

# Pairwise Trade, Payments, Asset Prices, and Monetary Policy\*

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November 17, 2008

## Abstract

We provide a monetary theory of asset returns that emphasizes the role that assets play as payments instruments. We adopt a search-theoretic model in which fiat money can coexist with real assets. The terms of trade in bilateral consumption matches are determined by a Pareto-efficient pricing mechanism. Our pricing mechanism replicates the liquidity constraints found in Kiyotaki and Moore (2005). We do not, however, place any restrictions on the use of assets as payments instruments. In an environment where agents are risk neutral, we show that fiat money can be valued, even though it is dominated in its rate of return, and that real assets can exhibit different rates of returns. In our model, an increase in inflation raises asset prices, lowers their returns and tends to widen the differences in rates of return of real assets, all of which is consistent with stylized facts. Finally, there is a range of inflation rates that can implement the first-best allocation.

*J.E.L. Classification:* E40, E50

*Keywords:* Search, money, bilateral trades, inflation, asset prices.

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\**PRELIMINARY DRAFT.* The views expressed herein are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

# 1 Introduction

Monetary policy makers seem to be interested in the impact that their actions have on financial markets, and financial market participants appear to closely follow the actions and utterings of policy makers. Or put another way, there is a widespread interest in understanding how inflation affects asset prices and the structure of assets' yields. Before we begin to address these interesting issues, however, a number of rather hard questions must be addressed. In particular, if inflation is a monetary phenomenon, why do people hold fiat money when interest-bearing assets also exist? Why does the difference between the rates of return on equity and risk-free government bonds appear to be abnormally high? And, why does the rate of return on risk-free government bonds appear to be abnormally low? We will argue that one can provide reasonable answers to all these questions when they are addressed within the context a monetary environment, where the "liquidity" of assets can be explicitly taken into account. In this paper, we provide a monetary theory of asset returns that emphasizes the role that assets play in facilitating payments. We then use this theory to study the implications of monetary policy (inflation) on asset returns.

We adopt the search-theoretic model of Lagos and Wright (2005), where anonymity and a lack of double coincidence of wants generates an explicit need for a medium of exchange. We also adhere to the Wallace (1996) dictum by placing no restrictions on what assets can and not be used as a means of payment. The dictum implies that both fiat money and real assets—which are "Lucas trees" that yield a flow of real dividends—can serve the role of media of exchange. We are able to generate the kind of rate of return differences across assets observed in reality by appealing to and generalizing the pricing mechanism of Zhu and Wallace (2007). This mechanism exploits an important feature of the model: If trade in consumption goods is required to be Pareto efficient, then the terms of trade are not uniquely pin downed.<sup>1</sup> So, for example, one can construct a Pareto-efficient pricing mechanism with the property that buyers obtain better terms of trade by using fiat money than interest-bearing assets in an environment where all agents behave rationally and use as many assets as is needed to exploit all the gains trade. Our pricing mechanism appears to replicate the liquidity constraint found in Kiyotaki and Moore (2005).<sup>2</sup> However, unlike Kiyotaki and Moore (2005) we do not impose such constraints in our environment; they arise endogenously.

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<sup>1</sup>Interestingly, the same feature of the labor search model is used by Hall (2005) to explain wage rigidity. See the discussion in Kocherlakota (2005).

<sup>2</sup>In an earlier version of their paper Zhu and Wallace (2007) called their pricing mechanism "cash-in-advance with a twist" because from the view point of the buyer, it is as if he faces a cash-in-advance constraint. To mirror this terminology, our pricing protocol could be called "Kiyotaki-Moore with a twist."

We have the following insights from our theory. First, fiat money can be held and valued despite being dominated in its rate of return by other assets. Our model can explain the so-called rate-of-return-dominance puzzle, (first pointed out in Hicks (1935)). Next, a monetary equilibrium will exist if the stock of the real assets and inflation are not too large. If real assets are sufficiently abundant, then an additional medium of exchange is not needed; if inflation is too high, then the cost of holding money balances outweigh its benefit. Hence, a monetary equilibrium is more likely to exist, the less liquid are real assets—less liquid in the sense that their terms of trade from a buyer’s perspective are much worse than that of fiat money. Finally, the real assets can exhibit different rates of returns even if they share the same risk characteristics or if agents are risk-neutral. These rate of return differences, which are anomalies from the standpoint of standard asset pricing theory (Mehra and Prescott (1985)), depend on the details of the pricing mechanism—which has implications for the implied liquidity of the assets—and monetary policy.

In Zhu and Wallace (2007), the non-monetary asset is completely illiquid; their pricing mechanism generates zero surplus to any consumer who uses the non-monetary to purchase the consumption good. For Zhu and Wallace, the price of the non-monetary asset is always equal to its intrinsic value, and is independent of the stance of monetary policy. In contrast, our pricing protocol admits a liquidity value for real assets since consumers are able to extract some surplus from trade when using real assets as media of exchange. Because of this liquidity effect, an increase in inflation will increase the price of real assets and will lower their returns. Our model predicts a negative relationship between inflation and assets’ returns, which is consistent with the data. The monetary transmission mechanism that is at work is quite simple. There is a one-to-one relationship between the cost of holding fiat money and inflation. As inflation increases agents want to reduce their (real) money holdings. Because of this, the liquidity value of all real assets increases, which raises their prices and lowers their returns. Furthermore, if different real assets have different liquidity properties, then, as inflation goes up, the demand for high liquidity assets will increase by more than the demand for the less liquid assets. So inflation tends to widen the rate of return differences between real assets. This prediction is also consistent with the positive relationship between inflation and the equity premium found in the data.

The Friedman rule maximizes a standard measure of social welfare and generates the first-best allocation. At the Friedman rule, all assets—including money—have the same rate of return. However, the optimal monetary policy is not unique. There exists a *range* of inflation rates that can implement the first-best

allocation. For these policies, the rates of return on assets are not equalized. While an increase of the inflation rate above the Friedman is neutral in terms of welfare, it does have redistributive consequences: the surpluses of buyers are reduced in bilateral meetings, while that of the sellers are increased.

## 1.1 Literature

Our paper is related to some recent macro literature that models a transaction role for assets. Much of this literature *assumes* the existence of either transactions costs or liquidity constraints in an attempt to explain asset pricing anomalies and the effects of monetary policy on asset returns. This particular literature does not adhere to the Wallace dictum. For example, Bansal and Coleman (1996) explain the risk-free rate and the equity premium puzzles in a pure exchange economy by assuming heterogeneous transactions costs that vary across assets, i.e., they assume that the costs associated with fiat money, government bonds or credit as a means of payments are different. Kiyoyaki and Moore (2005) consider an economy with two assets, land and capital, and assume that only a fraction of the capital stock can be used to as a means of payments to finance investment opportunities. In a similar vein, Lagos (2006), Shi (2006) and Lester, Postlewaite and Wright (2007), assume that an agent's non-monetary holdings can only be used in only a fraction of the meetings. In Telyukova and Wright (2008), credit can only be used in some markets, while in Aruoba and Wright (2003) and Aruoba, Waller and Wright (2006) capital cannot be used as means of payment in decentralized markets. A major difference between our model and any of the above is that we do not have to appeal to some sort of artificial liquidity restriction in order to explain rates of return anomalies.<sup>3</sup>

Even though our pricing mechanism endogenously generates what looks like a liquidity constraint that is found in the literature, the real allocations and welfare can be quite different from what they would be in an economy where an explicit liquidity constraint is imposed. For example, in all the papers mentioned so far, the Friedman rule is the unique optimal policy, and it eliminates the rate of return differentials between assets. In contrast, in our model has a range of inflation rates that are optimal, and the optimal policy can be associated with differences in rates of return between assets.

Finally, Lagos and Rocheteau (2006) and Geromichalos, Licari and Suarez-Lledo (2007) place no restrictions on the use of capital goods as means of payment. However—and not surprisingly—both models predict that capital and fiat money have the same rate of return. Moreover, in Geromichalos, Licari and

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<sup>3</sup>It should be pointed out that in the first part of his paper, Lagos (2006) does not appeal to liquidity constraints. But to better match the model with the data, he appeals to liquidity constraints later on.

Suarez-Lledo (2007), a necessary condition for money to be valued is that the inflation rate is negative.

## 2 The environment

Time is discrete and continues forever. The economy is populated with a  $[0, 1]$  continuum of infinitely-lived agents. As in Lagos and Wright (2005), each period is divided into two subperiods, called AM and PM. In the AM, trade takes place in decentralized markets, where agents are bilaterally matched in a random fashion. In the PM, trade takes place in competitive markets.

