(q_1^s)\left(q_1^b-q_1^n\right)\end{array}\right]\frac{dq_1^s}{d\alpha}\\
&-\left[\beta\pi\left(u'(q_1^n)-c'(q_1^s)\right)-(\gamma-\beta)c'(q_1^s)\right]\frac{dq_1^n}{d\alpha},
\end{aligned}
\tag{K.17}
$$

where $K_1 \equiv \frac{u''\left(q_1^n\right)c'\left(q_1^s\right)}{c''\left(q_1^s\right)u'\left(q_1^n\right)} < 0$ and $K_2 = \frac{u''\left(q_1^b\right)c'\left(q_1^s\right)}{c''\left(q_1^s\right)u'\left(q_1^b\right)} < 0$. Plugging (K.14), (K.15) and (K.16) into (K.17) yields

$$
\frac{dq_1^n}{d\alpha} = \frac{-\left[K_3+(1-\beta)\pi u''\left(q_1^b\right)q_1^b\right]\left(q_1^b-q_1^n\right)}{\alpha\left\{\frac{(1-\beta)\pi u''\left(q_1^b\right)q_1^b}{K_2}\left[(1-K_2)K_1+\frac{1-\alpha}{\alpha}K_2\right]+K_4K_1-K_3\left(\frac{K_1}{\pi\alpha}-\frac{1-\alpha}{\alpha}\right)+K_5\right\}},
\tag{K.18}
$$

where for $\pi \to 0$,

$$
\begin{aligned}
K_3 &\equiv \beta\pi\left(u'\left(q_1^b\right)-c'(q_1^s)\right)-\pi(\gamma-\beta)c'(q_1^s)-(1+r)(1-\beta)(1-\pi)c'(q_1^s)\\
&< 0
\end{aligned}
$$

$$
\begin{aligned}
K_4 &\equiv (1+r)(1-\beta)(1-\pi)c''(q_1^s)q_1^b+(\gamma-\beta)c''(q_1^s)\left(\pi q_1^b-q_1^n\right)+\beta\pi c''(q_1^s)\left(q_1^b-q_1^n\right)\\
&> 0
\end{aligned}
$$

(see the proof of Proposition 4 that shows $K_4 > 0$) and

$$
K_5 \equiv \beta\pi\left(u'(q_1^n)-c'(q_1^s)\right)-(\gamma-\beta)c'(q_1^s) < 0.
$$

When $\pi > 0$, the numerator is greater than 0 while the denominator is less than 0. Hence, $dq_1^n/d\alpha < 0$. Substituting (K.18) into (K.15) generates

$$
\frac{\left(q_1^b-q_1^n\right)}{\alpha}\left\{\frac{-\left[K_3+(1-\beta)\pi u''\left(q_1^b\right)q_1^b\right]\left(\frac{K_1}{\pi\alpha}-\frac{1-\alpha}{\alpha}\right)}{\frac{(1-\beta)\pi u''\left(q_1^b\right)q_1^b}{K_2}\left[(1-K_2)K_1+\frac{1-\alpha}{\alpha}K_2\right]+K_4K_1-K_3\left(\frac{K_1}{\pi\alpha}-\frac{1-\alpha}{\alpha}\right)+K_5}-1\right\}
$$

Denote the expression in the bracket by F. $F > 0$ if and only if

$$\begin{aligned}
&\frac{(1-\beta)\,\pi u''\left(q_1^b\right) q_1^b}{K_2}\left[(1-K_2)\,K_1+\frac{1-\alpha}{\alpha}K_2\right]+K_4K_1-K_3\left(\frac{K_1}{\pi\alpha}-\frac{1-\alpha}{\alpha}\right)+K_5 \\
< \quad & -\left[K_3+(1-\beta)\,\pi u''\left(q_1^b\right) q_1^b\right]\left(\frac{K_1}{\pi\alpha}-\frac{1-\alpha}{\alpha}\right).
\end{aligned}$$

This is true since

$$(1-\beta)\,\pi u''\left(q_1^b\right) q_1^b\frac{K_1}{\pi\alpha} > (1-\beta)\,\pi u''\left(q_1^b\right) q_1^b\frac{(1-K_2)\,K_1}{K_2}+K_4K_1+K_5,$$

because the left hand side is greater than 0 and the right hand side is less than 0. Thus $dq_1^b/d\alpha > 0$. It is easy to verify that $dq_1^s/d\alpha > 0$. When $\pi = 0$, it is trivial to see that all the derivatives are equal to 0. ■

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