

# The demand for liquid asset with uncertain lumpy expenditures

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# Introduction

Part of a project on *liquidity management* and *means of payments choice*

- Revisit classic inventory models for different economic agents
- Use simple cts time methods to think through the agents' decisions
- Confront / estimate models using micro datasets

# This paper: objective and contributions

Introduce Large purchases in models of Liquidity management

Examples: durable purchases for households, or M&A for firms

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Examples: durable purchases for households, or M&A for firms

Three contributions

- Theoretical predictions differ starkly from classic inventory models
  - large expenditures “drive” adjustments, not a threshold rule ( $M = 0$ )
- Solve tricky math concerning the optimal policy
  - usual boundary approach is not sufficient
- Provide scheme to interpret (new) empirical regularities
  - currency demand and size of purchases in cash
  - liquid asset and durables / non-durables purchases

# Applications: from Thai farmers to Wall Street

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- Currency
  - Survey of cash management practices Austria and Italy (cross section, retrospective questions)
  - Weekly Diary of cash purchases, and retrospective questions (Austria).
  - 10 year panel of monthly data from Thai villages (Townsend) net cash expenditures, withdrawals, deposits (in progress)
- Broad Liquid Asset management (say M2)
  - 35 months panel of 1,400 italian investors: administrative data on 26 accounts from Unicredit Bank.
- Demand of Liquid Asset by firms.

# Related Literature

- For comparison, classic model of inventory management:
  - Tobin (1956), Baumol (1952), Miller and Orr (1966),
  - Eppen and Fama (1969), Constantinides and Richard (1978),
  - Frenkel and Jovanovic (1980), Harrison, Sellke, and Taylor (1983),
  - Harrison and Taskar (1983), Sulem (1986),
  - Bar-Ilan (1990) , Alvarez and Lippi (2009).
- Inventory models with information costs
  - Duffie and Sun (1990), Abel, Eberly, and Panageas (2007), Alvarez, Guiso, and Lippi (2011)
- Models with Jumps in cumulated net cash consumption:  
Bar-Ilan, Perry, and Stadjé (2004) more general process for net cash.
- Early version of similar ideas: Whalen (1966).
- Alternative assumption about timing of shocks and withdrawals :  
Telyukova (2009) use it to explain hoarding more cash.

# Baumol-Tobin Model with Large Purchases

## Ingredients

- Continuous cash expenditures  $c$  per unit of time (small purchases) .
- Expenditures  $z$  every  $1/\kappa$  periods of time (large purchases) .
- Expected expenditures in cash per unit of time  $e \equiv c + \kappa z$  .
- $M$  is average cash with opportunity cost  $R$  .
- $n$  is number of adjustments per unit of time, each with fixed cost  $b$  .

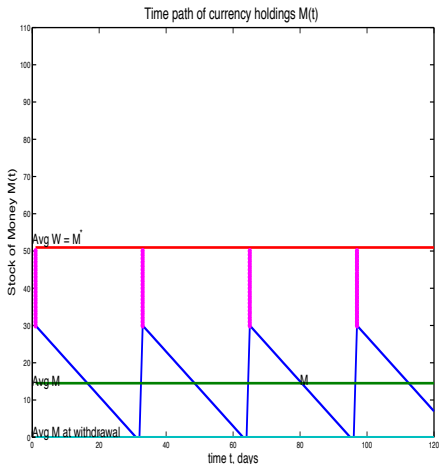
$$\text{Minimize } MR + bn$$



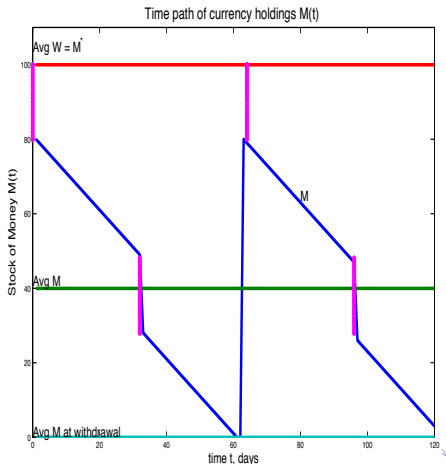
# BT with jumps: alternative policies

Period: 120 days  $c = 120, z = 20, \kappa = 4$

many transactions  $n = 4$



few transactions  $n = 2$



# BT with jumps: Three possible patterns

Recall Baumol-Tobin:  $\frac{W}{M} = 2$  and  $n_{BT} = \sqrt{\frac{R(c+z\kappa)}{2b}}$

- **withdraw more often than jumps:**  $n > \kappa$ , optimal when  $\underline{\kappa} < \bar{\kappa}$

$$n = \sqrt{\frac{Rc}{2b}} > \kappa, \quad n/n_{BT} < 1, \quad W/M = 2 \frac{c+\kappa Z}{c}$$

- **withdraw as often as jumps:**  $n = \kappa$ , optimal when  $\underline{\kappa} \leq \kappa \leq \bar{\kappa}(z)$ :

$$n = \kappa, \quad n/n_{BT} < 1, \quad W/M = 2 \frac{c+\kappa Z}{c}$$

- **withdraw less often than jumps:**  $n < \kappa$ , optimal when  $\kappa > \bar{\kappa}(z) > \underline{\kappa}$

$$n = \sqrt{\frac{R(c+z\kappa)}{2b}} < \kappa, \quad n/n_{BT} = 1, \quad W/M = 2 \frac{c+\kappa Z}{c+z(\kappa-n)}$$