In the AM decentralized market, agents produce and consume perishable goods that come in different varieties. The probability that an agent is matched with someone who produces a good he wishes to consume is  $\alpha \leq 1/2$ . Symmetrically, the probability that an agent meets someone who consumes the good he produces is  $\alpha \leq 1/2$ . For convenience, and without loss of generality, we rule out double-coincidence-of-wants meetings. In the PM subperiod, all agents are able to consume and produce a perishable (general) good.<sup>4</sup>

An agent's utility function is

$$\mathbb{E} \left[ \sum_{t=0}^{\infty} \beta^t [u(y_t^b) - c(y_t^s) + x_t - h_t] \right]$$

where  $y^b$  is consumption and  $y^s$  is production of the AM good,  $x$  is consumption of the general good,  $h$  is hours of work to produce the general good, and  $\beta = (1 + r)^{-1} \in (0, 1)$  is the discount factor across periods. We assume that  $u(y) - c(y)$  is continuously differentiable, strictly increasing and concave. In addition,  $c(0) = u(0) = 0$ ,  $u'(0) = +\infty$ ,  $u'(+\infty) = 0$  and there exists a  $y^* < +\infty$  such that  $u'(y^*) = c'(y^*)$ . The technology to produce general good is linear and one-to-one in hours, i.e.,  $h$  hours of work produces  $h$  units of the general good in the PM.<sup>5</sup>

Agents are unable to commit and their trading histories are private information. This implies that credit arrangements are infeasible. The infeasibility of credit, in conjunction with the specialization of agents' consumption and production in the AM decentralized markets, generates a role for a medium of exchange.

There are two storable and perfectly divisible assets in the economy, and both can serve as media of exchange. There is a real asset that is in fixed supply,  $A > 0$ . In each PM subperiod, one unit of the real

<sup>4</sup>We could assume that the same goods which are traded in the AM decentralized market are also traded in the PM competitive market. However, the specialization in terms of preferences and technologies is irrelevant in a complete information, competitive environment.

<sup>5</sup>Following Lagos and Wright (2005), we could adopt a more general utility function in the PM,  $U(x) - h$  with  $U'' < 0$ . Our results would not be affected provided that the nonnegativity constraint for the number of hours,  $h \geq 0$ , is not binding in equilibrium.

asset generates a dividend equal to  $\kappa > 0$  units of the general good. There also exists an intrinsically useless asset called fiat money. The money supply grows at the gross rate  $\gamma > \beta$ , where  $\gamma \equiv \frac{M_{t+1}}{M_t}$ , via lump-sum transfers or taxes in the PM subperiod. In the AM subperiod, producers and consumers in a bilateral match can exchange the assets for one other or for the consumption good. In the PM subperiod, assets are traded with the general good in competitive markets. No restrictions are place on assets regarding their roles as media of exchange.

The asset prices at date  $t$  are measured in terms of the general good in the date  $t$  PM subperiod. The price of money is denoted by  $\phi_t$  and the price of the real asset is denoted by  $q_t$ . In what follows, we will focus our attention on stationary equilibria, where  $\phi_t M_t$  and  $q_t$  are constants.

### 3 Pricing

In this section, we describe the determination of the terms of trade in bilateral meetings in the AM decentralized market. Before we do this, however, it will be useful to show some properties of an agent's value function in the PM subperiod,  $W$ , since it tells us how agents will value the assets they give up or receive in the AM decentralized market. The value function of an agent entering the PM competitive markets holding a portfolio of  $a$  units of real asset and  $z$  units of real balances is,

$$W(a, z) = \max_{x, h, a', z'} \{x - h + \beta V(a', z')\} \quad (1)$$

$$\text{s.t. } \gamma z' + qa' + x = z + h + a(q + \kappa) + T, \quad (2)$$

where  $T \equiv \phi_t(M_t - M_{t-1})$ , measured in terms of the general good, is the lump-sum transfer associated with money injection. At the start of each PM subperiod, each unit of the real asset generates  $\kappa$  units of the general good. Competitive markets then open, where the real asset can be bought or sold at price  $q$  and money can be bought or sold at price  $\phi_t$ . In a PM subperiod each agent chooses his net consumption,  $x - h$ , and a portfolio,  $(a', z')$ , that he brings into the subsequent decentralized market. Each unit of real balance acquired in the PM subperiod of date  $t$  will turn into  $\frac{\phi_{t+1}}{\phi_t} = \gamma^{-1}$  units of real balances in date  $t + 1$ . Hence, if an agent wants  $z'$  units of real balances next period, he must acquire  $\gamma z'$  units in the current period.

Substituting  $x - h$  from (2) into (1) gives

$$W(a, z) = z + a(q + \kappa) + T + \max_{a', z'} \{-\gamma z' - qa' + \beta V(a', z')\}. \quad (3)$$

From (3), the PM value function is linear in the agent’s wealth: this property will prove especially convenient in terms of simplifying the pricing problem in the AM decentralized market. Note also that the choice of the agent’s new portfolio,  $(a', z')$ , is independent of the portfolio that he brought into the PM subperiod,  $(a, z)$ , as a consequence of quasi-linear preferences.

Consider now a match in the AM decentralized market between a buyer holding portfolio  $(a, z)$  and a seller holding portfolio  $(a^s, z^s)$ . The terms of trade are given by the output  $y \geq 0$  produced by the seller and the transfer of assets  $(\tau_m, \tau_a) \in [-z^s, z] \times [-a^s, a]$  from the buyer to the seller, where  $\tau_m$  is the transfer of real balances and  $\tau_a$  is the transfer of real assets. (If the transfer is negative, then the seller is delivering assets to the buyer.) The procedure that determines the terms of trade in the AM decentralized market generalizes the one suggested by Zhu and Wallace (2007). The procedure has two steps. The first step generates a payoff or surplus for the buyer, denoted as  $\hat{U}^b$ , which is equal to what he would obtain in a bargaining game if he had all the bargaining power, but was facing *liquidity constraints*. Specifically, in this “virtual game” it is assumed that the buyer can at most transfer a fraction  $\theta$  of his real asset holdings, i.e.,  $\tau_a \leq \theta a$ . (Zhu and Wallace (2007) assume that  $\theta = 0$ .) In terms of real balance transfers, the buyer cannot transfer more than he holds, i.e.,  $\tau_m \leq z$ .<sup>6</sup> The liquidity constraint on real asset holdings in the virtual game is chosen purposely to be reminiscent to the one used in Kiyotaki and Moore (2005), where individuals can only use a fraction of their capital goods to finance investment opportunities.<sup>7</sup> From the buyer’s standpoint, it is as if he was trading in the Kiyotaki-Moore economy. The liquidity constraint is also reminiscent of the constraint in Lagos (2006), where  $\theta = 0$  in a fraction of the matches and  $\theta = 1$  in the remaining matches. Note that the output and wealth transfers that the first step generates are “virtual” in the sense that they are simply used to determine the buyer’s surplus,  $\hat{U}^b$ , in the match. The *actual* output and wealth transfer is determined in the second step and it generates a pairwise Pareto-efficient trade. The actual trade maximizes the seller’s surplus subject to the constraint that the buyer receives a surplus at least equal to  $\hat{U}^b$ . The only restrictions that are placed on the transfer of either asset in the second step is that an agent cannot transfer more than he has, i.e.,  $-a^s \leq \tau_a \leq a$  and  $-z^s \leq \tau_m \leq z$ .

The first step of this pricing protocol—which determines the buyer’s surplus  $\hat{U}^b$ —solves the following

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<sup>6</sup>We do not constraint the transfer of asset holdings of the seller, but this is with no loss in generality.

