The demand for liquid asset with uncertain lumpy expenditures

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Introduction

Part of a project on *liquidity management* and *means of payments choice*

- Revisit classic inventory models for different economic agents
- Use simple cts time methods to think through the agents’ decisions
- Confront / estimate models using micro datasets
This paper: objective and contributions

Introduce Large purchases in models of Liquidity management

Examples: durable purchases for households, or M&A for firms

Three contributions

- Theoretical predictions differ starkly from classic inventory models
- Solve tricky math concerning the optimal policy
- Provide scheme to interpret (new) empirical regularities
  - currency demand and size of purchases in cash
  - liquid asset and durables / non-durables purchases
This paper: objective and contributions

Introduce Large purchases in models of Liquidity management

Examples: durable purchases for households, or M&A for firms

Three contributions

- Theoretical predictions differ starkly from classic inventory models
  - large expenditures “drive” adjustments, not a threshold rule ($M = 0$)

- Solve tricky math concerning the optimal policy
  - usual boundary approach is not sufficient

- Provide scheme to interpret (new) empirical regularities
  - currency demand and size of purchases in cash
  - liquid asset and durables / non-durables purchases
Applications: from Thai farmers to Wall Street

Currency

- Survey of cash management practices Austria and Italy
  - Cross section, retrospective questions

- Weekly Diary of cash purchases, and retrospective questions (Austria)

- 10 year panel of monthly data from Thai villages (Townsend)
  - Net cash expenditures, withdrawals, deposits (in progress)

Broad Liquid Asset management (say M2)

- 35 months panel of 1,400 Italian investors:
  - Administrative data on 26 accounts from Unicredit Bank

Demand of Liquid Asset by firms.
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Broad Liquid Asset management (say M2)

- 35 months panel of 1,400 italian investors: administrative data on 26 accounts from Unicredit Bank.

Demand of Liquid Asset by firms.
For comparison, classic model of inventory management:
- Tobin (1956), Baumol (1952), Miller and Orr (1966),
- Eppen and Fama (1969), Constantinides and Richard (1978),
- Frenkel and Jovanovic (1980), Harrison, Sellke, and Taylor (1983),
- Harrison and Taskar (1983), Sulem (1986),

Inventory models with information costs

Models with Jumps in cumulated net cash consumption:

Early version of similar ideas: Whalen (1966).

Alternative assumption about timing of shocks and withdrawals: Telyukova (2009) use it to explain hoarding more cash.
Ingredients

- Continuous cash expenditures $c$ per unit of time (small purchases).
- Expenditures $z$ every $1/\kappa$ periods of time (large purchases).
- Expected expenditures in cash per unit of time $e \equiv c + \kappa z$.
- $M$ is average cash with opportunity cost $R$.
- $n$ is number of adjustments per unit of time, each with fixed cost $b$.

Minimize $MR + bn$
BT with jumps: alternative policies

Period: 120 days \( c = 120, z = 20, \kappa = 4 \)

many transactions \( n = 4 \)

few transactions \( n = 2 \)
Recall Baumol-Tobin: \( \frac{W}{M} = 2 \) and \( n_{BT} = \sqrt{\frac{R (c+z\kappa)}{2b}} \)

- **withdraw more often than jumps:** \( n > \kappa \), optimal when \( \kappa < \bar{\kappa} \)
  \[
  n = \sqrt{\frac{R c}{2b}} > \kappa , \quad n/n_{BT} < 1 \quad , \quad W/M = 2 \frac{c+\kappa z}{c}
  \]

- **withdraw as often as jumps:** \( n = \kappa \), optimal when \( \bar{\kappa} \leq \kappa \leq \bar{\kappa}(z) \):
  \[
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  \]

- **withdraw less often than jumps:** \( n < \kappa \), optimal when \( \kappa > \bar{\kappa}(z) > \bar{\kappa} \)
  \[
  n = \sqrt{\frac{R (c+z\kappa)}{2b}} < \kappa , \quad n/n_{BT} = 1 \quad , \quad W/M = 2 \frac{c+\kappa z}{c+z(\kappa-n)}
  \]

**Thresholds:**
\[
\bar{\kappa} \equiv \sqrt{\frac{R c}{2b}} < \bar{\kappa}(z) \equiv \frac{Rz+\sqrt{(Rz)^2+8bRc}}{4b}
\]
Main implication of lumpy purchases

- Some expenditures have infinite velocity
  - Break the BT link between withdrawal size and avg. liquidity holdings