Thresholds:  $\underline{\kappa} \equiv \sqrt{\frac{Rc}{2b}} < \bar{\kappa}(z) \equiv \frac{Rz + \sqrt{(Rz)^2 + 8bRc}}{4b}$

# Main implication of lumpy purchases

- Some expenditures have infinite velocity
  - Break the BT link between withdrawal size and avg. liquidity holdings
- Simple test of BT theory: use identity:  $n \equiv \frac{e}{W}$  and relation  $W = 2M$

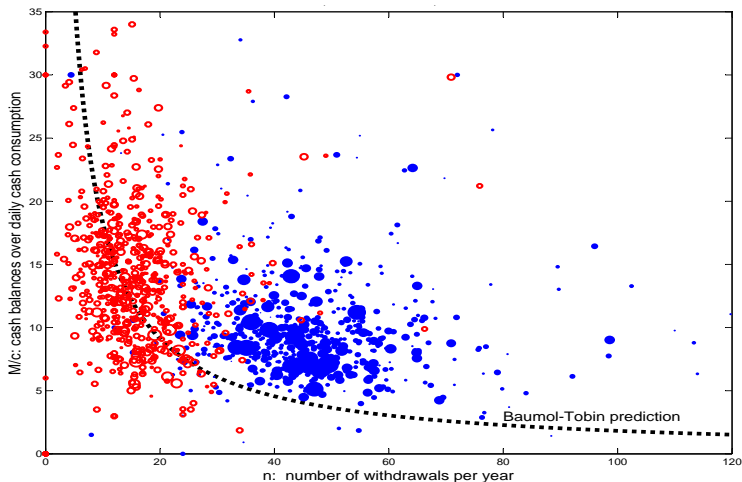
Basic Empirical prediction of BT:  $n_{BT} \equiv \frac{e}{2M}$

▶ currency data

▶ M2 data

# Narrow aggregates: Currency (Italian households)

plot  $M/e$  vs  $n$  ; BT prediction  $M/e = 1/(2n)$



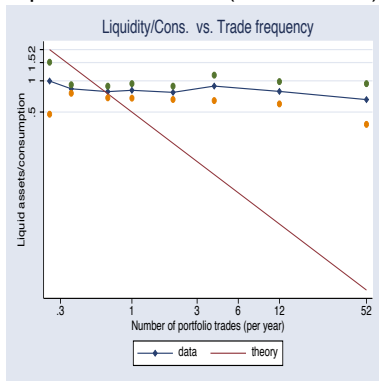
Source: Alvarez Lippi (Ecta 2009)

[▶ back](#)

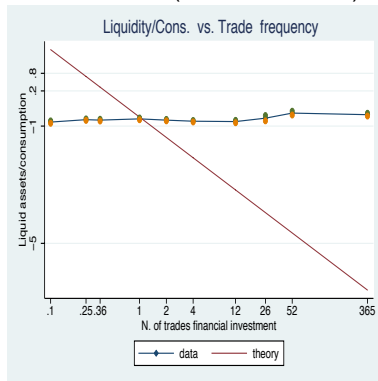
# Broad aggregates: M2 (Italian investors)

plot  $M/e$  vs  $n$  ; BT prediction  $M/e = 1/(2n)$

representative HH (SHIW 2004)



investor HH (Unicredit 2003)



Source: Alvarez Guiso Lippi (AER 2011)

▶ back

# Currency management statistics in Italy and Austria

● recall:  $n/n_{BT} < 1 \iff W/M > 2$

▶ identity  $nW/e = 1$  in data

	ATM Card	Italy (2002)	Austria (2005)
Expenditure share paid w. currency	w/o	0.65	0.96
	w.	0.52	0.73
Currency: $M/e$ ( $e$ per day)	w/o	17	15
	w.	13	15
$M$ per Household	w/o	410	332
	w.	330	206
Currency at withdrawals $\underline{M}/M$	w/o	0.46	0.22
	w.	0.41	0.26
Withdrawal: $W/M$	w/o	2.0	2.4
	w.	1.3	1.6
# of withdrawals: $n$ (per year)	w/o	23	21
	w.	58	68
Normalized: $\frac{n}{n_{BT}} = \frac{n}{e/(2M)}$ ( $e$ per year)	w/o	1.7	1.4
	w.	3.9	5.4
Fraction of households with $W/M > 2$	w/o	0.25	0.29
	w.	0.13	0.19
Fraction of households with $\frac{n}{n_{BT}} \equiv \frac{n}{e/(2M)} < 1$	w/o	0.50	0.57
	w.	0.19	0.31
# of observations	w/o	2,275	153
	w.	3,729	895

# Stochastic Model w/Large expenditures

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  - Poisson  $dN = 1$  with probability  $\kappa dt$ , and zero otherwise,
  - Brownian Motion  $B$ , so  $\frac{1}{dt}\mathbb{E}(dB) = 0$ ,  $\frac{1}{dt}\mathbb{E}(dB^2) = 1$
  - Expected net cash consumption:  $\frac{1}{dt}\mathbb{E}(dC) = c + \kappa z \equiv e$ ,
  - $dC$  can be positive or negative (inflow of cash) if  $\sigma > 0$ .



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  - $dC$  can be positive or negative (inflow of cash) if  $\sigma > 0$ .
- adjust cash with NO cost with probability  $p dt$  during period  $dt$ :
- Opportunity cost of cash  $R$  ; Fixed adjustment cost  $b$

# Optimal adjustment rule & Bellman equation

- withdraw if  $m$  hits zero , adjust cash to  $m^*$  .
- deposit if  $m$  hits  $m^{**}$  , adjust cash to  $m^*$  .
- if hit by free adjustment, withdraw or deposit and adjust cash to  $m^*$  .