<sup>7</sup>Kiyotaki and Moore (2005) consider an economy with two assets, capital and land. Land is in fixed supply while capital is accumulated. Both assets are inputs in the production of the final good. Individuals receive random opportunities to invest. In order to finance investment, they can use all their land—land is “completely liquid”—but only a fraction  $\theta$  of their capital holdings. So land is analogous to money in our formulation, while capital is similar to our real asset.

problem,

$$\begin{aligned} \hat{U}^b(a, z) &= \max_{y, \tau_m, \tau_a} [u(y) + W(a - \tau_a, z - \tau_m) - W(a, z)] \\ \text{s.t.} \quad &-c(y) + W(a^s + \tau_a, z^s + \tau_m) \geq W(a^s, z^s) \\ &\tau_m \in [-z^s, z], \quad \tau_a \in [-a^s, \theta a] \end{aligned}$$

The buyer in this virtual game maximizes his surplus, subject to the participation constraint of the seller and the constraints on the transfer of his asset holdings: while the buyer can transfer all his money balances, he can only hand over a fraction  $\theta$  of his real asset.<sup>8</sup> Using the linearity of  $W(a, z)$ , the above problem can be rewritten as

$$\hat{U}^b(a, z) = \max_{y, \tau_m, \tau_a} [u(y) - \tau_m - \tau_a(q + \kappa)] \quad (4)$$

$$\text{s.t.} \quad -c(y) + \tau_m + \tau_a(q + \kappa) \geq 0 \quad (5)$$

$$-z^s - a^s(q + \kappa) \leq \tau_m + \tau_a(q + \kappa) \leq z + \theta a(q + \kappa) \quad (6)$$

From this formulation, note that what matters is the total value of the transfer of assets,  $\tau_m + \tau_a(q + \kappa)$ , and not its composition in terms of money and real asset. Moreover, from the seller's participation constraint,  $y \geq 0$  requires that  $\tau_m + \tau_a(q + \kappa) \geq 0$ . Thus, the constraint that says that the seller cannot transfer more than his wealth is irrelevant. Therefore, the buyer's payoff is independent of  $(a^s, z^s)$ . From this, it is easy to see that the buyer's payoff is a function of only his "liquid wealth," the wealth he can use in the virtual game to maximize his payoff,  $z + \theta a(q + \kappa)$ . We now describe some of the properties of buyer's surplus function,  $\hat{U}^b(a, z)$ .

**Lemma 1** *The buyer's payoff is uniquely determined and satisfies,*

$$\hat{U}^b(a, z) = \begin{cases} u(y^*) - c(y^*) & \text{if } z + \theta a(q + \kappa) \geq c(y^*) \\ u \circ c^{-1}[z + \theta a(q + \kappa)] - z - \theta a(q + \kappa) & \text{otherwise} \end{cases} \quad (7)$$

*If  $z + \theta a(q + \kappa) < c(y^*)$ , then  $\hat{U}^b(a, z)$  is strictly increasing and strictly concave with respect to each of its arguments. Moreover,  $\hat{U}^b(a, z)$  is jointly concave (but not strictly) with respect to  $(a, z)$ .*

**Proof.** The solution to (4)-(6) is  $y = y^*$  and  $\hat{U}^b = u(y^*) - c(y^*)$  iff  $z + \theta a(q + \kappa) \geq c(y^*)$ ; otherwise,  $y = c^{-1}[z + \theta a(q + \kappa)]$  and  $(\tau_m, \tau_a) = (z, \theta a)$ .

<sup>8</sup>In principle,  $\hat{U}^b$  should also have  $z^s$  and  $a^s$  as arguments. Here we anticipate on the result according to which the terms of trade in this virtual game are independent of the seller's portfolio.

If  $z + \theta a(q + \kappa) < c(y^*)$ , then

$$\hat{U}_a^b = \frac{\partial \hat{U}^b}{\partial a} = \theta(q + \kappa) \left[ \frac{u'(\omega)}{c'(\omega)} - 1 \right] > 0 \quad (8)$$

$$\hat{U}_z^b = \frac{\partial \hat{U}^b}{\partial z} = \frac{u'(\omega)}{c'(\omega)} - 1 > 0, \quad (9)$$

where  $\omega = c^{-1}[z + \theta a(q + \kappa)]$ ;  $\hat{U}^b(a, z)$  is increasing with respect to each of its arguments. As well,

$$\begin{aligned} \hat{U}_{zz}^b &= \frac{u''(\omega)c'(\omega) - u'(\omega)c''(\omega)}{[c'(\omega)]^3} < 0 \\ \hat{U}_{za}^b &= \theta(q + \kappa) \left[ \frac{u''(\omega)c'(\omega) - u'(\omega)c''(\omega)}{[c'(\omega)]^3} \right] < 0 \\ \hat{U}_{aa}^b &= [\theta(q + \kappa)]^2 \left[ \frac{u''(\omega)c'(\omega) - u'(\omega)c''(\omega)}{[c'(\omega)]^3} \right] < 0; \end{aligned}$$

$\hat{U}^b(a, z)$  is strictly concave with respect to each of its arguments and  $\hat{U}_{aa}^b \hat{U}_{zz}^b - (\hat{U}_{za}^b)^2 = 0$ . Hence,  $\hat{U}^b(a, z)$  is jointly concave, but not strictly jointly concave. ■

The second step of the pricing protocol determines the seller's surplus,  $\hat{U}^s(a, z)$ , and the actual terms of trade,  $(y, \tau_m, \tau_a)$ , as functions of the *buyer's* portfolio in the match,  $(a, z)$ . By construction, the terms of trade are chosen so that the allocation is pairwise Pareto-efficient. The allocation solves the following problem,

$$\hat{U}^s(a, z) = \max_{y, \tau_m, \tau_a} [-c(y) + \tau_m + \tau_a(q + \kappa)] \quad (10)$$

$$\text{s.t. } u(y) - \tau_m - \tau_a(q + \kappa) \geq \hat{U}^b(a, z) \quad (11)$$

$$-z^s \leq \tau_m \leq z, \quad -a^s \leq \tau_a \leq a \quad (12)$$

Notice that in this problem, the use of the real asset as means of payment is not restricted. Moreover,  $\hat{U}^s(a, z) \geq 0$  since the allocation determined in the first step of the pricing protocol is still feasible in the second step. It is straightforward to characterize the solution to the seller's problem.

**Lemma 2** *If  $z + a(q + \kappa) \geq u(y^*) - \hat{U}^b(a, z)$ , then the terms of trade in bilateral meetings satisfy*

$$y = y^* \quad (13)$$

$$\tau_m + \tau_a(q + \kappa) = u(y^*) - \hat{U}^b(a, z); \quad (14)$$

otherwise,

$$y = u^{-1} \left[ z + a(q + \kappa) + \hat{U}^b(a, z) \right] \quad (15)$$

$$(\tau_a, \tau_m) = (a, z). \quad (16)$$

The seller's payoff and output are uniquely determined. The composition of the payment between money and the real asset is unique if the output is strictly less than the efficient level,  $y^*$ . If, however,  $z + a(q + \kappa) > u(y^*) - \hat{U}^b(a, z)$ , then there are a continuum of transfers  $(\tau_a, \tau_m)$  that achieve (14).

Consider the case where  $z + a(q + \kappa) < u(y^*) - \hat{U}^b(a, z)$ , the allocation depends on the composition of the buyer's portfolio. From (15)-(16) one can compute the quantity of output a buyer can acquire with an additional unit of wealth. If a buyer accumulates an additional unit of real balances, his consumption in the AM increases by

$$\frac{\partial y}{\partial z} = \frac{u'(\omega)}{c'(\omega)u'(y)},$$

where  $\omega = c^{-1}[z + \theta a(q + \kappa)]$  and, from the proof of Lemma 1,  $\partial \hat{U}^b / \partial z = u'(\omega) / c'(\omega) - 1$ . If a buyer accumulates an additional unit of the real asset, which promises  $q + \kappa$  units of output in the next PM, then

$$(q + \kappa)^{-1} \frac{\partial y}{\partial a} = \frac{1 + \theta \left[ \frac{u'(\omega)}{c'(\omega)} - 1 \right]}{u'(y)}.$$

If  $\theta < 1$ , then a claim of one unit of PM output buys more output in the AM if it takes the form of fiat money instead of the real asset. It can also be checked from Lemma 1 that

$$\frac{\partial \hat{U}^b(a, z)}{\partial a} = \theta(q + \kappa) \frac{\partial \hat{U}^b(a, z)}{\partial z}.$$

One unit of the real asset generates an increase of the buyer's surplus that is  $\theta$  times the one associated with  $q + \kappa$  units of real balances. In this sense,  $\theta$  is a measure of the liquidity of the real asset, i.e., its ability to buy AM output at favorable terms of trade.

