Approaches to Current Stock Market Valuations

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- *The application of commonly used valuation techniques to stock markets in the United States and Canada suggests that market values (measured at the end of February 2000) could be sustained only by rapid growth of dividends in the future or by the continued assumption of an uncharacteristically low risk premium on equity.*
- *If the technology sector is excluded, however, one does not need to assume as high a growth of dividends or as low a risk premium for the remaining portion of the stock market.*
- *Several explanations for the decline in risk premiums on equity are considered. While short-term volatility in the stock market has, if anything, increased in recent years, low inflation and improved economic performance, along with demographics and investor preferences, may have contributed to a decline in the risk premium demanded by investors.*
- *A scenario of rapid growth of dividends in the near term slowing to historical norms in the longer term is examined. It can go some way towards explaining high stock market valuation but requires assumptions that are outside historical experience.*

The increase in North American stock prices in 1999 and early 2000 has sparked interest in the valuation assumptions that would make these price levels sustainable. This article looks at some simple methods of valuing stocks. The relationships among earnings yields, dividend yields, and interest rates in Canada and the United States are examined. Real interest rates (that is, rates on price-index-linked securities) are shown to provide the best comparators to yields on stocks. The valuation measures for the stock markets excluding the technology sector are then reviewed.

The framework of the dividend-discount model, which expresses stock prices as the present value of the stream of future dividends, is then used to evaluate relationships between two important determinants of stock market values: the expected growth rate of dividends and the risk premium on equity. The article concludes by looking at explanations for a decline in the equity-risk premium and at the role that near-term rapid growth in dividends could play in explaining high stock market valuations.

The Comparative-Yield Approach

Asset allocation among broad classes of securities such as stocks, bonds, and low-risk liquid assets has an important impact on portfolio performance. Yield relationships are used by portfolio managers to determine the relative attractiveness of these asset classes in investment portfolios. This type of analysis has a long history, stemming from Graham and Dodd's approach to security analysis (Graham et al. 1962). Modern models of asset allocation, designed by

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investment managers, usually incorporate similar indicators to assess the relative values of stocks and bonds. These indicators help portfolio managers determine the timing for the switching of funds among stocks, bonds, and liquid assets in order to add value relative to buy-and-hold and constant-asset-mix portfolios.

Two yield measures are commonly applied to equities. The *dividend yield*—the ratio of dividends over the last year to the current stock price—is a measure of recent cash income in the form of dividends paid out to stockholders. The *earnings yield* is the ratio of the last year's corporate earnings (accruing to common stockholders) to the current stock price; it is the reciprocal of the price-earnings ratio. Sometimes these measures are difficult to apply to individual stocks; for example, in the case of companies that do not pay dividends or that are experiencing losses (negative earnings). They are, however, suited to the analysis of broader market indexes such as the Toronto Stock Exchange (TSE) 300 index in Canada and the Standard and Poor's (S&P) 500 index in the United States, as is done here.

Clearly, bond yields are not strictly comparable to these yield indicators for equities. While bond yields are forward-looking and give some idea of total returns over the term of the bond (abstracting from default and reinvestment risk), the dividend yield represents merely the (often relatively small) cash payout that the board of a corporation has distributed to shareholders over the last year, divided by the current stock price. This payout can change from quarter to quarter, depending on the decisions of management and the board. Most of the total return on stocks usually comes from capital gains, rather than from dividends. The earnings yield also suffers from several deficiencies. Like the dividend yield, it is a backward-looking measure. Shareholders have only an indirect claim on earnings, the use of which tends to be controlled by management. Earnings are regularly affected by transitory write-offs, gains, accounting conventions, and non-cash items, which make their interpretation difficult.[1]

In spite of these deficiencies, dividends and earnings do convey some information about stock valuation. A dividend increase is taken as a signal that sustainable earnings and cash flow, and consequently, the value of the firm, have increased. Higher earnings provide more funds from which dividends can be paid to shareholders or which can be reinvested in the firm, generating more internal growth and equity value.