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Bellman equation in inaction region (expected discounted cost)

$$rV(m) = Rm - cV'(m) + \frac{\sigma^2}{2}V''(m) + \rho[V(m^*) - V(m)] \\ + \kappa \min[b + V(m^*) - V(m), V(m-z) - V(m)]$$

$$\text{for all } 0 \leq m \leq m^{**}$$

boundary cond.

$$V(m^{**}) = V(m^*) + b \text{ and } V'(m^{**}) = 0 : \text{ pay cost and adjust,}$$

$$V(0) = V(m^*) + b : \text{ pay cost and adjust,}$$

$$m^* = \arg \min_m V'(m) : \text{ choice of } m^* \text{ is optimal .}$$

# The Bellman Equation

- Economics: for  $m < z$  must withdraw after large consumption

Mathematics: for  $m < z$  Bellman Eqn is ODE

$$(r + \rho + \kappa)V(m) = Rm - cV'(m) + \frac{\sigma^2}{2}V''(m) + (\rho + \kappa)V(m^*) + \kappa b$$

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- Algorithm: solve DDE as a system of ODEs, recursively in segments:  
 $[0, z]$ ,  $[z, 2z]$ ,  $[2z, 3z]$ , ...,  $[zJ, m^{**}]$

# The role of the Brownian shocks: $\sigma > 0$

Presence of  $\sigma > 0$  allows inflows of cash. Appropriate for firms  
Miller and Orr (1966)), farmers (Alvarez - Townsend , 2011)

- Implies deposits:  $m^{**} - m^*$ , or smaller if hit by free adjustment.

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- Households in Italy: very small frequency of deposits:
  - Not self-employed  $\frac{\# \text{ deposits}}{\# \text{ withdrawals}} = \frac{n_D}{n} = 0.007$
  - Self-employed  $\frac{\# \text{ deposits}}{\# \text{ withdrawals}} = \frac{n_D}{n} = 0.058$



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  - Self-employed  $\frac{\# \text{ deposits}}{\# \text{ withdrawals}} = \frac{n_D}{n} = 0.058$
- Model with  $\sigma = 0$ , simplifies since agent keeps  $m \in [0, m^*]$ .

# Large & Infrequent purchases (with $\sigma = 0$ )

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Define benchmark with NO large purchases s.t.  $e_0 = c$  and  $p_0 = p + \kappa$

Several statistics as in benchmark case:

$$m^* = m_0^* , \quad M = M_0 , \quad \underline{M} = \underline{M}_0 , \quad n = n_0 > p + \kappa$$

Larger withdrawals:  $W = W_0 + \frac{\kappa}{n}z$  hence “fewer” withdrawals  $\frac{n}{n_{BT}} < \frac{n_0}{n_{BT,0}}$ .

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comparative statics: optimal  $m_0^*$  decreasing in  $p_0$  , increasing  $e_0$

# Large & infrequent purchases: testable moments

Observations on  $\left\{ \frac{M}{e}, \frac{W}{M}, \frac{M}{M}, n \right\}$  identify 3 params  $\left\{ \frac{\kappa Z}{e}, \kappa + \rho, \frac{R}{b/c} \right\}$ .

$$\frac{\kappa Z}{\kappa Z + c} = 1 + \frac{1}{W/M} \left( \frac{\frac{M}{M}}{\frac{\log(1 - \frac{M}{M})}{\frac{M}{M}} + 1} \right),$$

$$\kappa \leq \rho + \kappa = n \frac{M}{M} \leq n,$$

$$z \geq m^* = M \left( \frac{\log(1 - \frac{M}{M})}{1 + \frac{\log(1 - \frac{M}{M})}{\frac{M}{M}}} \right) \quad \text{and} \quad \frac{m^*}{c} = f \left( \frac{b/c}{R}, \rho + \kappa \right)$$

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$z \geq m^*$  holds in Austrian diary dataset where  $z > 400 \text{ €}$  are recorded

# Austria: is the model consistent with evidence?

Model predicts  $n/n_{BT}$  decreasing in  $z$

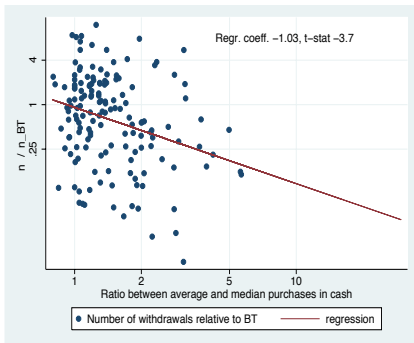
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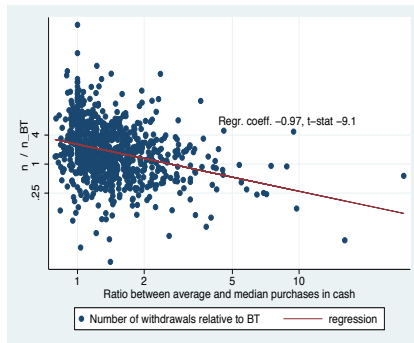
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w/o ATM Card