In Figure 1 the determination of the terms of trade is illustrated. The surpluses of the buyer and the seller are denoted  $U^b$  and  $U^s$ , respectively. This figure is constructed assuming that the first-best level of output is incentive-feasible in the sense that  $z + a(q + \kappa) > c(y^*)$ . There are two Pareto frontiers: the lower (dashed) frontier corresponds to the pair of utility levels in the first step of the pricing protocol, where the buyer cannot spend more than a fraction  $\theta$  of his real asset *and he has insufficient liquid resources to purchase output  $y^*$* , i.e.,  $z + \theta a(q + \kappa) < c(y^*)$ . The upper frontier corresponds to the pair of utility levels in the second step of

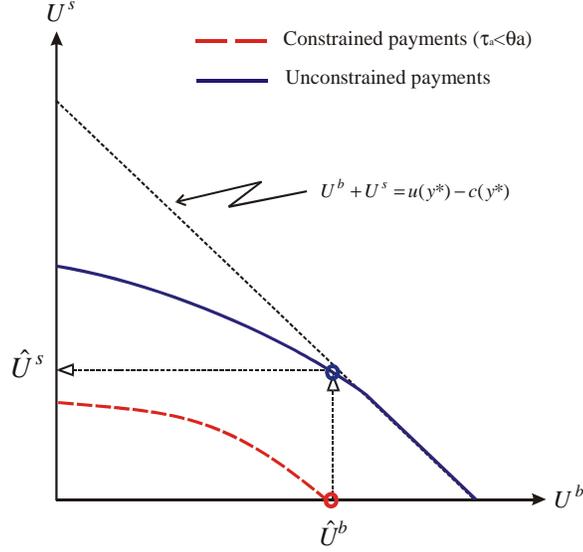


Figure 1: Pricing mechanism

the procedure, where payments are unconstrained, (it will also correspond to the first step of the procedure if the buyer has sufficient liquid resources to purchase output  $y^*$ , i.e., if  $z + \theta a(q + \kappa) \geq c(y^*)$ ). Along the linear portion of the upper frontier—moving in a north-west direction—output remains at  $y^*$  but the wealth transferred to the seller from the buyer increases. The linear portion “ends” when the wealth transference to the seller equals  $z + a(q + \kappa)$ ; beyond that point on the upper frontier, the seller’s surplus increases by having him receive all of the buyer’s wealth in exchange for producing successively smaller amounts of the consumption good. If the buyer has sufficient liquid resources to purchase  $y^*$ , then the seller’s surplus (in the two-stage pricing protocol) is zero and the buyer receives maximum surplus. This outcome is given the the intersection of the upper frontier and the  $x$ -axis in figure 1. If the buyer does not have sufficient liquid resources to purchase  $y^*$ , then in the first step he will offer his entire liquid portfolio,  $z + \theta a(q + \kappa)$ , the maximum amount of output that the seller is willing to supply,  $c^{-1}(z + \theta a(q + \kappa))$ . In Figure 1, the buyer’s surplus associated with such an offer is given by  $\hat{U}^b$ . In the second step, the seller’s surplus is chosen so that the agreement  $(\hat{U}^b, \hat{U}^s)$  lies on the upper frontier in Figure 1 so that the trade is Pareto efficient. Given the configuration of Figure 1, the seller will receive all of the buyer’s wealth and will produce a level of output  $y < y^*$  that provides the buyer with a surplus equal to  $\hat{U}^b$ .

## 4 Equilibrium

We incorporate the pricing mechanism, described in Section 3, in our general equilibrium model. Let  $y(a, z)$ ,  $\tau_a(a, z)$  and  $\tau_m(a, z)$ , represent the output and transfer outcomes from the pricing mechanism, when the buyer in the match has portfolio  $(a, z)$ . The value to the agent of holding portfolio  $(a, z)$  at the beginning of the AM subperiod,  $V(a, z)$ , is given by

$$\begin{aligned} V(a, z) &= \alpha \{u[y(a, z)] + W[a - \tau_a(a, z), z - \tau_m(a, z)]\} \\ &\quad + \alpha \mathbb{E} \{-c[y(\tilde{a}, \tilde{z})] + W[a + \tau_a(\tilde{a}, \tilde{z}), z + \tau_m(\tilde{a}, \tilde{z})]\} \\ &\quad + (1 - 2\alpha)W(a, z). \end{aligned} \tag{17}$$

With probability  $\alpha$ , the agent is the buyer in a match. He consumes  $y(a, z)$  and delivers the assets  $[\tau_a(a, z), \tau_m(a, z)]$  to the seller.<sup>9</sup> As established in Lemmas 1 and 2, the terms of trade  $(y, \tau_a, \tau_m)$  only depend on the portfolio of the buyer in the match.<sup>10</sup> With probability  $\alpha$  the agent is the seller in the match. He produces  $y$  and receives  $(\tau_a, \tau_m)$  from the buyer where  $(y, \tau_a, \tau_m)$  is a function of the buyer's portfolio  $(\tilde{a}, \tilde{z})$ . The expectation is taken with respect to  $(\tilde{a}, \tilde{z})$ , since the distribution of asset holdings might be non-degenerate, assuming that the seller's partner is chosen at random from the whole population of potential buyers. Finally, with probability  $1 - 2\alpha$  the agent is neither a buyer nor a seller. Using the linearity of  $W(a, z)$  and the expressions for the buyer's and the seller's surpluses, (17) can be rewritten as

$$V(a, z) = \alpha \hat{U}^b(a, z) + \alpha \mathbb{E} \hat{U}^s(\tilde{a}, \tilde{z}) + W(a, z). \tag{18}$$

Notice that if the agent was living in an economy with exogenous liquidity constraints, where he can only transfer a fraction  $\theta$  of his real asset holdings—as in Kiyotaki and Moore (2005) or Lagos (2006)—then the main difference would be that  $\mathbb{E} \hat{U}^s(\tilde{a}, \tilde{z}) = 0$ , assuming that the buyer makes a take-it-or-leave-it-offers. Consequently, the buyer's choice of portfolios in both economies would be similar.

Substituting  $V(a, z)$ , as given by (18), into (3), and simplifying, an agent's portfolio  $(a, z)$  solves:

$$(a, z) = \arg \max_{a, z} \left\{ -\gamma z - qa + \beta \left[ \alpha \hat{U}^b(a, z) + z + a(q + \kappa) \right] \right\}. \tag{19}$$

<sup>9</sup>Recall from Lemma 2 that even though the terms of trade  $(\tau_a, \tau_m)$  may not be uniquely determined, the total transfer of wealth,  $\tau_m + \tau_a(\kappa + q)$ , is unique.

<sup>10</sup>For given  $(a, z)$  the transfer  $(\tau_a, \tau_m)$  might not be unique but this indeterminacy is irrelevant for the payoffs of the buyer and the seller.

The portfolio is chosen so as to maximize the expected discounted utility of the agent if he happens to be a buyer in the next AM market minus the effective cost of the portfolio. The cost of holding an asset is its purchase price minus its discounted resale price plus its dividend. Rearranging (19), it simplifies further to

$$(a, z) = \arg \max_{a, z} \left\{ -iz - ar(q - q^*) + \alpha \hat{U}^b(a, z) \right\}, \quad (20)$$

where  $i \equiv \frac{\gamma - \beta}{\beta}$  represents the cost of holding real balances,  $r(q - q^*)$  is the cost of holding the real asset where  $q^* \equiv \frac{\kappa}{r}$  is the discounted sum of the real asset's dividends. The first-order (necessary and sufficient) conditions for this (concave) problem are:

$$-i + \alpha \left[ \frac{u'(\omega)}{c'(\omega)} - 1 \right]^+ \leq 0, \quad \text{"=" if } z > 0 \quad (21)$$

$$-r(q - q^*) + (q + \kappa)\theta\alpha \left[ \frac{u'(\omega)}{c'(\omega)} - 1 \right]^+ \leq 0, \quad \text{"=" if } a > 0 \quad (22)$$

where  $[x]^+ = \max(x, 0)$  and  $\omega = c^{-1}[z + \theta a(q + \kappa)]$ . The term  $\left[ \frac{u'(\omega)}{c'(\omega)} - 1 \right]^+$  in (21) represents the liquidity return of real balances, i.e., the increase in the buyer's surplus from holding an additional unit of money. From (22), the liquidity return of  $\frac{1}{q + \kappa}$  units of the real asset is  $\theta$  times the liquidity return of real balances.

Finally, the asset price is determined by the market clearing condition

$$\int_{[0,1]} a(j) dj = A, \quad (23)$$

where  $a(j)$  is the asset choice of agent  $j \in [0, 1]$ .

**Definition 1** *An equilibrium is a list  $\{[a(j), z(j)]_{j \in [0,1]}, [y(a, z), \tau_a(a, z), \tau_m(a, z)], q\}$  that satisfies (13)-(16), (20) and (23). The equilibrium is monetary if  $z(j) > 0$  for some  $j$ .*

Consider first a nonmonetary equilibrium, where the real asset is the only means of payment in the AM market.<sup>11</sup> In this case,  $z(j) = 0$  for all  $j$ .