Simple test of BT theory: use identity: \( n \equiv \frac{e}{W} \) and relation \( W = 2M \)

Basic Empirical prediction of BT: \( n_{BT} \equiv \frac{e}{2M} \)
Key statistics

Narrow aggregates: Currency (Italian households)

plot $M/e$ vs $n$ ; BT prediction $M/e = 1/(2n)$

Source: Alvarez Lippi (Ecta 2009)
**Key statistics**

**Broad aggregates: M2 (Italian investors)**

plot $M/e$ vs $n$; BT prediction $M/e = 1/(2n)$

**Liquidity/Cons. vs. Trade frequency**

- **representative HH (SHIW 2004)**
- **investor HH (Unicredit 2003)**

Source: Alvarez Guiso Lippi (AER 2011)
### Currency management statistics in Italy and Austria

**Recall:** \( \frac{n}{n_{BT}} < 1 \iff \frac{W}{M} > 2 \)

<table>
<thead>
<tr>
<th>Key statistic</th>
<th>ATM Card</th>
<th>Italy (2002)</th>
<th>Austria (2005)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expenditure share paid w. currency</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/o</td>
<td>0.65</td>
<td>0.96</td>
<td></td>
</tr>
<tr>
<td>w.</td>
<td>0.52</td>
<td>0.73</td>
<td></td>
</tr>
<tr>
<td>Currency: ( M/e ) (( e ) per day)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/o</td>
<td>17</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>w.</td>
<td>13</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>( M ) per Household</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/o</td>
<td>410</td>
<td>332</td>
<td></td>
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<tr>
<td>w.</td>
<td>330</td>
<td>206</td>
<td></td>
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<tr>
<td>Currency at withdrawals ( M/M )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w/o</td>
<td>0.46</td>
<td>0.22</td>
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<tr>
<td>w.</td>
<td>0.41</td>
<td>0.26</td>
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<td>Withdrawal: ( W/M )</td>
<td></td>
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<tr>
<td>w/o</td>
<td>2.0</td>
<td>2.4</td>
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<tr>
<td>w.</td>
<td>1.3</td>
<td>1.6</td>
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<tr>
<td># of withdrawals: ( n ) (per year)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>w/o</td>
<td>23</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>w.</td>
<td>58</td>
<td>68</td>
<td></td>
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<tr>
<td>Normalized: ( \frac{n}{n_{BT}} = \frac{n}{e/(2M)} ) (( e ) per year)</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>w/o</td>
<td>1.7</td>
<td>1.4</td>
<td></td>
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<tr>
<td>w.</td>
<td>3.9</td>
<td>5.4</td>
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<tr>
<td>Fraction of households with ( W/M &gt; 2 )</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>w/o</td>
<td>0.25</td>
<td>0.29</td>
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<tr>
<td>w.</td>
<td>0.13</td>
<td>0.19</td>
<td></td>
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<tr>
<td>Fraction of households with ( \frac{n}{n_{BT}} = \frac{n}{e/(2M)} &lt; 1 )</td>
<td></td>
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<tr>
<td>w/o</td>
<td>0.50</td>
<td>0.57</td>
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<tr>
<td>w.</td>
<td>0.19</td>
<td>0.31</td>
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<tr>
<td># of observations</td>
<td></td>
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</tr>
<tr>
<td>w/o</td>
<td>2,275</td>
<td>153</td>
<td></td>
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<tr>
<td>w.</td>
<td>3,729</td>
<td>895</td>
<td></td>
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</table>
In a period of length $dt$, net cash consumption: $dC = c\, dt + z\, dN + \sigma\, dB$
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- Poisson $dN = 1$ with probability $\kappa \, dt$, and zero otherwise,
- Brownian Motion $B$, so $\frac{1}{dt} \mathbb{E}(dB) = 0, \frac{1}{dt} \mathbb{E}(dB^2) = 1$
- Expected net cash consumption: $\frac{1}{dt} \mathbb{E}(dC) = c + \kappa z \equiv e$,
- $dC$ can be positive or negative (inflow of cash) if $\sigma > 0$. 
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adjust cash with NO cost with probability $p\, dt$ during period $dt$:

Opportunity cost of cash $R$; Fixed adjustment cost $b$
withdraw if \( m \) hits zero, adjust cash to \( m^* \).

deposit if \( m \) hits \( m^{**} \), adjust cash to \( m^* \).

if hit by free adjustment, withdraw or deposit and adjust cash to \( m^* \).
Optimal adjustment rule & Bellman equation

- withdraw if $m$ hits zero, adjust cash to $m^*$. 
- deposit if $m$ hits $m^{**}$, adjust cash to $m^*$. 
- if hit by free adjustment, withdraw or deposit and adjust cash to $m^*$. 