w. ATM card





# Two technical issues

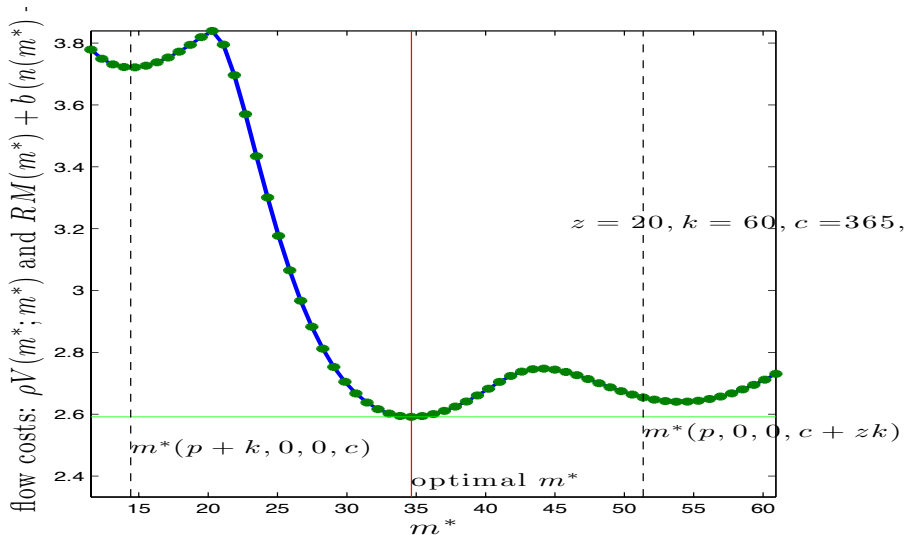
- Shape of inaction set for general  $sS$  problem: union of disjoint intervals
  - Characterization and (counter) examples. Scarf K-convexity:  
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  - This paper: inaction is an interval  
due to continuous time & jumps in one direction
- Determination of threshold  $m^*$ 
  - Local vs global minimum for  $m^*$
  - No verification theorem (unlike the continuous path case)
  - Optimal value of  $m^*$  depend on size of  $\kappa$

# Deceptive first order conditions.....

Conditional value function  $\rho V^*(m^*; m^*)$  vs. threshold  $m^*$  at  $m = m^*$ .



# Broader aggregates (Alvarez Guiso Lippi, AER 11)

- Baumol-Tobin-Merton-Duffie-Sun applied to:
  - Expenditures only **Non-durable** goods
  - Financial trades and Liquid assets behave as in BT
- Grossman Laroque + CIA
  - Expenditures only **Durable** goods: lumpy and infrequent
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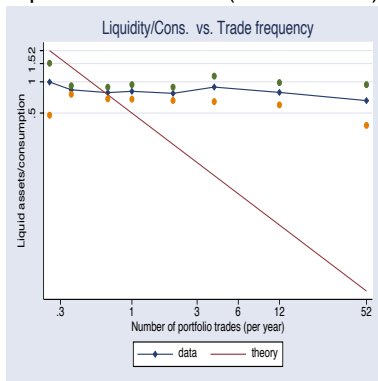
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  - Expenditures only **Durable** goods: lumpy and infrequent
  - No BT here: Zero holding of liquid assets!
- Model in this paper has BOTH type of expenditures but
  - Exogenous process for expenditures.
  - Simple structure of adjustment costs.

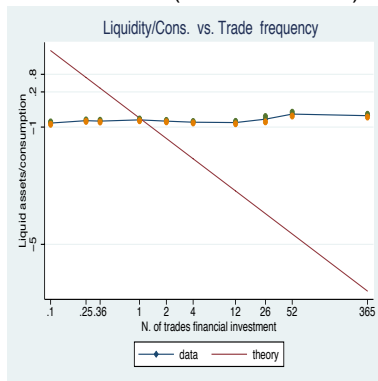
# Broad aggregates: M2 (Italian investors)

M2 / Non-Dur Cons vs Trading Frequency ; BT prediction  $M/e = 1/(2n)$

representative HH (SHIW 2004)



investor HH (Unicredit 2003)



Source: Alvarez Guiso Lippi (AER 2011)

▶ back

# Broad aggregates (M2); multivariate analysis

Theory: Liquidity  $M/e = 1/(2 \times \text{portfolio trade freq.})$

Dependent variable: (log)  $M/e$  ; Regressor: (log) asset trade frequency

	<i>Shiw data</i>		<i>Unicredit data</i>	
	bivariate <sup>a</sup>	Multivariate <sup>b</sup>	bivariate <sup>a</sup>	Multivariate <sup>b*</sup>
<i>All investors</i>	<i>(2,808 obs.)</i>		<i>(1,365 obs.)</i>	
Trade freq. (log)	0.005 (0.02)	0.03 (0.02)	0.10 (0.02)	0.13 (0.02)
<i>Equity investors</i>	<i>(1,535 obs.)</i>		<i>(875 obs.)</i>	
Trade freq. (log)	0.06 (0.03)	0.06 (0.03)	0.08 (0.03)	0.11 (0.03)

2004 SHIW and 2003 Unicredit surveys.  $M/c = M2 / \text{non-durable consumption}$ ; for the Unicredit consumption is imputed. <sup>a</sup>Regression coefficient of bivariate OLS. <sup>b</sup> controls (all in logs): household income, age, size, income risk dummy, self-employed, gov. employment

# Italian data: Liquid Asset and Sale of Financial Assets

$$\Delta M_{jt} = \sum_{k=0}^4 \beta_k F_{jt-k}^S + \sum_{k=0}^4 \gamma_k F_{jt-k}^P + \delta SFA_{jt} + h_j + u_{jt}$$

- $\Delta M_{jt}$  change in Liquid Asset of investor  $j$  at  $t$ .
- $F_{jt}^S$  sales of financial asset of investor  $j$  at  $t$
- $F_{jt}^P$  purchases of financial asset of investor  $j$  at  $t$
- $SFA_{jt}$  investor  $j$  total financial assets (level)
- $h_j$  investor  $j$  fixed effect