**Proposition 1** *There is a nonmonetary equilibrium such that  $q \in [q^*, +\infty)$ .*

(i) *If  $\theta = 0$ , then  $q = q^*$ .*

(ii) *If  $\theta > 0$ , then  $q$  is the unique solution to*

$$\frac{q^* + \kappa}{q + \kappa} + \frac{\theta\alpha}{r} \left[ \frac{u' \circ c^{-1}[\theta A(q + \kappa)]}{c' \circ c^{-1}[\theta A(q + \kappa)]} - 1 \right]^+ = 1. \quad (24)$$

*Finally, if  $\theta A(q^* + \kappa) \geq c(y^*)$ , then  $q = q^*$ ; otherwise  $q > q^*$ .*

<sup>11</sup>As is standard in models of money, there always exists a non-monetary equilibrium, where money is not valued.

**Proof.** Define the demand correspondence for the real asset as

$$A^d(q) = \left\{ \int_{[0,1]} a(j) dj : a(j) = \arg \max \left\{ -ar(q - q^*) + \alpha \hat{U}^b(a, 0) \right\} \right\}. \quad (25)$$

The market clearing condition (23) can then be re-expressed as  $A \in A^d(q)$ . First, suppose  $\theta = 0$ . From (7)  $\hat{U}^b(a, 0) = 0$  for all  $a \geq 0$ . Then,  $A^d(q) = \{0\}$  for all  $q > q^*$  and  $A^d(q^*) = [0, +\infty)$ . Consequently, the unique solution to  $A \in A^d(q)$  is  $q = q^*$ . Note that the agent's problem (25) has no solution if  $q < q^*$ .

Next, suppose  $\theta > 0$ . In order to characterize  $A^d(q)$  we distinguish three cases:

1. If  $q > q^*$  then  $A^d(q) = \{a\}$  where  $a < \frac{c(y^*)}{\theta(q+\kappa)}$  is the unique solution to

$$r(q - q^*) = \alpha \hat{U}_a^b(a, 0). \quad (26)$$

To see, recall from Lemma 1 that  $\hat{U}^b(a, 0)$  is strictly concave with respect to its first argument over the domain  $\left[0, \frac{c(y^*)}{\theta(q+\kappa)}\right]$ ,  $\hat{U}_a^b(0, 0) = \infty$  and  $\hat{U}_a^b(a, 0) = 0$  for all  $a \geq \frac{c(y^*)}{\theta(q+\kappa)}$ . Substituting  $\hat{U}_a^b$  by its expression given by (8) into (26) and rearranging, one obtains

$$\frac{q^* + \kappa}{q + \kappa} + \frac{\theta\alpha}{r} \left[ \frac{u' \circ c^{-1}[\theta a(q + \kappa)]}{c' \circ c^{-1}[\theta a(q + \kappa)]} - 1 \right] = 1. \quad (27)$$

Note that the left-hand side is strictly decreasing in both  $q$  and  $a$ ; hence,  $a$  is strictly decreasing in  $q$ . So, for all  $q > q^*$ ,  $A^d(q)$  is single-valued and strictly decreasing. Moreover, as  $q \rightarrow q^*$ ,  $a \rightarrow \frac{c(y^*)}{\theta(q^*+\kappa)}$  and as  $q \rightarrow \infty$ ,  $a \rightarrow 0$ .

2. If  $q = q^*$ , then  $A^d(q^*) = \operatorname{argmax}_{a \geq 0} \left\{ \hat{U}^b(a, 0) \right\} = \left[ \frac{c(y^*)}{\theta(q^*+\kappa)}, +\infty \right)$ .
3. If  $q < q^*$ , then the agent's problem has no solution.

In summary,  $A^d(q)$  is upper hemi-continuous over  $[q^*, \infty)$  and its range is  $[0, \infty)$ . Hence, a solution  $A \in A^d(q)$  exists. Furthermore, any selection from  $A^d(q)$  is strictly decreasing in  $q \in [q^*, \infty)$ , so there is a unique  $q$  such that  $A \in A^d(q)$ . If  $A \geq \frac{c(y^*)}{\theta(q^*+\kappa)}$  then  $A \in A^d(q)$  implies  $q = q^*$ . If  $A < \frac{c(y^*)}{\theta(q^*+\kappa)}$  then  $A \in A^d(q)$  implies that  $q$  solves (27) with  $a = A$ , i.e., (24). ■

If  $\theta = 0$ , then the real asset is fully illiquid in the sense that holding the asset does not allow the buyer to extract a surplus from his trade in the decentralized AM. In this case, the asset is priced at its fundamental value,  $q = q^*$ . Notice that it does not imply that the asset is not used as a medium of exchange in bilateral trades.

In contrast, if  $\theta > 0$  then the buyer can obtain a positive surplus from holding the asset in the decentralized market. If either the intrinsic value of the stock of the real asset,  $A(q^* + \kappa)$ , is sufficiently high, or if it is not too illiquid—i.e.,  $\theta$  is not too low—then the buyer can extract the entire surplus of the match. An additional unit of asset does not affect the buyer’s trade surplus in the AM. This implies that the asset has no liquidity value—an additional unit of the asset does not increase surplus—and its price corresponds to its fundamental price,  $q = q^* = \kappa/r$ . In this case, the distribution of asset holdings is not uniquely determined. But this indeterminacy is payoff irrelevant since the output traded in all matches in the AM is  $y^*$ .

In contrast, if either the intrinsic value of the asset is low or the asset is very illiquid—i.e.,  $\theta$  is low but positive—then the price of the asset raises above its fundamental value. Here, an additional unit of the asset will increase buyer surplus in the AM. In this case, the equilibrium and the distribution of asset holdings—which is degenerate—are unique.

It is rather interesting, and important, to note from (24) that the allocation can be socially efficient,  $y = y^*$ , even when  $q > q^*$ . This can happen since asset prices are a function of the buyer’s *liquid* portfolio,  $\theta a(q + \kappa)$ . So it can be the case that the value of the buyer’s liquid portfolio is insufficient value to purchase  $y^*$  in the decentralized AM subperiod—i.e.,  $\theta a(q + \kappa) < c(y^*)$ —but the total value of the buyer’s portfolio can be sufficient to support an output of  $y^*$ .

Let’s now turn to monetary equilibria.

**Proposition 2** *There exists a monetary equilibrium iff*

$$A < \frac{(r - i\theta)}{\theta\kappa(1 + r)}\ell(i) \quad (28)$$

where  $\ell(i)$  is unique and implicitly defined by

$$1 + \frac{i}{\alpha} = \frac{u' \circ c^{-1}(\ell)}{c' \circ c^{-1}(\ell)}. \quad (29)$$

In a monetary equilibrium, the asset price is uniquely determined by

$$q = \frac{\kappa(1 + i\theta)}{r - i\theta} \geq q^*. \quad (30)$$

**Proof.** With a slight abuse of notation, define  $\hat{U}^b(\ell) = \hat{U}^b(a, z)$  where  $\ell = z + \theta a(q + \kappa)$ . (Notice from Lemma 1 that  $(a, z)$  matters for the buyer’s payoff only through  $z + \theta a(q + \kappa)$ .) Then, the agent’s portfolio problem (20) can be re-expressed as

$$(a, \ell) = \arg \max_{a, \ell} \left\{ -i\ell - a[(r - i\theta)q - \kappa(1 + i\theta)] + \alpha \hat{U}^b(\ell) \right\} \text{ s.t. } \theta a(q + \kappa) \leq \ell. \quad (31)$$

Define the demand correspondence for the real asset as

$$A^d(q) = \left\{ \int_{[0,1]} a(j) dj : \exists \ell \text{ s.t. } [a(j), \ell] \text{ is solution to (31)} \right\}.$$

The market-clearing condition (23) requires  $A \in A^d(q)$  for some  $q$ .

In order to characterize  $A^d(q)$  we distinguish three cases:

1. If  $(r - i\theta)q > \kappa(1 + i\theta)$  then, from (31),  $A^d(q) = \{0\}$ .
2. If  $(r - i\theta)q < \kappa(1 + i\theta)$ , then constraint  $\theta a(q + \kappa) \leq \ell$  must bind and  $z = 0$ , i.e., the equilibrium is non-monetary. Hence, from the proof of Proposition 1,  $A^d(q^*) = [\frac{c(y^*)}{\theta(q^* + \kappa)}, \infty)$  and, for all  $q \in (q^*, \frac{\kappa(1+i\theta)}{r-i\theta})$ ,  $A^d(q) = \{a\}$  where  $a$  solves

$$r(q - q^*) = \alpha\theta(q + \kappa) \left[ \frac{u' \circ c^{-1}[\theta a(q + \kappa)]}{c' \circ c^{-1}[\theta a(q + \kappa)]} - 1 \right]. \quad (32)$$

3. If  $(r - i\theta)q = \kappa(1 + i\theta)$ , then  $A^d(q) = [0, \frac{\ell}{\theta(q + \kappa)}]$  where, from (31),  $\ell$  solves (29).