Bellman equation in inaction region (expected discounted cost)

$$rV(m) = Rm - cV'(m) + \frac{\sigma^2}{2} V''(m) + p[V(m^*) - V(m)]$$

$$+ \kappa \min [b + V(m^*) - V(m), V(m - z) - V(m)]$$

for all $0 \leq m \leq m^{**}$

boundary cond.

$$V(m^{**}) = V(m^*) + b$$ and $$V'(m^{**}) = 0$$ : pay cost and adjust, 

$$V(0) = V(m^*) + b$$ : pay cost and adjust, 

$$m^* = \arg \min_m V'(m)$$ : choice of $m^*$ is optimal.
The Bellman Equation

- Economics: for $m < z$ must withdraw after large consumption

- Mathematics: for $m < z$ Bellman Eqn is ODE

\[
(r + p + \kappa) V(m) = Rm - cV'(m) + \frac{\sigma^2}{2} V''(m) + (p + \kappa) V(m^*) + \kappa b
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- Economics, for $m \geq z$ don’t adjust after large consumption.
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- Algorithm: solve DDE as a system of ODEs, recursively in segments:
  \([0, z], [z, 2z], [2z, 3z], \ldots, [zJ, m^{**}]\)
The role of the Brownian shocks: $\sigma > 0$

Presence of $\sigma > 0$ allows inflows of cash. Appropriate for firms Miller and Orr (1966), farmers (Alvarez - Townsend, 2011)

- Implies deposits: $m^{**} - m^*$, or smaller if hit by free adjustment.
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- Households in Italy: very small frequency of deposits:
  - Not self-employed: $\frac{\text{# deposits}}{\text{# withdrawals}} = \frac{n_D}{n} = 0.007$
  - Self-employed: $\frac{\text{# deposits}}{\text{# withdrawals}} = \frac{n_D}{n} = 0.058$
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- Model with $\sigma = 0$, simplifies since agent keeps $m \in [0, m^*]$. 
Model with $\sigma = 0$

Large & Infrequent purchases (with $\sigma = 0$)

- Small & frequent purchases
  
  $z \downarrow 0$, $\kappa \uparrow \infty$ while $\kappa z/e$ constant $\implies$ standard model.
Large & Infrequent purchases (with $\sigma = 0$)

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- **Large & infrequent purchases**; parameters such that: $z > m^*$

  Define benchmark with NO large purchases s.t. $e_0 = c$ and $p_0 = p + \kappa$

  Several statistics as in benchmark case:

  $m^* = m_0^*$, $M = M_0$, $\underline{M} = \underline{M}_0$, $n = n_0 > p + \kappa$

  Larger withdrawals: $W = W_0 + \frac{\kappa}{n}z$ hence “fewer” withdrawals $\frac{n}{n_{BT}} < \frac{n_0}{n_{BT,0}}$. 
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Larger withdrawals: $W = W_0 + \frac{\kappa}{n} z$ hence “fewer” withdrawals $\frac{n}{n_{BT}} < \frac{n_0}{n_{BT,0}}$.

Comparative statics: optimal $m_0^*$ decreasing in $p_0$, increasing $e_0$. 

Alvarez, Lippi (U. Chicago, U. Sassari EIEF)
Observations on $\left\{ \frac{M}{e}, \frac{W}{M}, \frac{M}{M}, n \right\}$ identify 3 params $\left\{ \frac{\kappa Z}{e}, \kappa + p, \frac{R}{b/c} \right\}$.

\[
\frac{\kappa Z}{\kappa Z + c} = 1 + \frac{1}{W/M} \left( \frac{M/M}{\log(1-M/M)} + 1 \right), \\
\kappa \leq p + \kappa = n \frac{M}{M} \leq n, \\
z \geq m^* = M \left( \frac{\log(1 - M/M)}{1 + \frac{\log(1 - M/M)}{M/M}} \right) \quad \text{and} \quad \frac{m^*}{c} = f \left( \frac{b/c}{R}, p + \kappa \right)
\]
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\[
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\]

$z \geq m^*$ holds in Austrian diary dataset where $z > 400$ \(\text{€}\) are recorded.
Austria: is the model consistent with evidence?