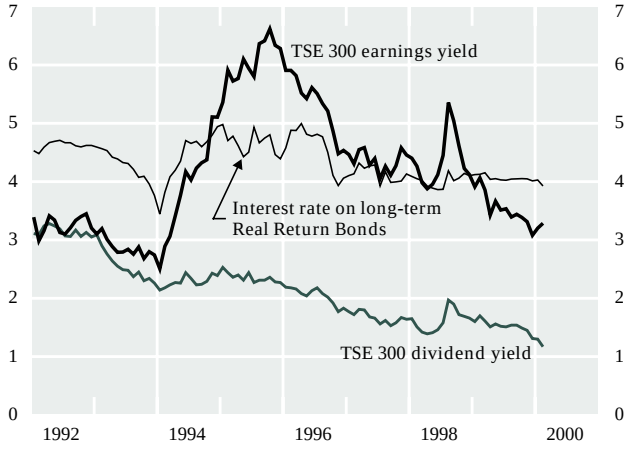
Traditionally, investment analysts have used nominal interest rates in performing these comparisons. However, in their work on the effects of inflation on equity valuation, Modigliani and Cohn (1979) have shown that it is inappropriate to compare current yields on equities with nominal interest rates in periods of inflation. Equity earnings and dividends are variable nominal dollar payments that can be expected to rise with increases in prices; in that sense, they are linked to inflation. Therefore it is appropriate to compare earnings and dividend yields with yields on bonds linked to inflation, which is done in Charts 1 to 5.[2]

As seen in Charts 1 and 2, since 1992 the dividend yields on both the TSE 300 index and the S&P 500 index have trended down steadily from about 3 per cent to well under 2 per cent recently. The earnings yield, in contrast, is considerably more volatile and cyclical. It reached low points in both markets in the early 1990s and then recovered strongly with renewed economic growth. At the end of February 2000, the earnings yield stood near 3 per cent in both Canada and the United States. When the technology sector[3] is separated from the rest of the Canadian market, as is

Chart 1

TSE Earnings Yield, Dividend Yield, and the Interest Rate on Long-Term Real Return Bonds

Per cent



1. For the purpose of stock valuation, alternatives to earnings have been proposed, such as free cash flow. The free cash flow of firms can be defined as the cash flow that remains after all investments with positive net present values have been made.

2. Kennedy et al. (1998) also compare dividend yields to real interest rates.

3. The technology sector includes both hardware and software firms, plus Bell Canada Enterprises.

Chart 2

S&P 500 Earnings Yield, Dividend Yield, and the Long-Term Real Interest Rate

Per cent

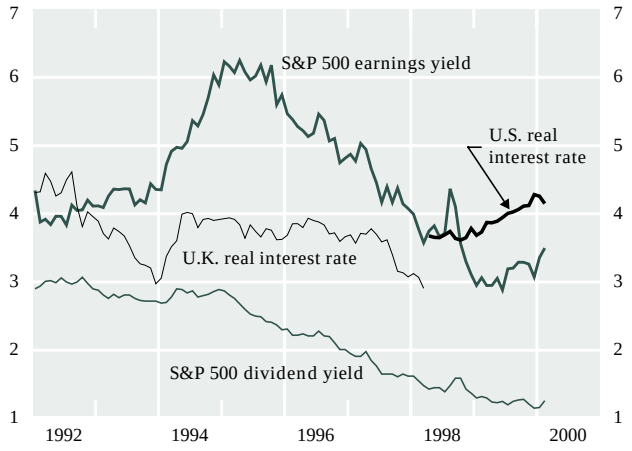
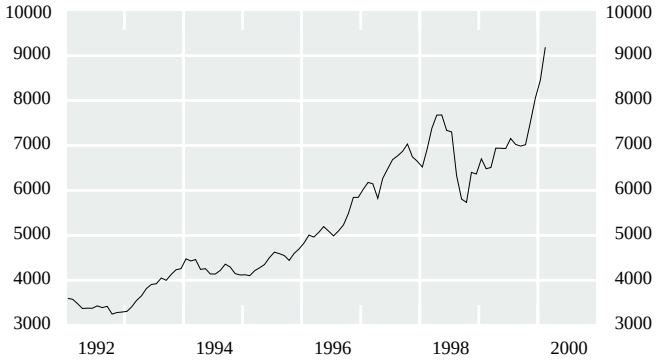