In conjunction with (29), it can be checked that the solution to (32) for  $a$  approaches  $\frac{\ell}{\theta(q + \kappa)}$  as  $q \nearrow \frac{\kappa(1+i\theta)}{r-i\theta}$ . Hence,  $A^d(q)$  is upper hemi-continuous on  $[q^*, +\infty)$  with range  $(0, +\infty)$ . Moreover, any selection from  $A^d(q)$  is strictly decreasing. Therefore, there is a unique  $q \in [q^*, +\infty)$ , such that  $A \in A^d(q)$ . Moreover, if  $r > i\theta$  then  $q \in [q^*, \frac{\kappa(1+i\theta)}{r-i\theta}]$ . See Figure 2.

A monetary equilibrium exists if  $z(j) > 0$  for a positive measure of agents. From the discussion above, a monetary exists if and only if  $r > i\theta$  and  $A < \frac{\ell(i)}{\theta(q + \kappa)}$  where  $\ell(i)$  is defined by (29) and  $q = \frac{\kappa(1+i\theta)}{r-i\theta}$ , i.e., equation (30). This gives (28). ■

Since the right-hand side of (28) is decreasing in  $i$ , by taking the limit as  $i$  approaches 0 we obtain the following necessary condition for the existence of a monetary equilibrium:  $A < \frac{c(y^*)}{\theta(q^* + \kappa)}$ . If this condition holds, (28) can be restated as  $i < i_0$ , where  $i_0$  is the unique solution to<sup>12</sup>

$$\frac{(r - i_0\theta)}{\theta\kappa(1+r)}\ell(i_0) = A. \quad (34)$$

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<sup>12</sup>In a monetary equilibrium,  $\ell(i) > A \frac{\theta\kappa(1+r)}{r-i\theta}$  or, from (29),

$$1 + \frac{i}{\alpha} < \frac{u' \circ c^{-1}\left(A \frac{\theta\kappa(1+r)}{r-i\theta}\right)}{c' \circ c^{-1}\left(A \frac{\theta\kappa(1+r)}{r-i\theta}\right)}. \quad (33)$$

Since the left-hand side is increasing and continuous in  $i$  and the right-hand side is decreasing and continuous in  $i$ , there exists a unique  $i_0$  such that (33) holds at equality, which implies that for any  $i < i_0$ , condition (33) holds and, hence, a monetary equilibrium exists.

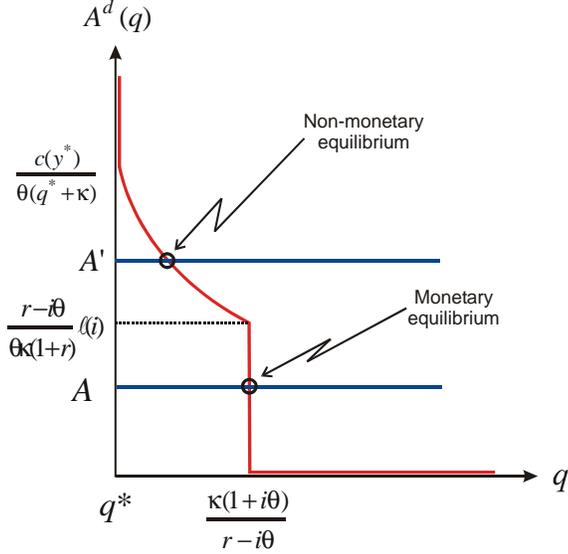


Figure 2: Asset demand correspondence

Thus, a monetary equilibrium exists whenever the asset price in the nonmonetary equilibrium is greater than its fundamental value, provided that the inflation rate is not too large. If money is valued, then the asset price is still greater than its fundamental value if  $\theta i > 0$ , the real asset is not fully illiquid and the cost of holding real balances is positive.<sup>13</sup>

The determination of the equilibrium is characterized in Figure 2. The price  $q$  is at the intersection of the constant supply,  $A$ , and the downward-sloping demand,  $A^d(q)$ . It is uniquely determined. The equilibrium is monetary whenever the supply intersects the demand in its vertical portion.

A rather important result from above is that, if  $A < \frac{c(y^*)}{\theta(q^* + \kappa)}$ , then there always exists a  $i > 0$ , such that an monetary equilibrium exists. Diagrammatically speaking, it is easy to see this result. Suppose that  $A < \frac{c(y^*)}{\theta(q^* + \kappa)}$ , but  $A > \frac{r - \theta i}{\theta \kappa (1 + r)} \ell(i)$  for a given value of  $i$ ; in figure 2, let  $A = A'$ . This implies that at the current inflation rate, the equilibrium is non-monetary, as  $A'$  intersects the strictly downward portion of  $A^d(q)$ , see figure 2. However, since  $\frac{\kappa(1 + \theta i)}{r - \theta i}$  is increasing in  $i$ , by decreasing  $i$  from its current level, the

<sup>13</sup>If the AM trading mechanism is chosen optimally, money is not essential whenever  $A(q^* + \kappa) \geq c(q^*)$ . In our model, however, money can improve matters when  $A(q^* + \kappa)/c(q^*) \in (1, 1/\theta)$ . However, our objective here is not to design an optimal trading mechanism. Rather, it is to consider the implications for asset prices and monetary policy of a particular mechanism that does not constrain the use of assets in payments, and that is (pairwise) Pareto efficient. The same comment would apply if we would use the generalized Nash solution provided that buyers do not have all the bargaining power: fiat money could be valued despite being inessential. See, e.g., Geromichalos, Licari and Suarez-Lledo (2007) or Lester, Postlewaite and Wright (2007).

vertical portion of  $A^d(q)$  will “move” to the left. If  $i$  is decreased sufficiently,  $A'$  will intersect the vertical portion of  $A^d(q)$ .

Since in a monetary equilibrium the “liquid wealth” of the buyer,  $\ell(i)$ , is uniquely determined, the buyer’s payoff in the decentralized market is unique. However, there are infinitely many ways to combine  $a$  and  $z$  to obtain a given  $\ell(i)$ . As a consequence, the terms of trade  $(y, \tau_a, \tau_z)$  and a seller’s payoff need not be unique. So, even though the asset price is unique, the real allocation may be indeterminate. In the following we restrict our attention to symmetric steady-state equilibria so that the equilibrium is unique. All agents hold  $A$  of the real asset and  $z(i) = \ell(i) - \theta A(q + \kappa)$  real balances.

## 5 Asset prices and monetary policy

Let us turn to asset pricing considerations and the implications for monetary policy. We have established in Proposition 2 that in any monetary equilibrium the asset price satisfies  $q = \frac{\kappa(1+\theta i)}{r-\theta i}$ , which can be rewritten as

$$q = q^* + \theta i \frac{q^* + \kappa}{r - \theta i}. \quad (35)$$

If  $\theta = 0$ , then the asset price is equal to its fundamental value, and it is unaffected by the money growth rate. This is the result of Zhu and Wallace (2007). In contrast, if the real asset is at least partially liquid, i.e.,  $0 < \theta \leq 1$ , then the asset price is above its fundamental value.

We define the liquidity premium of the asset as the difference between  $q$  and  $q^*$ . From (35), it is equal to

$$\mathcal{L} = \theta i \frac{q^* + \kappa}{r - \theta i}. \quad (36)$$

This liquidity premium arises because (an additional unit of) the real asset allows the buyer in a bilateral match in the AM to capture some (additional) gains from trade. The real asset has value as a means of payment from the buyer’s standpoint.

Monetary policy affects the asset price through this liquidity premium. The asset price increases with inflation, i.e.,  $\partial q / \partial i > 0$ . As the cost of holding money gets higher, agents will attempt to reduce their real balance holdings in favor of the real asset. This result reflects the fact that money and the real asset are substitutes as means of payment in bilateral meetings. The price of the real asset will, therefore, increase. As the cost of holding real balances is driven to zero,  $i \rightarrow 0$ , then the liquidity premium vanishes and the asset price approaches its fundamental value. In this limiting case, agents can use fiat money to extract all

the gain from trade in the decentralized AM market, and hence the real asset has no extra value beyond the one generated by its dividend stream.

In a monetary equilibrium, the gross rate of return of the real asset is  $R = \frac{q+\kappa}{q}$  or, from (35),

$$R = \frac{1+r}{1+\theta i}. \quad (37)$$

From (37), the rate of return of the asset depends on preferences,  $r$ , monetary policy,  $i$ , and the characteristics of the pricing mechanism,  $\theta$ .