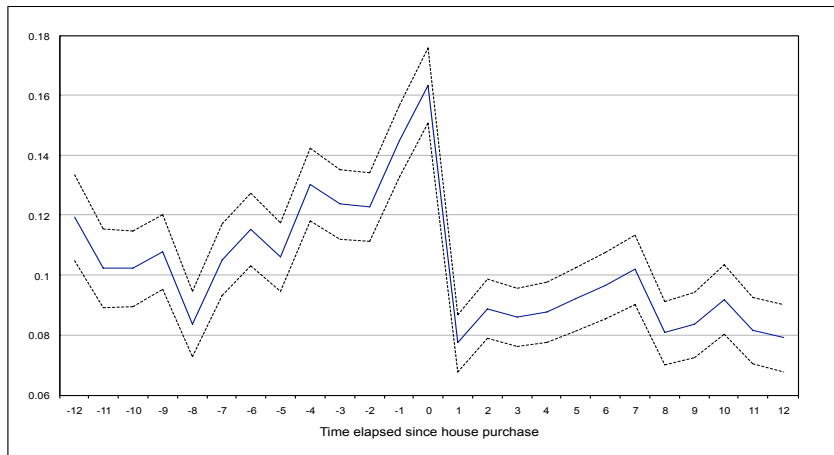
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  - $F_{jt}^S$  sales of financial asset of investor  $j$  at  $t$
  - $F_{jt}^P$  purchases of financial asset of investor  $j$  at  $t$
  - $SFA_{jt}$  investor  $j$  total financial assets (level)
  - $h_j$  investor  $j$  fixed effect
- Investors trade assets about every 6 - 9 months....
  - .....yet two months after asset sale 65 cents are spent (vs 16 - 32 cents)

# Mortgage approval and financial asset sales (Italy)

vertical axis: fraction that sell financial assets



--- line  $\pm$  one standard error bands

Source: *Unicredit* monthly administrative records of 26 accounts for each of 40,000 investors.

# Concluding remarks

Simple model to improve our understanding of liquidity management

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- Bates, Kahle, and Stulz (2009) panel regressions of liquid asset to total asset for of U.S. manufacturing firms from 1980 to 2006....  
.... find negative coefficient on the ratio of acquisitions to assets... we interpret this as measure of  $z$



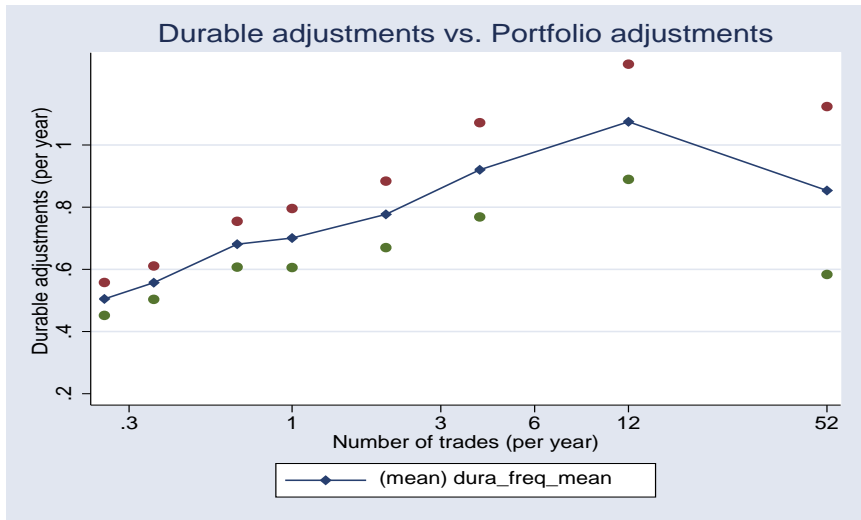
**Table:** Fraction of Investors who adjusted the durables stock in 2004

	Precious & Antiques	Cars & other <sup>c</sup>	Furniture & appliances	All	Housing <i>a,c</i>
All investors					
% purchases <sup>b</sup>	0.09 97	0.15 83	0.37 -	0.47 95	-
By Investor type					
<i>Portfolio adjust.</i>					
< 1 per year	0.07	0.13	0.34	0.42	0.05
≥ 1 per year	0.12	0.20	0.45	0.57	0.07

Source: Bank of Italy survey - SHIW 2004, <sup>a</sup> during the last 5 years, among homeowners

<sup>b</sup> adjustment that are purchases, <sup>c</sup> survey only ask about purchases.

# Adjustment frequency: durable vs. portfolio



# Durable trade vs. asset trade frequency

Dependent variable: (log) durable trade freq. on (log) asset trade frequency

*All investors (2,808 obs.)*

*Equity investors (1,535 obs.)*

	<i>All investors (2,808 obs.)</i>		<i>Equity investors (1,535 obs.)</i>	
	bivariate <sup>a</sup>	Multivariate <sup>b</sup>	bivariate <sup>a</sup>	Multivariate <sup>b</sup>
Trade freq. (log)	0.27 (0.03)	0.16 (0.03)	0.16 (0.04)	0.09 (0.04)

—<sup>a</sup>Regression coefficient of bivariate OLS. —<sup>b</sup>Includes (all in logs): household income, age, size.

Durable adjustment frequency: add buys and sells of each type of durable (0, 1, ..., 4) .