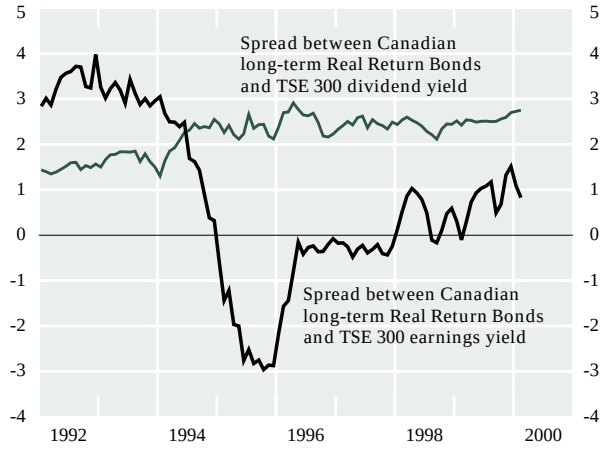
Chart 3

TSE 300 Index



Yield Spreads on Earnings and Dividends

Per cent



done in Chart 4, the most striking feature is the sharp upward movement in Canadian technology stock prices in 1999.[4]

The cyclical behaviour of earnings over this period is much more evident in Canada than in the United States. In 1992–93, earnings yields on Canadian stocks fell below the dividend rate. This did not happen in the United States. The disparity reflected the greater severity of the recession in Canada and the volatility of earnings in the resource sector, which is relatively more important in Canada than in the United States. For example, at times over the period, the metals and forest products industries recorded sector-wide losses. Also clearly evident from the charts is the steady pay-out of dividends in the face of variable earnings. Corporations tend to set dividends based on their perception of their longer-run earnings and are reluctant to cut dividends unless it is necessary to conserve cash.

Charts 3, 4, and 5 illustrate the spreads between real interest rates and these stock index yields. While a long-term real interest rate is available for Canada since 1992, a similar rate is available for the United States only since 1998. The real interest rate on U.K. index-linked gilts is used as a representative real rate comparator for the previous period.[5] As seen in Chart 5, in the United States, the spreads between the real interest rate and stock yields moved higher through the second half of the 1990s, reaching a peak in 1999. In Canada (Charts 3 and 4), the earnings yield spread has moved in a wider range because of the greater cyclical volatility of earnings. In the early 1990s, while earnings and earnings yields were low, investors were anticipating an economic recovery and better profits in coming years. This expectation, which was indeed fulfilled, supported stock prices relative to their earnings at the time. The spread between the real interest rate and the dividend yield has trended very slightly higher over the period.

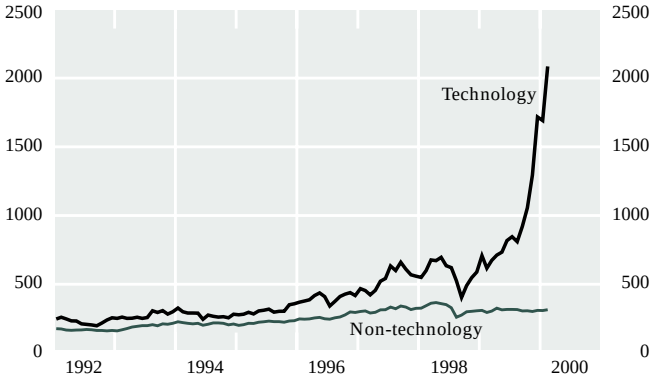
4. Nortel Networks is by far the largest company in this group.

5. With high capital mobility between the relatively sophisticated U.S. and U.K. capital markets, it can be argued that a U.K. real interest rate is a reasonable, although imperfect, proxy for the unobserved U.S. real interest rate prior to 1998.