Model predicts $n/n_{BT}$ decreasing in $z$

$$z \approx \frac{e_a}{e_m}, \text{ ratio of average to median cash expenditure}$$
Austria: is the model consistent with evidence?

Model predicts $n/n_{BT}$ decreasing in $z$

$$z \approx \frac{e_a}{e_m}, \text{ ratio of average to median cash expenditure}$$

![Graph](image1.png)

Regr. coeff. $-1.03$, t-stat $-3.7$

![Graph](image2.png)

Regr. coeff. $-0.97$, t-stat $-9.1$
Two technical issues

- Shape of inaction set for general sS problem: union of disjoint intervals
  - Characterization and (counter) examples. Scarf K-convexity:
    Neave MS 70, Bar-Ilan IER 90, Chien-Levy PEIS 09
  - This paper: inaction is an interval
due to continuous time & jumps in one direction
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due to continuous time & jumps in one direction

- Determination of threshold $m^*$
  - Local vs global minimum for $m^*$
  - No verification theorem (unlike the continuous path case)
  - Optimal value of $m^*$ depend on size of $\kappa$
Deceptive first order conditions.....

Conditional value function $\rho V^*(m^*; m^*)$ vs. threshold $m^*$ at $m = m^*$.

flow costs: $\rho V(m^*; m^*)$ and $R.M(m^*) + b(n(m^*) - \text{optimal } m^*)$

$z = 20, k = 60, c = 365$,
Broader aggregates (Alvarez Guiso Lippi, AER 11)

- Baumol-Tobin-Merton-Duffie-Sun applied to:
  - Expenditures only **Non-durable** goods
  - Financial trades and Liquid assets behave as in BT

- Grossman Laroque + CIA
  - Expenditures only **Durable** goods: lumpy and infrequent
  - No BT here: Zero holding of liquid assets!
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  - Expenditures only Durable goods: lumpy and infrequent
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- Model in this paper has BOTH type of expenditures but
  - Exogenous process for expenditures.
  - Simple structure of adjustment costs.
M2 / Non-Dur Cons vs Trading Frequency; BT prediction $M/e = 1/(2n)$

**Liquid asset management with Large Purchases**

**Source:** Alvarez Guiso Lippi (AER 2011)
Theory: Liquidity $M/e = 1/(2 \times \text{portfolio trade freq.})$

Dependent variable: (log) $M/e$; Regressor: (log) asset trade frequency

<table>
<thead>
<tr>
<th></th>
<th><strong>Shiw data</strong></th>
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<th><strong>Unicredit data</strong></th>
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<tbody>
<tr>
<td></td>
<td><strong>bivariate</strong></td>
<td><strong>Multivariate</strong></td>
<td><strong>bivariate</strong></td>
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<tr>
<td><strong>All investors</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade freq. (log)</td>
<td>0.005</td>
<td>0.03</td>
<td>0.10</td>
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<tr>
<td></td>
<td>(0.02)</td>
<td>(0.02)</td>
<td>(0.02)</td>
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<tr>
<td><strong>Equity investors</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Trade freq. (log)</td>
<td>0.06</td>
<td>0.06</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

2004 SHIW and 2003 Unicredit surveys. $M/c = M2 / \text{non-durable consumption}$; for the Unicredit consumption is imputed. $^a$Regression coefficient of bivariate OLS. $^b$ controls (all in logs): household income, age, size, income risk dummy, self-employed, gov. employment.
$\Delta M_{jt} = \sum_{k=0}^{4} \beta_k F_{jt-k}^S + \sum_{k=0}^{4} \gamma_k F_{jt-k}^P + \delta SFA_{jt} + h_j + u_{jt}$

- $\Delta M_{jt}$: change in Liquid Asset of investor $j$ at $t$.
- $F_{jt}^S$: sales of financial asset of investor $j$ at $t$.
- $F_{jt}^P$: purchases of financial asset of investor $j$ at $t$.
- $SFA_{jt}$: investor $j$ total financial assets (level).
- $h_j$: investor $j$ fixed effect.
Italian data: Liquid Asset and Sale of Financial Assets

\[
\Delta M_{jt} = \sum_{k=0}^{4} \beta_k F_{jt-k}^S + \sum_{k=0}^{4} \gamma_k F_{jt-k}^P + \delta SFA_{jt} + h_j + u_{jt}
\]