Source: Bank of Italy survey - SHIW 2004



# Portfolio Trades in Unicredit Data

- Use *administrative data Unicredit investors*:
  - “one stop banking“: commercial + investment banking,
  - 35 months, 1500 investors (same as survey),
  - end month balance on 26 bank accounts,
  - distinguish flows vs valuation.
- Liquid Asset = Checking account (+ time deposits)
- Financial Asset = Remaining 25 accounts (equity, bonds, m. funds, ...)

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- Financial Asset = Remaining 25 accounts (equity, bonds, m. funds, ...)
- In the model, all Financial Trades have net cash flows.
- In the Unicredit/SHIW surveys Financial Trades may include rebalancing.

# Statistics on annual number of asset sales

	All Asset Sales $N_{Sj}$		Asset sales $\geq 500$		Asset sales $\geq 1000$	
	Median	Mean (sd)	Median	Mean (sd)	Median	Mean (sd)
Total sample	1.03	1.40 (1.29)	1.03	1.17 (1.11)	0.70	1.06 (1.03)
Stockholders (total)	1.71	1.81 (1.28)	1.37	1.53 (1.13)	1.02	1.40 (1.07)
Stockholders (direct)	1.71	1.97 (1.30)	1.37	1.69 (1.19)	1.37	1.55 (1.12)

- UCS adm data: 35 months, 26 accounts, 1500 investors.
- Asset sale = at least 1 of 25 financial asset (accounts) sold in month.
- Trade = some asset sold or some asset purchased.
- Infrequent and large trades.

# Statistics on annual number of total trades

Liquid Assets =	Median		Mean		Std Dev
	Checking	Broad	Checking	Broad	Checking
All trades ( $N_{Tj}$ )	3.4	3.4	4.5	4.5	3.7
Of which asset Sales ( $N_{Sj}$ )	1.4	1.4	2.0	2.0	2.0
Of which asset Purchases ( $N_{Pj}$ )	2.4	2.4	3.6	3.5	3.6
Stockholders ( $N_{Tj}$ ) (direct+indirect)	5.1	5.1	5.8	5.8	3.6
Stockholders ( $N_{Tj}$ ) (direct)	5.8	5.5	6.0	6.0	3.4

- Broad Measure of Liquid Asset = Checking + Time Deposits ( $\approx$  M2)

# Ratio of rebalancing trades on total trades

	Number of trades with some rebalancing			Number of trades with only rebalancing		
	Median	Mean	Std dev	Median	Mean	Std dev
Whole sample	0.13	0.18	0.21	0	0.017	0.07
Stockholders (direct+indirect)	0.21	0.25	0.22	0	0.022	0.09
Stockholders (direct)	0.21	0.25	0.22	0	0.018	0.08

- Trade some rebalancing : simultaneous sale and purchase of asset.
- Trade only rebalancing : value of sale = value purchase of assets.

▶ [back to table](#)

# Panel of bank account of 40,000 Unicredit investors

- Same panel data used for rebalancing but for 40,000 Investors
- Indicator of final approval of Mortgage (about 800 in 35 months)
- Compute fraction of asset sales prior and after obtaining mortgage.
- Compute average asset sales prior and after obtaining mortgage.
- **Prior to mortgage approval more frequent and larger sales of assets.**
- Robust to controls (Probit and Tobit regressions)

▶ back to test

Table: Timing of assets sales and house purchases [▶ back](#)

Regressor	Probit estimates for asset sale decision		Tobit estimates of size of asset sold	
	Coefficient	Standard error	Coefficient	Standard error
Obtained mortgage:				
$\alpha_0$ : current	0.070***	0.012	34413.8***	3889.2
$\alpha_1$ : lag 1	0.053***	0.012	26985.6***	4075.6
$\alpha_2$ : lag 2	0.029***	0.011	12338.9***	4437.1
$\alpha_3$ : lag 3	0.027***	0.011	14200.6***	4457.9
$\alpha_4$ : lag 4	0.030***	0.011	15696.9***	4489.7
$\alpha_{-1}$ : lead 1	-0.010	0.010	-6863.4	5103.3
$\alpha_{-2}$ : lead 2	0.003	0.010	3993.9	4868.3
$\alpha_{-3}$ : lead 3	-0.001	0.010	288.3	5015.2
Time dummies and				
$\beta$ : Investor total assets	1.51e-07***	1.24e-08	0.124***	0.006
$\lambda$ : Stockholder	0.094***	0.003	42174.19***	2294.0
N. observations	31247		31247	
Pseudo $R^2$	0.07		0.0164	