Chart 4

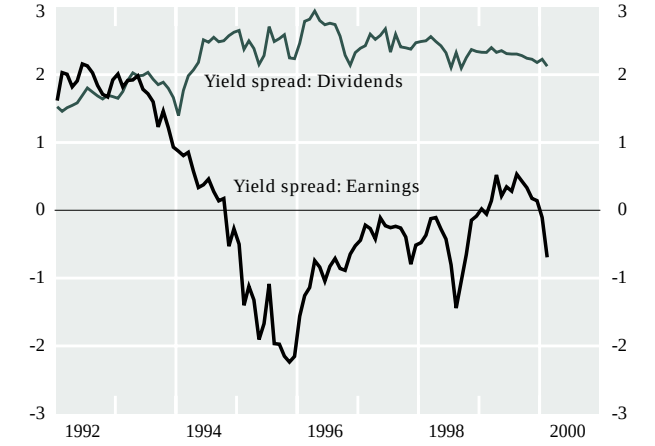
TSE 300 Index: Technology and Non-Technology Sectors

1989=100



Yield Spreads on Earnings and Dividends for the Non-Technology Sector

Per cent



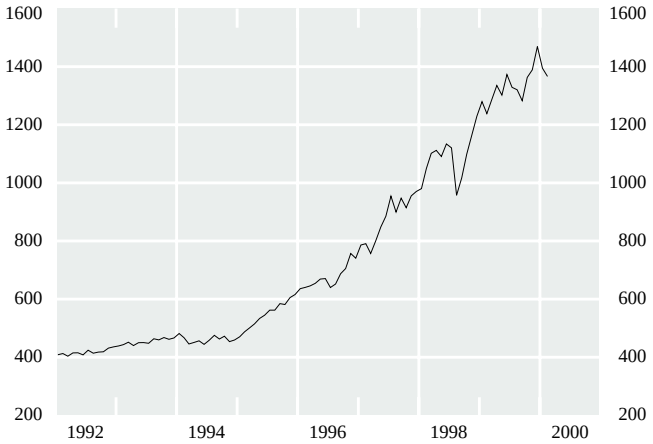
Source: Nesbitt Burns

A wide yield spread between the real interest rate and the equity yield may indicate overvaluation; that is, earnings and dividend yields that are too low (stock prices too high) relative to interest rates. Since early 1998, real interest rates have moved above 4 per cent in the United States and have remained near 4 per cent in Canada, before declining this year. However, dividend yields in both stock markets have declined. Earnings yields have recovered somewhat in the United States but have declined in Canada. Consequently, spreads of these yields against interest rates have generally increased, except for the Canadian dividend yield spread. In the United States, the earnings spread exceeds that observed before the financial market turmoil in the late summer of 1998 by a significant margin (Chart 5). In Canada, the earnings spread has risen somewhat, but has been much more volatile owing to fluctuations in earnings yields (Chart 3).

The spectacular rise in the value of technology stocks through 1999 has contributed to the movements in these valuation measures. Chart 4 shows the divergence in the prices of the Canadian technology sector and the rest of the market in 1999 and illustrates the spreads between the real interest rate and non-technology stock market yields. The dividend yield spread

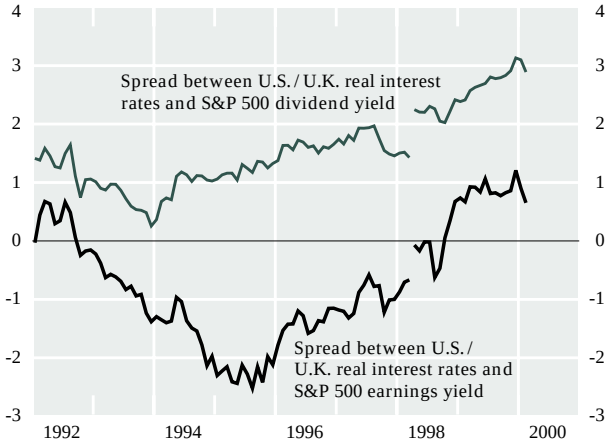
Chart 5

S&P 500 Index



Yield Spreads on Earnings and Dividends

Per cent



Note: The break in series in April 1998 relates to the use of the U.K. gilt real interest rate prior to that time. Technical factors in the U.K. gilt real interest rate market may result in a downward bias in the series.

is lower than for the overall market, and the earnings yield spread in 1999 and early 2000 moved decidedly lower, suggesting that this portion of the market may not be overvalued at present. Many non-technology sectors within the U.S. S&P 500 index also appear to be more reasonably valued now.