**Proposition 3** *In any monetary equilibrium,  $R \leq 1+r$ , with a strict inequality if  $\theta i > 0$ . Moreover,  $\partial R/\partial i < 0$  and  $\partial R/\partial \theta < 0$ .*

**Proof.** Immediate from equation (37). ■

If  $\theta i = 0$  then the liquidity premium of the asset is 0 and hence its rate of return is equal to the rate of time preference. In contrast, if  $\theta i > 0$  then the asset price exhibits a liquidity premium and its rate of return is smaller than the rate of time preference. The model predicts a negative correlation between the rate of return of the real asset and inflation. As inflation increases, agents substitute the real asset for real money balances and, as a consequence, the asset price increases and its return decreases.<sup>14</sup> The rate of return of the asset also decreases with its liquidity as captured by  $\theta$  for much the same reason: as the liquidity of the asset, increases, the value of the asset for transactions purposes increases. Hence, the asset price increases and its return decreases.

The absolute value of the elasticity of the asset rate of return with respect to  $i$  is given by

$$\eta_{R/i} = \frac{\partial R/R}{\partial i/i} = \frac{\theta i}{1+\theta i}.$$

This elasticity is less than one and is increasing with  $\theta$ . Hence, if the asset becomes more liquid, as measured by an increase in  $\theta$ , its return becomes more sensitive to inflation. Also, in high-inflation environments the rate of return of the asset is more sensitive to changes in monetary policy.

Let's now turn to the rate of return differential between the real asset and money. Since the gross rate of return of fiat money is  $\gamma^{-1}$ , the rate of return differential is

$$R - \gamma^{-1} = \frac{1}{\gamma} \left( \frac{1+i}{1+\theta i} - 1 \right). \quad (38)$$

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<sup>14</sup>This finding is in accordance with the empirical evidence. See, e.g., Marshall (1992).

**Proposition 4** *In any monetary equilibrium, the real asset dominates money in its rate of return iff  $i > 0$  and  $\theta < 1$ .*

**Proof.** Immediate from equation (38). ■

If  $\theta = 1$ , as in Lagos and Rocheteau (2006) or Geromichalos, Licari and Suarez-Lledo (2007), the model is unable to explain the rate of return differential between money and the real asset. Since both capital and money are “equally liquid,” in order for the two media of exchange to coexist, they must have the same rate of return. As well, since  $R \geq 1$ , a monetary equilibrium cannot exist if inflation is positive, i.e., if  $\gamma > 1$ . Both of these results are counterfactual.

In our model, if  $i = 0$ , then fiat money and the real asset will have the same rate of return. By running the Friedman rule, the monetary authority can satiate agents’ need for liquidity, in which case the rate of return of the real asset is equal to the rate of time preference—since the asset has no value as a medium of exchange—which is also the rate of return of fiat money.

However, if  $\theta < 1$  and  $i > 0$ , then our model delivers a rate of return differential between the real asset and money. For given  $i$ , this differential decreases with  $\theta$ : as  $\theta$  increases, the value of the real asset increases owing to its increased benefit as a medium of exchange. As a result, its rate of return declines. One can relate the rate of return differential and the elasticity of the asset return with respect to inflation, i.e.,

$$R - \gamma^{-1} = \frac{(1+i)(1 - \eta_{R/i}) - 1}{\gamma}.$$

There is a negative relationship between the rate of return differential and the elasticity of the asset rate of return with respect to inflation.

We conclude this section by investigating the optimal monetary policy.

**Proposition 5** *Assume  $\theta A(q^* + \kappa) < c(y^*)$  and  $0 < \theta < 1$ . Then there is  $\hat{i} > 0$  such that for all  $i \in [0, \hat{i}]$ ,  $y = y^*$  at the symmetric monetary equilibrium.*

**Proof.** From Proposition 2, since  $\theta A(q^* + \kappa) < c(y^*)$ , there is a symmetric monetary equilibrium, provided that  $i < i_0$  (where  $i_0$  is defined in (34)). Moreover,  $q(i) = \frac{\kappa(1+\theta i)}{(r-\theta i)}$ ,  $\ell(i)$  solution to  $1 + \frac{i}{\alpha} = \frac{u'_{oc^{-1}}(\ell)}{c'_{oc^{-1}}(\ell)}$  and  $z(i)$  solution to

$$z(i) = \ell(i) - \theta A \frac{\kappa(1+r)}{(r-\theta i)} \tag{39}$$

are all continuous in  $i$ . Define

$$\Gamma(i) \equiv A(q(i) + \kappa) + z(i) + \hat{U}^b[\ell(i)] - u(y^*).$$

The function  $\Gamma(i)$  is continuous over  $[0, i_0)$  and, from Lemma 2,  $y = y^*$  whenever  $\Gamma(i) > 0$ . Substitute  $z(i)$  by its expression given by (39) to get

$$\Gamma(i) \equiv (1 - \theta)A \left( \frac{\kappa(1+r)}{r - \theta i} \right) + \ell(i) + \hat{U}^b[\ell(i)] - u(y^*).$$

As  $i \rightarrow 0$ ,  $\ell(i) \rightarrow c(y^*)$  and  $\hat{U}^b[\ell(i)] \rightarrow u(y^*) - c(y^*)$ . Since  $\theta < 1$ ,

$$\lim_{i \rightarrow 0} \Gamma(i) = (1 - \theta)A \frac{\kappa(1+r)}{r} > 0$$

By continuity, there exists a nonempty interval  $[0, \hat{i}]$  such that  $\Gamma(i) > 0$ . ■

In most monetary models with a single asset, the Friedman rule is optimal and it achieves the first best (provided that there are no externalities, no distortionary taxes, and the pricing is well-behaved); we have that result too.<sup>15</sup> However, in contrast to standard monetary models, a small deviation from the Friedman rule is neutral in terms of welfare in our model. Hence, a small inflation is (weakly) optimal. The only effect of increasing the inflation above the Friedman rule is to increase asset prices.

This finding has the following implications. First, a small inflation will have no welfare cost. Hence, we conjecture that moderate inflation (let say, 10 percent) will have a lower welfare cost than in standard models. Second, even though asset prices respond to monetary policy, these movements do not correspond to changes in society's welfare. Hence, asset prices may not be a very good indicator of society's welfare or monetary policy effectiveness. Third, since there is a range of inflation rate that generates the first-best allocation, the optimal monetary policy is consistent with a rate of return differential between fiat money and the real asset.

## 6 Liquidity structure of asset yields

In this section, we extend the model to allow for multiple real assets. We will show that the same model that can explain the rate of return dominance puzzle—that fiat money has a lower rate of return than risk-free

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<sup>15</sup>In search monetary models, the Friedman rule can be suboptimal because of search externalities (Rocheteau and Wright, 2005) or distortionary taxes (Aruoba and Chugh, 2007). Also, if the coercive power of the government is limited then the Friedman rule might not be incentive-feasible (Andolfatto, 2007).

bonds—can also deliver a non-degenerate distribution of assets' yields despite agents being risk neutral. We will investigate how this structure of yields is affected by monetary policy.

Suppose that there are a finite number  $K \geq 1$  of infinitely-lived real assets indexed by  $k \in \{1, \dots, K\}$ . Denote  $A_k > 0$  as the fixed stock of the asset  $k \in \{1, \dots, K\}$ ,  $\kappa_k$  its expected dividend, and  $q_k$  its price. Agents learn the realization of the dividend of an asset at the beginning of the PM centralized market. Consequently, the terms at which the asset is traded in the AM decentralized market only depend on the expected dividend  $\kappa_k$ . Moreover, since agents are risk-neutral with respect to their consumption in the PM centralized market, the risk of an asset has no consequence for its price.<sup>16</sup>

Consider a buyer in the AM with a portfolio  $(\{a_k\}_{k=1}^K, z)$ , where  $a_k$  is the quantity of the  $k^{\text{th}}$  real asset. The pricing mechanism is a straightforward generalization of the one studied in the previous sections. The buyer's payoff is given by,

$$\hat{U}^b = \max_{y, \tau_m, \{\tau_k\}} \left[ u(y) - \tau_m - \sum_{k=1}^K \tau_k (q_k + \kappa_k) \right] \quad (40)$$

$$\text{s.t.} \quad -c(y) + \tau_m + \sum_{k=1}^K \tau_k (q_k + \kappa_k) \geq 0 \quad (41)$$

$$\tau_m + \sum_{k=1}^K \tau_k (q_k + \kappa_k) \leq z + \sum_{k=1}^K \theta_k a_k (q_k + \kappa_k), \quad (42)$$

where  $\theta_k \in [0, 1]$  for all  $k$ . According to (40)-(42), the buyer's payoff is the same as the one he would get in an economy where he can make a take-it-or-leave-if-offer to the seller, but where he is constrained not to spend more than a fraction  $\theta_k$  of the real asset  $k$ .