- \(\Delta M_{jt}\) change in Liquid Asset of investor \(j\) at \(t\).
- \(F_{jt}^S\) sales of financial asset of investor \(j\) at \(t\)
- \(F_{jt}^P\) purchases of financial asset of investor \(j\) at \(t\)
- \(SFA_{jt}\) investor \(j\) total financial assets (level)
- \(h_j\) investor \(j\) fixed effect

- Investors trade assets about every 6 - 9 months....
- .....yet two months after asset sale 65 cents are spent (vs 16 - 32 cents)
Mortgage approval and financial asset sales (Italy)

vertical axis: fraction that sell financial assets

Source: *Unicredit* monthly administrative records of 26 accounts for each of 40,000 investors.
Concluding remarks

Simple model to improve our understanding of liquidity management

- uncertainty & lumpiness (large purchases) seem necessary ingredients
- preliminary empirical application of these ideas help interpreting the data

Bates, Kahle, and Stulz (2009) panel regressions of liquid asset to total asset for of U.S. manufacturing firms from 1980 to 2006...
Concluding remarks

Simple model to improve our understanding of liquidity management

- uncertainty & lumpiness (large purchases) seem necessary ingredients

- preliminary empirical application of these ideas help interpreting the data

  .... find negative coefficient on the ratio of acquisitions to assets... we interpret this as measure of $z$
Table: Fraction of Investors who adjusted the durables stock in 2004

<table>
<thead>
<tr>
<th></th>
<th>Precious &amp; Antiques</th>
<th>Cars &amp; other&lt;sup&gt;c&lt;/sup&gt;</th>
<th>Furniture &amp; appliances</th>
<th>All % purchases</th>
<th>Housing&lt;sup&gt;a,c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>All investors</td>
<td>0.09</td>
<td>0.15</td>
<td>0.37</td>
<td>0.47</td>
<td>-</td>
</tr>
<tr>
<td>% purchases&lt;sup&gt;b&lt;/sup&gt;</td>
<td>97</td>
<td>83</td>
<td>-</td>
<td>95</td>
<td></td>
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<tr>
<td>By Investor type</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Portfolio adjust.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>&lt; 1 per year</td>
<td>0.07</td>
<td>0.13</td>
<td>0.34</td>
<td>0.42</td>
<td>0.05</td>
</tr>
<tr>
<td>≥ 1 per year</td>
<td>0.12</td>
<td>0.20</td>
<td>0.45</td>
<td>0.57</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Source: Bank of Italy survey - SHIW 2004, <sup>a</sup> during the last 5 years, among homeowners <sup>b</sup> adjustment that are purchases, <sup>c</sup> survey only ask about purchases.
Durable adjustments vs. Portfolio adjustments

Source: Bank of Italy survey - SHIW 2004
Durable trade vs. asset trade frequency

<table>
<thead>
<tr>
<th></th>
<th>All investors (2,808 obs.)</th>
<th>Equity investors (1,535 obs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade freq. (log)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bivariate(^a)</td>
<td>0.27</td>
<td>0.16</td>
</tr>
<tr>
<td>Multivariate(^b)</td>
<td>0.16</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.04)</td>
</tr>
</tbody>
</table>

\(^a\)Regression coefficient of bivariate OLS. \(^b\)Includes (all in logs): household income, age, size.

Durable adjustment frequency: add buys and sells of each type of durable (0, 1, ..., 4).

Source: Bank of Italy survey - SHIW 2004
Use administrative data Unicredit investors:

- “one stop banking“: commercial + investment banking,
- 35 months, 1500 investors (same as survey),
- end month balance on 26 bank accounts,
- distinguish flows vs valuation.

Liquid Asset = Checking account (+ time deposits)

Financial Asset = Remaining 25 accounts (equity, bonds, m. funds, ...)
Use *administrative data Unicredit investors*: 

- “one stop banking”: commercial + investment banking,
- 35 months, 1500 investors (same as survey),
- end month balance on 26 bank accounts,
- distinguish flows vs valuation.

Liquid Asset = Checking account (+ time deposits)

Financial Asset = Remaining 25 accounts (equity, bonds, m. funds, ...)

In the model, all Financial Trades have net cash flows.

In the Unicredit/SHIW surveys Financial Trades may include rebalancing.
UCS adm data: 35 months, 26 accounts, 1500 investors.

Asset sale = at least 1 of 25 financial asset (accounts) sold in month.

Trade = some asset sold or some asset purchased.