The Dividend-Discount Model Approach

In the dividend-discount model (DDM), current equity values are expressed as the present value of the stream of future dividends. This dividend flow is discounted to a present value by an appropriate discount rate for equity capital, consisting of a risk-free rate plus a risk premium. In practice, the risk-free rate is usually measured as a government bond rate.

When dividends are expected to grow at a constant rate *g*, the model can be represented as:

$$P = D/(r-g), r > g \qquad (1)$$

where *P* is the current price of equity, *D* is dividends,[6] and *r* is the discount rate for dividends received from equity capital.

It can be seen that stock prices are positively related to dividends and their growth rate (and by implication to earnings, out of which dividends can be paid on a sustainable basis) and are negatively related to the equity discount rate (which is partly related to market interest rates).

Other interesting relationships are apparent. By rearranging equation (1), the dividend yield *d* (equal to D/P) is seen as the difference between *r* and *g*.

$$d = r - g. \qquad (2)$$

If *r* is separated into its risk-free rate component, *rf*, and the equity-risk premium, *re*, then the relationship between the risk premium and other variables can be expressed as:

$$re = g - rf + d, \; g < re + rf. \qquad (3)$$

The risk-free rate, *rf*, and the dividend yield, *d*, are readily observable. The other two variables, the equity-risk premium, *re*, and the expected growth of dividends, *g*, are not, but one can assess the reasonableness of the range of values implied for them using this model, and by implication, assess the level of stock market prices, as is done in Table 2 later in the article. The variables *g* and *rf* can be specified in nominal or real terms; in Table 2 real variables are used.

The Equity-Risk Premium

Investors require compensation for holding risky assets,[7] over and above the return they could earn on risk-free investments. For example, one can get a fairly good measure of the risk premium on corporate bonds by looking at the yield spread between them and government bonds of the same term. For stocks, the current risk premium is not observable. Over a long period, however, the ex post risk premium realized by investors can be observed as the difference between the total return on government bonds relative to that on stocks. Such a comparison is shown in Table 1.

Table 1

Equity-Risk Premiums Based on 40-Year Annualized Total Returns

Per cent

	Stocks	Bonds	Difference: Risk premium
United States	12.0	7.2	4.5
Canada	10.2	8.2	1.9

Sources: Ibbotson Associates, Inc. (1998); Canadian Institute of Actuaries (1998)

Notes: The data represent the total annualized nominal returns realized on equities and bonds for the 40 years ended 1998. The choice of a time period is arbitrary; it should be long enough to even out unexpected shocks to the economy, but very long periods are less useful for current analysis. Over earlier periods, ex post realized risk premiums in both countries were higher. The stocks series are based on the S&P 500 index and the TSE 300 index, respectively. The bonds were based on long Canada nominal bonds and long U.S. Treasuries. The difference between the stock and bond return is calculated geometrically.

The size of the realized equity-risk premium in the United States over this 40-year period, at 4.5 per cent, is within the range considered normal by investment policy professionals. It is higher than that observed for Canada, reflecting the relative performances of stock markets and the positive fixed-income yield spreads

6. For notational simplicity, *D* is assumed to be the dividend yield one period ahead. If *D* is the yield for the current period, the correct formula is $P = D(1+g)/(r-g)$.