Table: Cash Management and Large Purchases in Austria

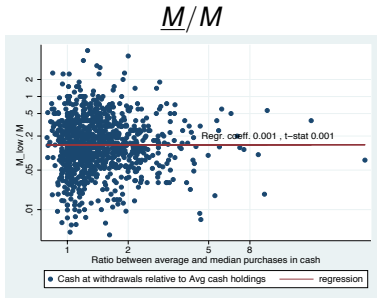
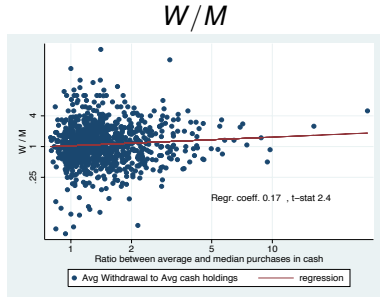
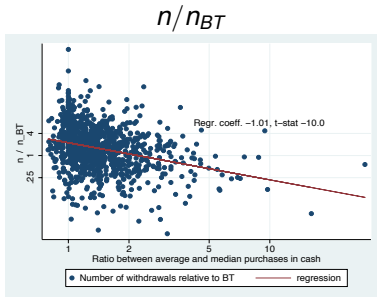
	All (1048 Obs)		w/ATM card (895 Obs.)		w/o ATM card (153 Obs.)	
Those that usually make large purchases (> 400 euros) in cash <sup>a</sup>						
% persons that use cash for large purchase	46%		37%		96%	
	mean	median	mean	median	mean	median
Withdrawal to Money: $W/M$	2.0	1.1	1.9	1.0	2.1	1.3
# withdrawals relative to BT <sup>b</sup> : $n/n_{BT}$	3.5	1.2	4.4	1.5	1.5	0.7
Normalized cash at withdrawals $n\bar{M}/M^c$	13.4	4.5	17.5	6.3	4.0	2.6
Those that usually do NOT make large purchases (> 400 euros) in cash <sup>a</sup>						
% persons that do not use cash for large purchase	54%		63%		4% (6 obs!!)	
	mean	median	mean	median	mean	median
Withdrawal to Money: $W/M$	1.6	1.0	1.5	1.0	10.4	2.1
# withdrawals relative to BT <sup>b</sup> : $n/n_{BT}$	5.9	1.9	6.0	1.9	0.9	0.7
Normalized cash at withdrawals $n\bar{M}/M^c$	20.6	7.8	20.8	8.0	3.8	2.9

- <sup>a</sup> Based on a question about how individual usually paid for items that cost more than 400 euros. Two options are available, either currency or other payment methods. Total number of respondents is 1048.

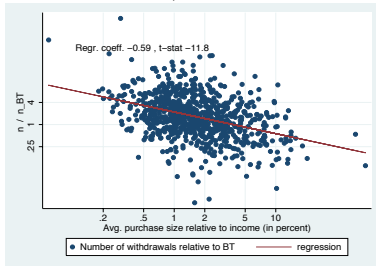
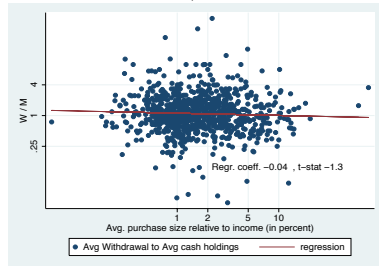
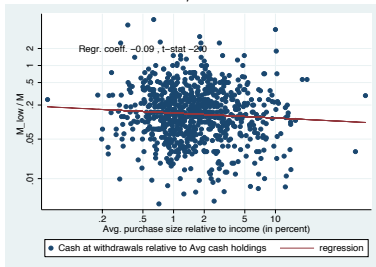
- <sup>b</sup> # of withdrawals  $n$  relative to Baumol-Tobin benchmark,  $n_{BT} = e/(2M)$  Based on a diary of all transactions during a week. This the week is right after the month corresponding to the question on large transactions above.

- <sup>c</sup> the variable  $n\bar{M}/M$  is the the product of the number of withdrawals  $n$  and the ratio of the average cash at the time of withdrawal,  $\bar{M}$  to the average cash holdings.



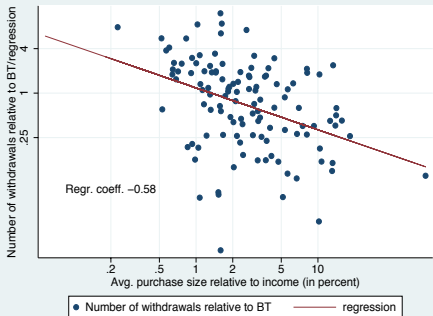
Figure: Austria: the inventory model statistics vs.  $e_a/e_m$ 

$$e_a/e_m = \text{avg} / \text{median purchase in cash}$$

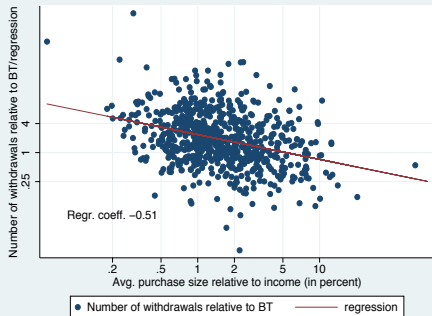
Figure: Austria: the inventory model statistics vs.  $e_a/y$  $n/n_{BT}$  $W/M$  $M/M$ 

$$e_a/y = \text{avg purchase} / \text{income}$$

## w/o ATM Card



## w. ATM card



# Solving the BE: recursion

- Bellman equation given  $\{m^*, m^{**}\}$  solves in each  $J$  segments:

- $V(m) = V_j(m)$  for  $m \in [z_j, \min\{z_{j+1}, m^{**}\}]$ ,  $j = 0, \dots, J-1$
- $V_j(m) = A_j + D_j(m - z_j) + \sum_{k=1,2} \sum_{i=0}^j B_{j,i}^k e^{\lambda_k(m-z_j)} (m - z_j)^i$
- $\lambda_k$  solves  $r + \rho + k = -c\lambda + \frac{\sigma^2}{2}\lambda^2$  for  $k = 1, 2$ .
- coefficients  $\{A_j, D_j, B_{j,i}^k\}_{j=0,1,2,\dots,J-1, i=1,\dots,j, k=1,2}$  solve block recursive system of linear equations.