Infrequent and large trades.
### UCS portfolio dataset

#### Statistics on annual number of total trades

<table>
<thead>
<tr>
<th>Liquid Assets =</th>
<th>Median</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Checking</td>
<td>Broad</td>
<td>Checking</td>
</tr>
<tr>
<td>All trades ($N_{Tj}$)</td>
<td>3.4</td>
<td>3.4</td>
<td>4.5</td>
</tr>
<tr>
<td>Of which asset Sales ($N_{Sj}$)</td>
<td>1.4</td>
<td>1.4</td>
<td>2.0</td>
</tr>
<tr>
<td>Of which asset Purchases ($N_{Pj}$)</td>
<td>2.4</td>
<td>2.4</td>
<td>3.6</td>
</tr>
<tr>
<td>Stockholders ($N_{Tj}$) (direct+indirect)</td>
<td>5.1</td>
<td>5.1</td>
<td>5.8</td>
</tr>
<tr>
<td>Stockholders ($N_{Tj}$) (direct)</td>
<td>5.8</td>
<td>5.5</td>
<td>6.0</td>
</tr>
</tbody>
</table>

- **Broad Measure of Liquid Asset = Checking + Time Deposits ($\approx$ M2)**
## Ratio of rebalancing trades on total trades

<table>
<thead>
<tr>
<th></th>
<th>Number of trades with some rebalancing</th>
<th>Number of trades with only rebalancing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>Mean</td>
</tr>
<tr>
<td>Whole sample</td>
<td>0.13</td>
<td>0.18</td>
</tr>
<tr>
<td>Stockholders (direct+indirect)</td>
<td>0.21</td>
<td>0.25</td>
</tr>
<tr>
<td>Stockholders (direct)</td>
<td>0.21</td>
<td>0.25</td>
</tr>
</tbody>
</table>

- **Trade some rebalancing**: simultaneous sale and purchase of asset.
- **Trade only rebalancing**: value of sale = value purchase of assets.
Same panel data used for rebalancing but for 40,000 Investors

Indicator of final approval of Mortgage (about 800 in 35 months)

Compute fraction of asset sales prior and after obtaining mortgage.

Compute average asset sales prior and after obtaining mortgage.

Prior to mortgage approval more frequent and larger sales of assets.

Robust to controls (Probit and Tobit regressions)
## Table: Timing of assets sales and house purchases

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Probit estimates for asset sale decision</th>
<th>Tobit estimates of size of asset sold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>Standard error</td>
</tr>
<tr>
<td>Obtained mortgage:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_0$ : current</td>
<td>0.070***</td>
<td>0.012</td>
</tr>
<tr>
<td>$\alpha_1$ : lag 1</td>
<td>0.053***</td>
<td>0.012</td>
</tr>
<tr>
<td>$\alpha_2$ : lag 2</td>
<td>0.029***</td>
<td>0.011</td>
</tr>
<tr>
<td>$\alpha_3$ : lag 3</td>
<td>0.027***</td>
<td>0.011</td>
</tr>
<tr>
<td>$\alpha_4$ : lag 4</td>
<td>0.030***</td>
<td>0.011</td>
</tr>
<tr>
<td>$\alpha_{-1}$ : lead 1</td>
<td>-0.010</td>
<td>0.010</td>
</tr>
<tr>
<td>$\alpha_{-2}$ : lead 2</td>
<td>0.003</td>
<td>0.010</td>
</tr>
<tr>
<td>$\alpha_{-3}$ : lead 3</td>
<td>-0.001</td>
<td>0.010</td>
</tr>
<tr>
<td>Time dummies and $\beta$ : Investor total assets</td>
<td>1.51e-07***</td>
<td>1.24e-08</td>
</tr>
<tr>
<td>$\lambda$: Stockholder</td>
<td>0.094***</td>
<td>0.003</td>
</tr>
<tr>
<td>N. observations</td>
<td>31247</td>
<td></td>
</tr>
<tr>
<td>Pseudo $R^2$</td>
<td>0.07</td>
<td></td>
</tr>
</tbody>
</table>
Table: Cash Management and Large Purchases in Austria