7. Broadly speaking, there is a spectrum of risk premiums across financial assets, ranging from essentially zero on short-term, risk-free government securities; low premiums on investment-grade bonds; higher premiums on real estate and stocks; to the highest, associated with venture capital and private equity.

between Canada and the United States over most of this period. The spectacular total returns on U.S. stocks in recent years stand in marked contrast to these figures, which appear conservative by comparison. Although the historical excess returns to equity in the two markets differ over this period, it is difficult to see why there would have been a significant difference in the expected risk premium over this historical period or looking forward from the present. In the North American investment industry, equity-risk premiums in the range of 2 per cent to 6 per cent in both markets bracket the ranges typically used when looking forward for purposes of investment planning.[8]

Linking the Risk Premium, Dividend Growth, and Market Valuation

The constant-growth dividend-discount model discussed above provides an analytic linkage among the equity-risk premium, the expected growth of dividends, and the level of stock market prices. Table 2 shows the implied dividend growth assuming equity-risk premiums of 2 per cent, 4 per cent, and 6 per cent (the typical historical range), as well as zero and one per cent, given the dividend yields and interest rates on long-term government bonds for February 2000. All variables are measured in real terms. The implied dividend growth, *g*, is equal to the risk-free rate (proxied by the government bond rate) plus the assumed equity-risk premium,[9] minus the observed dividend yield, from equation (3):

$$g = rf + re - d. \qquad (4)$$

Historical dividend growth in both countries and expected growth of earnings in the United States, out of which sustainable dividends can be paid, are shown for comparison.

First, it is notable that almost all the implied dividend growth rates for the positive equity-risk premiums shown are higher than sustainable long-run real economic growth of perhaps a little over 3 per cent.[10] While the shares of income accounted for by earnings and dividends may vary in the short and medium term, it is not reasonable to expect them to rise without limit in the long term. Only the lower range of the equity-risk-premium assumptions (zero to 2 per cent) generates reasonable future real dividend growth rates, and these generally still exceed historical growth rates. In Canada, the higher dividend yield for the non-technology sector implies a lower growth rate for that sector's dividends as shown in the last row of Table 2, but, at 4.2 per cent (assuming an equity-risk premium of 2 per cent), this is still greater than overall long-run GDP growth.[11]

Another indicator of expected growth is provided in the last column of Table 2, the expected 5-year earnings growth of companies in the S&P 500 index. In the long run, dividends can be paid only out of sustainable earnings. The 15.2 per cent annual expected earnings growth would appear to support significant dividend increases, or if retained, good internal growth in the value of firms. But this growth rate appears excessive when compared with the prospects for economy-wide growth in nominal income (2 to 3 per cent inflation and 2 to 4 per cent real output growth).

Table 2

Dividend Growth and Equity-Risk Premiums[a]

Per cent

	Implied long-run real dividend growth, assuming equity-risk premiums of:					Historical real dividend growth[b]	Historical real GDP growth[b]	Analysts' expected 5-year nominal earnings growth[c]
	0%	1%	2%	4%	6%			
United States	3.0	4.0	5.1	7.2	9.2	0.7	3.1	15.2
Canada:								
Total	2.6	3.6	4.7	6.8	8.8	-1.1	3.2	na
Excl. tech. stocks	2.1	3.1	4.2	6.3	8.3			

a. For the United States, *rf* (the yield on inflation-protected Treasury securities) was 4.15 per cent and the dividend yield was 1.20 per cent; for Canada, *rf* (the yield on Real Return Bonds) was 3.93 per cent and the dividend yield was 1.20 per cent. For the non-technology sector, the dividend yield was 1.80 per cent.
b. 30 years ended 1998; dividends based on the S&P 500 and TSE 300 indexes, respectively, deflated by the CPI.
c. S&P 500 earnings estimates as of 4 April 2000 provided by Zacks.

The low historical growth rate of dividends, particularly in Canada, is worth special mention. In Canada, dividends paid by companies in the TSE index have

8. A discussion of the equity-risk premium is found in Siegel (1999).

9. The sum used in the table is the geometric sum, calculated as $(1+rf)(1+re)-1$.

10. There is considerable uncertainty surrounding both estimates of the level and growth of potential output. Based on recent productivity growth in the United States, estimates of growth in potential output range up to 4 per cent.