One can generalize Lemma 1 in the obvious way; in particular,

$$\hat{U}^b(\ell) = \begin{cases} u(q^*) - c(q^*) & \text{if } \ell \geq c(y^*) \\ u \circ c^{-1}(\ell) - \ell & \text{otherwise} \end{cases}, \quad (43)$$

where  $\ell = z + \sum_{k=1}^K \theta_k a_k (q_k + \kappa_k)$  is the buyer's liquid portfolio. Assume that  $\theta_1 \geq \theta_2 \geq \dots \geq \theta_K$ . Then,

$$(q_k + \kappa_k)^{-1} \frac{\partial \hat{U}^b}{\partial a_k} = \theta_k \frac{\partial \hat{U}^b}{\partial z}.$$

So  $1/(q_k + \kappa_k)$  units of the  $k^{\text{th}}$  asset, which yields one unit of PM output, allows the buyer to raise his surplus in the AM decentralized market by a fraction  $\theta_k$  of what he would obtain by accumulating one additional

<sup>16</sup>The result that the price of an asset does not depend on its risk would no longer be true if the realization of the dividend was known in the AM when agents trade in bilateral matches. See Lagos (2006) and Rocheteau (2008).

unit of real balances instead. The parameter  $\theta_k$  can then be interpreted as a measure of the liquidity of the asset  $k$ , that is, the extent to which it can be used to financed consumption opportunities in the AM at favorable terms of trade. Given our ranking, the asset 1 is the most liquid one and the asset  $K$  is the least.

The second step of the pricing procedure is a generalization of (10)-(12). The seller's payoff and the actual terms of trade are determined by

$$\begin{aligned} \hat{U}^s &= \max_{y, \tau_m, \{\tau_k\}} \left[ -c(y) + \tau_m + \sum_{k=1}^K \tau_k (q_k + \kappa_k) \right] \\ \text{s.t. } & u(y) - \tau_m - \sum_{k=1}^K \tau_k (q_k + \kappa_k) \geq \hat{U}^b \\ & -z^s \leq \tau_m \leq z, \quad -a_k^s \leq \tau_k \leq a_k \end{aligned}$$

In the PM agents choose the portfolio,  $(\{a_k\}, z)$ , that they will bring into the decentralized market. The portfolio problem becomes

$$(\{a_k\}, z) = \arg \max_{\{a_k\}, z} \left\{ -iz - r \sum_{k=1}^K a_k (q_k - q_k^*) + \alpha \hat{U}^b \left( z + \sum_{k=1}^K \theta_k a_k (q_k + \kappa_k) \right) \right\}, \quad (44)$$

and  $q_k^* = \kappa_k/r$ . According to (44), the agent maximizes his expected utility of being a buyer in the AM decentralized market, net of the cost of the portfolio. The cost of holding asset  $k$  is the difference between the price of the asset and its fundamental value (expressed in flow terms), while the cost of holding real balances is  $i = \frac{\gamma - \beta}{\beta}$ , approximately the sum of the inflation rate and the rate of time preference. An agent's portfolio choice problem, (44), can be rewritten as

$$\max_{\{a_k\}, \ell} \left\{ -i\ell + \sum_{k=1}^K a_k [i\theta_k (q_k + \kappa_k) - r (q_k - q_k^*)] + \alpha \hat{U}^b(\ell) \right\} \quad (45)$$

$$\text{s.t. } \sum_{k=1}^K \theta_k a_k (q_k + \kappa_k) \leq \ell. \quad (46)$$

As in Section 4, in a monetary equilibrium, constraint (46) does not bind,  $\ell$  solves (29), and the asset prices must satisfy  $i\theta_k (q_k + \kappa_k) - r (q_k - q_k^*) = 0$  or

$$q_k = \frac{1 + i\theta_k}{r - i\theta_k} \kappa_k, \quad \forall k \in \{1, \dots, K\} \quad (47)$$

for all  $k \in \{1, \dots, K\}$ .<sup>17</sup> Note that the price of the real asset  $k$  increases with inflation, provided that  $\theta_k > 0$ .

<sup>17</sup>The proof to these claims are almost identical to the proof of Proposition 2. Specifically, if  $r (q_k - q_k^*) > i\theta_k (q_k + \kappa_k)$ , then  $A^d(q_k) = \{0\}$ , which cannot be an equilibrium; if  $r (q_k - q_k^*) < i\theta_k (q_k + \kappa_k)$ , then constraint (46) binds and  $z = 0$ , i.e., the equilibrium is non-monetary; and if  $r (q_k - q_k^*) = i\theta_k (q_k + \kappa_k)$ , then  $\sum_k A_k^d(q_k) \theta_k (q_k + \kappa_k) \in [0, \ell]$ , where  $\ell$  solves (29).

From (46) and (47), a monetary equilibrium exists if

$$\sum_{k=1}^K \theta_k A_k \left( \frac{1+r}{r-i\theta_k} \right) \kappa_k < \ell(i). \quad (48)$$

For money to be valued, the total stock of real “liquid” assets in the economy must not be too large.

The rate of return of asset  $k$  is given by

$$R_k = \frac{\kappa_k + q_k}{q_k} = \frac{1+r}{1+i\theta_k}, \quad \forall k \in \{1, \dots, K\} \quad (49)$$

Provided that the nominal interest rate is strictly positive, the extended model is able to generate differences in the rates of return of the real assets, where the ordering depends on the liquidity coefficients  $\{\theta_k\}$ .

**Proposition 6** *In any monetary equilibrium,  $R_K \geq R_{K-1} \geq \dots \geq R_1 \geq \gamma^{-1}$ . Moreover,*

$$R_{k'} - R_k = (1+r) \frac{i(\theta_k - \theta_{k'})}{(1+i\theta_k)(1+i\theta_{k'})} > 0. \quad (50)$$

**Proof.** Direct from (49). ■

The differences between the rates of return across assets emerge even if the assets are risk-free. They result from the pricing mechanism according to which different assets are traded at different terms of trade in the decentralized market. These differences would be viewed as anomalies by standard asset pricing theory.

Let us turn to monetary policy. A change in inflation affects the entire structure of asset returns. Denote  $\{R_k^i\}_{k=1}^K$  the structure of asset yields when the cost of holding fiat money is equal to  $i$ .

**Proposition 7** *In any monetary equilibrium,  $\{R_k^i\}_{k=1}^K$  dominates  $\{R_k^{i'}\}_{k=1}^K$  in a first-order stochastic sense whenever  $i' > i$ . Moreover,  $\frac{\partial(R_{k'} - R_k)}{\partial i} > 0$  if and only if  $(\theta_k - \theta_{k'}) (1 - i^2 \theta_k \theta_{k'}) > 0$ .*

**Proof.** The first part of the proposition is direct from (49). From (50), if  $\theta_k = \theta_{k'}$  then  $R_{k'} - R_k = 0$  which is independent of  $i$ . With no loss, assume  $\theta_k > \theta_{k'}$  and  $i > 0$  so that  $R_{k'} - R_k > 0$ . From (50), differentiate  $\ln(R_{k'} - R_k)$  to get

$$\frac{\partial \ln(R_{k'} - R_k)}{\partial i} = \frac{1 - i^2 \theta_k \theta_{k'}}{i(1+i\theta_k)(1+i\theta_{k'})}.$$

■

An increase in inflation raises the rates of return of all real assets because agents substitute the real assets for real balances which are more costly to hold. Moreover, the premia paid to the less liquid assets increase provided that  $i$  is not too large.

## 6.1 An example

To illustrate the ability of the model to generate rate of return anomalies, consider a simple example with two real assets. Call asset 1 Treasury bills, and asset 2 stocks. The expected real rate of return of stocks is  $R_2 = 1.07$  while the expected rate of return of Treasury bills is  $R_1 = 1.01$ . The growth rate of money supply is  $\gamma = 1.03$  which corresponds to an inflation rate of 3 percent. The minimum rate of time preference consistent with these returns is  $r = 0.07$  and it corresponds to  $\theta_2 = 0$ . So stocks are fully illiquid from the buyer's standpoint. In order to obtain a rate of return of 1 percent for Treasury bills, one needs to set  $\theta_1 \simeq 0.5$ . So the buyer's payoff is the one he would get in an economy where he can spend about half of his portfolio of bonds.

## 7 Conclusion

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