<table>
<thead>
<tr>
<th></th>
<th>All (1048 Obs)</th>
<th>w/ATM card (895 Obs.)</th>
<th>w/o ATM card (153 Obs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Those that usually make large purchases (&gt; 400 euros) in cash&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% persons that use cash for large purchase</td>
<td>46%</td>
<td>37%</td>
<td>96%</td>
</tr>
<tr>
<td>Withdrawal to Money: $W/M$</td>
<td>2.0 1.1</td>
<td>1.9 1.0</td>
<td>2.1 1.3</td>
</tr>
<tr>
<td># withdrawals relative to BT&lt;sup&gt;b&lt;/sup&gt;: $n/n_{BT}$</td>
<td>3.5 1.2</td>
<td>4.4 1.5</td>
<td>1.5 0.7</td>
</tr>
<tr>
<td>Normalized cash at withdrawals $nM/M$&lt;sup&gt;c&lt;/sup&gt;</td>
<td>13.4 4.5</td>
<td>17.5 6.3</td>
<td>4.0 2.6</td>
</tr>
<tr>
<td>Those that usually do NOT make large purchases (&gt; 400 euros) in cash&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% persons that do not use cash for large purchase</td>
<td>54%</td>
<td>63%</td>
<td>4% (6 obs!!)</td>
</tr>
<tr>
<td>Withdrawal to Money: $W/M$</td>
<td>1.6 1.0</td>
<td>1.5 1.0</td>
<td>10.4 2.1</td>
</tr>
<tr>
<td># withdrawals relative to BT&lt;sup&gt;b&lt;/sup&gt;: $n/n_{BT}$</td>
<td>5.9 1.9</td>
<td>6.0 1.9</td>
<td>0.9 0.7</td>
</tr>
<tr>
<td>Normalized cash at withdrawals $nM/M$&lt;sup&gt;c&lt;/sup&gt;</td>
<td>20.6 7.8</td>
<td>20.8 8.0</td>
<td>3.8 2.9</td>
</tr>
</tbody>
</table>

<sup>a</sup> Based on a question about how individual usually paid for items that cost more than 400 euros. Two options are available, either currency or other payment methods. Total number of respondents is 1048.

<sup>b</sup> # of withdrawals $n$ relative to Baumol-Tobin benchmark, $n_{BT} = e/(2M)$ Based on a diary of all transactions during a week. This the week is right after the month corresponding to the question on large transactions above.

<sup>c</sup> the variable $nM/M$ is the product of the number of withdrawals $n$ and the ratio of the average cash at the time of withdrawal, $M$ to the average cash holdings.
Figure: Austria: the inventory model statistics vs. $e_a/e_m$

### $n/n_{BT}$
- Ratio between average and median purchases in cash
- **Regr. coeff.** $-1.01$, **t-stat** $-10.0$

### $W/M$
- Ratio between average and median purchases in cash
- **Regr. coeff.** $0.17$, **t-stat** $2.4$

### $M/M$
- Ratio between average and median purchases in cash
- **Regr. coeff.** $0.001$, **t-stat** $0.001$

### $e_a/e_m = \text{avg} / \text{median purchase in cash}$
Figure: Austria: the inventory model statistics vs. $e_a/y$

\[
\frac{n}{n_{BT}}
\]

Regr. coeff. $-0.59$, t-stat $-11.8$

Avg. purchase size relative to income (in percent)

\[
\frac{W}{M}
\]

Regr. coeff. $-0.04$, t-stat $-1.3$

Avg. purchase size relative to income (in percent)

\[
\frac{M}{M_{low}}
\]

Regr. coeff. $-0.09$, t-stat $-2.0$

Avg. purchase size relative to income (in percent)

$e_a/y = \text{avg purchase / income}$
more austrian data

w/o ATM Card

Regr. coeff. $\approx -0.58$

Number of withdrawals relative to BT/regression

Avg. purchase size relative to income (in percent)

w. ATM card

Regr. coeff. $\approx -0.51$

Number of withdrawals relative to BT/regression

Avg. purchase size relative to income (in percent)
Solving the BE: recursion

- Bellman equation given \( \{m^*, m^{**}\} \) solves in each \( J \) segments:

\[
\begin{align*}
V(m) &= V_j(m) \text{ for } m \in [z_j, \min \{z(j+1), m^{**}\}], \quad j = 0, \ldots, J-1 \\
V_j(m) &= A_j + D_j(m - z_j) + \sum_{k=1,2} \sum_{i=0}^{j} B_{j,i}^k e^{\lambda_k(m - z_j)} (m - z_j)^i \\
\lambda_k &\text{ solves } r + p + k = -c\lambda + \frac{\sigma^2}{2}\lambda^2 \text{ for } k = 1, 2.
\end{align*}
\]

- To solve for \( \{m^*, m^{**}\} \) use form of Bellman equation:

\[
V'(m^*) = 0 \text{ and } V'(m^{**}) = 0 \quad (\text{necessary, but not sufficient conditions}).
\]
## Currency vs Cons paid cash and Withdrawals