11. This comparison may not necessarily mean a relatively better valuation for the non-technology sector, since that sector may be expected to grow more slowly than the technology sector.

grown at only 4.1 per cent per year in nominal terms over the last 30 years, less than half the rate for nominal GDP. This translates into negative real growth as shown in Table 2. Dividends paid by companies on U.S. exchanges have grown more rapidly but still by less than U.S. GDP. One factor retarding the growth of dividends in recent years in Canada has been the low levels of commodity prices. To conserve cash flow, companies in the resource sector have opted to cut dividends.

Another reason for the low growth of dividends is the ascent of "new economy" companies, which tend to pay little or no dividends.[12] Such companies retain more of their earnings for research and product development, thereby generating more internal growth than former mainstays of the index such as utilities, financial institutions, and consumer-product companies, which pay out a significant portion of their earnings in dividends. In the technology sector of the Canadian market, the dividend yield is currently only 0.3 per cent. Moreover, it is generally accepted that the traditional valuation procedures used in this article are not suited to high-growth technology companies in a current loss position.

The "new economy" has another characteristic that bears on the results of this analysis—expectations for potential long-run, economy-wide growth. If, as advocates of the new economy suggest, this potential is significantly higher than 3 per cent because of a permanently higher track for productivity growth, then higher-than-historical assumptions about earnings growth would be more plausible. At the corporate level, we have seen some large U.S. technology companies deliver annualized nominal earnings growth well into the double digits steadily for the last decade.

The remainder of this article looks at two issues related to the current valuation puzzle: recent experience with stock market volatility and the equity-risk premium, and the possibility that near-term rapid growth of dividends could justify the current high valuations.

Volatility and the Risk Premium

Investors' recent experience with stock market volatility affects their views about stock market risk and the risk premium (in terms of added return) they will demand from the market. If volatility has declined, this would give some credence to the view that market participants have reduced the risk premium embedded in the discount rate attached to equity cash flows, as suggested by the results of the previous DDM analysis.

Chart 6 shows the annualized 30-day volatility of the TSE 300 and S&P 500 indexes. The general trend of volatility has been higher since 1995, and the trend has been punctuated by short periods of high volatility during the Asian and Russian crises in 1997 and 1998, respectively. This does not suggest that experience with recent volatility has given investors any reason to accept a low risk premium.

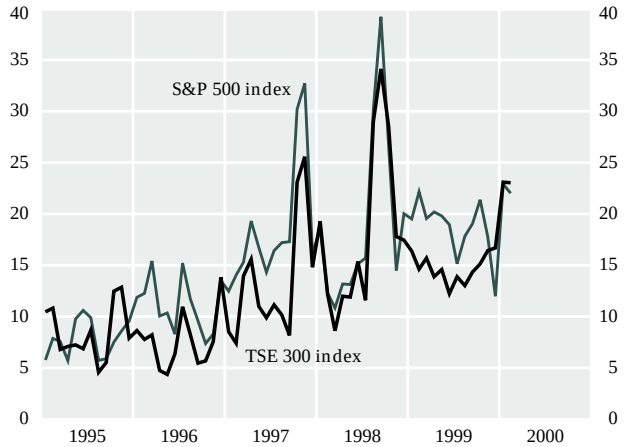
Some discussion about declining risk premiums has centred on demographics and the changing preferences of investors. The flow of funds into equity-based retirement savings from baby boomers anticipating retirement may be associated with some lowering of the risk premium on equity. Investor preferences in recent years, both institutional and individual, may also be responding to more and better information and understanding about risk in financial markets and stock markets. The greater availability of investment vehicles, resulting in larger and better-diversified equity portfolios may also have influenced such flows and contributed to a declining risk premium.