- To solve for  $\{m^*, m^{**}\}$  use form of Bellman equation:

$$V'(m^*) = 0 \text{ and } V'(m^{**}) = 0 \text{ (necessary, but not sufficient conditions).}$$

# Currency vs Cons paid cash and Withdrawals

Dependent Variable $\log(M/c)$				
	<i>without ATM card</i>		<i>with ATM card</i>	
	bivariate	multivariate	bivariate	multivariate
$\log n$	-0.24***	-	-0.25***	-
$\log n$	-	-0.22***	-	-0.24***
	(900 obs.)	(900 obs.)	(2326 obs.)	(2325 obs.)
Dependent Variable $\log(W/c)$				
	<i>without ATM card</i>		<i>with ATM card</i>	
	bivariate	multivariate	bivariate	multivariate
$\log n$	-0.39***	-	-0.52***	-
$\log n$	-	-0.40***	-	-0.52***
	(2250 obs.)	(2249 obs.)	(1256 obs.)	(1255 obs.)

2004 SHIW and 2003 Unicredit surveys.  $M/c = M2 / \text{non-durable consumption}$ ; for the Unicredit consumption is imputed.  $-^a$ Regression coefficient of bivariate OLS.  $-^b$  controls (all in logs): household income, age, size.

Table: Temporal pattern of changes in the liquid and investments assets

Change in liquid asset in a month $\Delta M$		
Regressors	Coefficient	Standard Error
Flow of investment sales:		
$\beta_0$ : current	0.703***	0.0057
$\beta_1$ : lag 1	-0.23***	0.0062
$\beta_2$ : lag 2	-0.16***	0.0065
$\beta_3$ : lag 3	0.002	0.006
$\beta_4$ : lag 4	-0.03	0.0065
Flow of investment purchases:		
$\gamma_0$ : current	-0.65***	0.0065
$\gamma_1$ : lag 1	0.020***	0.007
$\gamma_2$ : lag 2	-0.076***	0.007
$\gamma_3$ : lag 3	0.056***	0.007
$\gamma_4$ : lag 4	-0.011**	0.006
Investor total assets:		
$\delta$ :	0.092***	0.0025
N. observations	31622	
$R^2$	0.47	



# Free withdrawal opportunities $\rho$

- We argue elsewhere (Alvarez-Lippi Eca 09) that:
  - Small and frequent withdrawal,  $\frac{W}{M} < 2$ ,  $\frac{n}{n_{BT}} > 1$  and
  - Substantial cash at hand at time of withdrawal,  $\frac{M}{M} \gg 0$
- Are consistent with introducing  $\rho$ :  
average free withdrawals opportunities per year.
- Agent withdraw everytime that it is free, regardless of level of cash, so:  
 $\underline{M} > 0$  and  $W/M < 2$ .



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 $M > 0$  and  $W/M < 2$ .
- We think this feature helps understand the "average" values, and the difference between those with and without ATM cards.
- Yet, we found other form of heterogeneity interesting,  
we explore if it is due to large purchases.

- Abel, Andrew B., Janice C. Eberly, and Stavros Panageas. 2007. "Optimal Inattention to the Stock Market." *American Economic Review* 97 (2):244–249.
- Alvarez, Fernando E., Luigi Guiso, and Francesco Lippi. 2011. "Durable consumption and asset management with transaction and observation costs." *American Economic Review*, forthcoming .
- Alvarez, Fernando E. and Francesco Lippi. 2009. "Financial Innovation and the Transactions Demand for Cash." *Econometrica* 77 (2):363–402.
- Bar-Ilan, A. 1990. "Overdrafts and the Demand for Money." *The American Economic Review* 80 (5):1201–1216.
- Bar-Ilan, A., D. Perry, and W. Stadje. 2004. "A generalized impulse control model of cash management." *Journal of Economic Dynamics and Control* 28 (6):1013–1033.
- Baumol, William J. 1952. "The transactions demand for cash: An inventory theoretic model." *Quarterly Journal of Economics* 66 (4):545–556.
- Constantinides, George and Scott F. Richard. 1978. "Existence of Optimal Simple Policies Discounted-Cost Inventory and Cash Management in Continuous Time." *Operations Research* 26 (4):620–636.
- Duffie, Darrell and Tong-sheng Sun. 1990. "Transactions costs and portfolio choice in a discrete-continuous-time setting." *Journal of Economic Dynamics and Control* 14 (1):35–51.

- Eppen, Gary D and Eugene F Fama. 1969. "Cash Balance and Simple Dynamic Portfolio Problems with Proportional Costs." *International Economic Review* 10 (2):119–33.
- Frenkel, Jacob A. and Boyan Jovanovic. 1980. "On transactions and precautionary demand for money." *The Quarterly Journal of Economics* 95 (1):25–43.
- Harrison, Michael and Michael I. Taskar. 1983. "Instantaneous control of Brownian motion." *Mathematics of Operations Research* 8 (2):439–453.
- Harrison, Michael J., Thomas M. Sellke, and Allison J. Taylor. 1983. "Impulse Control of Brownian Motion." *Mathematics of Operations Research* 8 (3):454–466.
- Miller, Merton and Daniel Orr. 1966. "A model of the demand for money by firms." *Quarterly Journal of Economics* 80 (3):413–435.
- Sulem, Agnes. 1986. "A Solvable One-Dimensional Model of a Diffusion Inventory System." *Mathematics of Operations Research* 11 (1):125–133.
- Telyukova, Irina A. 2009. "Household Need for Liquidity and the Credit Card Debt Puzzle." MPRA Paper 6674, University Library of Munich, Germany.
- Tobin, James. 1956. "The interest elasticity of transactions demand for money." *Review of Economics and Statistics* 38 (3):241–247.
- Whalen, E.L. 1966. "A rationalization of the precautionary demand for cash." *The Quarterly Journal of Economics* 80 (2):314–324.