<table>
<thead>
<tr>
<th>Dependent Variable log( (M/c) )</th>
<th>without ATM card</th>
<th>with ATM card</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bivariate</td>
<td>multivariate</td>
</tr>
<tr>
<td>log ( n )</td>
<td>-0.24***</td>
<td>-</td>
</tr>
<tr>
<td>log ( n )</td>
<td>-</td>
<td>-0.22***</td>
</tr>
<tr>
<td>(900 obs.)</td>
<td>(900 obs.)</td>
<td>(2326 obs.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dependent Variable log( (W/c) )</th>
<th>without ATM card</th>
<th>with ATM card</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bivariate</td>
<td>multivariate</td>
</tr>
<tr>
<td>log ( n )</td>
<td>-0.39***</td>
<td>-</td>
</tr>
<tr>
<td>log ( n )</td>
<td>-</td>
<td>-0.40***</td>
</tr>
<tr>
<td>(2250 obs.)</td>
<td>(2249 obs.)</td>
<td>(1256 obs.)</td>
</tr>
</tbody>
</table>

2004 SHIW and 2003 Unicredit surveys. \( M/c = M2 / \) non-durable consumption; for the Unicredit consumption is imputed. \( ^{a} \)Regression coefficient of bivariate OLS. \( ^{b} \) controls (all in logs): household income, age, size.
## Table: Temporal pattern of changes in the liquid and investments assets

<table>
<thead>
<tr>
<th>Regressors</th>
<th>Coefficient</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Flow of investment sales:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$: current</td>
<td>0.703***</td>
<td>0.0057</td>
</tr>
<tr>
<td>$\beta_1$: lag 1</td>
<td>-0.23***</td>
<td>0.0062</td>
</tr>
<tr>
<td>$\beta_2$: lag 2</td>
<td>-0.16***</td>
<td>0.0065</td>
</tr>
<tr>
<td>$\beta_3$: lag 3</td>
<td>0.002</td>
<td>0.006</td>
</tr>
<tr>
<td>$\beta_4$: lag 4</td>
<td>-0.03</td>
<td>0.0065</td>
</tr>
<tr>
<td><strong>Flow of investment purchases:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_0$: current</td>
<td>-0.65***</td>
<td>0.0065</td>
</tr>
<tr>
<td>$\gamma_1$: lag 1</td>
<td>0.020***</td>
<td>0.007</td>
</tr>
<tr>
<td>$\gamma_2$: lag 2</td>
<td>-0.076***</td>
<td>0.007</td>
</tr>
<tr>
<td>$\gamma_3$: lag 3</td>
<td>0.056***</td>
<td>0.007</td>
</tr>
<tr>
<td>$\gamma_4$: lag 4</td>
<td>-0.011**</td>
<td>0.006</td>
</tr>
<tr>
<td><strong>Investor total assets:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.092***</td>
<td>0.0025</td>
</tr>
<tr>
<td>N. observations</td>
<td>31622</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.47</td>
<td></td>
</tr>
</tbody>
</table>
Read errors of fit as measurement error.

A property of inventory models (Accounting identity): \( \log \left( n \frac{W}{e} \right) = 0 \)
We argue elsewhere (Alvarez-Lippi Eca 09) that:

- Small and frequent withdrawal, \( \frac{W}{M} < 2 \), \( \frac{n}{n_{BT}} > 1 \) and
- Substantial cash at hand at time of withdrawal, \( \frac{M}{M} >> 0 \)

Are consistent with introducing \( p \):
average free withdrawals opportunities per year.

Agent withdraw everytime that it is free, regardless of level of cash, so:
\( M > 0 \) and \( \frac{W}{M} < 2 \).
Free withdrawal opportunities $p$

- We argue elsewhere (Alvarez-Lippi Eca 09) that:
  - Small and frequent withdrawal, $\frac{W}{M} < 2$, $\frac{n}{n_{BT}} > 1$ and
  - Substantial cash at hand at time of withdrawal, $\frac{M}{M} >> 0$

- Are consistent with introducing $p$:
  average free withdrawals opportunities per year.

- Agent withdraw everytime that it is free, regardless of level of cash, so: $M > 0$ and $W/M < 2$.

- We think this feature helps understand the "average" values, and the difference between those with and without ATM cards.

- Yet, we found other form of heterogeneity interesting, we explore if it is due to large purchases.