Investors may also be responding to a more fundamental change in their experience with financial

Chart 6

TSE 300 and S&P 500 Indexes: 30-Day Volatility

Per cent



Source: Bloomberg, 30-day annualized index standard deviation as a percentage of the index

12. For example, Microsoft and Cisco, among the largest companies (by capitalization) in the S&P 500 index, do not pay cash dividends.

market risk in recent years. Sustained economic growth, improved government fiscal positions, and low and controlled inflation have all contributed to an improvement in investor confidence compared with a decade ago.

Medium-Term Growth of Dividends

Earnings and dividends cannot be expected to grow faster than nominal economy-wide income in the long run; however, they may do so over the medium term for several years, particularly if they start from an unusually low level. This higher growth rate can arise both from real growth of the economy that is higher than potential and from a rise in the share of profits in national income, from which dividends can be paid. This section examines the possibility that such growth might explain high valuations. The standard constant-growth DDM shown earlier is modified to accommodate near-term, super-normal growth of dividends.

The standard constant-growth DDM, $P = D/(r-g)$ from equation (1), can be modified to incorporate near-term variable dividend growth as follows (a derivation is provided in the Appendix):

$$P = \frac{D^*}{(r-g)} + \left(\frac{D - D^*}{(\alpha + r - g)}\right) \qquad (5)$$

where D is the actual level of dividends, D^* is the normal long-run path of dividends, and α is the speed of adjustment of actual towards long-run dividend levels.

When divided through by P, the relationship is expressed in terms of dividend yields; multiplying both sides by r-g, and expressing r as $rf + re$, allows us to write:

$$re = g + d^* + \frac{(d - d^*)(re + rf - g)}{(\alpha + re + rf - g)} - rf\,. \qquad (6)$$

With this expression, we can re-examine the relationship between the dividend growth rate and the risk premium on equity, *re*, as we did in Table 2. The equation looks very much like that used to generate Table 2, except that the long-run dividend yield, d^*, rather than the actual dividend yield appears. A new add-on term related to the difference between current dividends and long-run dividends, $d - d^*$, and to the adjustment term, α, also appears. If d^* is high, and if α is high (rapid convergence), then the equity-risk premium will be higher, other things being equal.[13]

Table 3

Equity-Risk Premiums and the Modified DDM

Per cent

	Modified DDM		Standard DDM
	α = 10%	α = 30%	
United States	2.1	2.5	0.3
Canada	2.3	2.6	0.6

In Table 3, levels of the equity-risk premium calculated using equation (6) are shown. The calculations are based on the following assumptions about g, d, d^*, and α:

- For the long-run real dividend growth rate, g, 3.25 per cent is chosen for both Canada and the United States, paralleling real long-run growth in potential output.
- For long-run dividend yields, d^*, 3.5 per cent is chosen—the approximate average of the past 30 years. For current yields, d, 1.3 per cent is chosen for Canada and 1.2 per cent for the United States. These were the dividend yields on the TSE 300 index and the S&P 500 index, respectively, for February 2000.
- The current risk-free real interest rates, as specified by yields on long-term government bonds, are 3.9 per cent in Canada and 4.2 per cent in the United States.
- For the speed of adjustment, α, two scenarios of 10 per cent per year and 30 per cent per year are chosen. In the latter case, after 5 years, about 83 per cent of the convergence to the long-run growth rate has occurred.

The last column also shows results calculated using the standard DDM (using current dividend yields, $\alpha = 0$, and long-run g).

13. Although *re* is also on the right-hand side of this equation, its importance there is secondary, and the calculations reported in Table 3 are easily done by iteration.

The main determinants of the risk premium are the long-run dividend yield, its growth rate, and the risk-free interest rate. Variations in the rate of convergence have a significant but smaller effect on the equity-risk premium. While still low, the risk premiums calculated from the modified DDM are plausible.

While the modified DDM can help to explain high stock market valuations, the assumptions needed to do so can be readily challenged. For example, a long-run real growth rate of 3.25 per cent implies continued strong productivity growth, and reversion to a 3.5 per cent dividend yield on stocks in several years implies a rapid pickup in dividend payouts, which, as yet, shows little sign of occurring. The dividend growth generated by the adjustment process is quite rapid and is inconsistent with past corporate dividend-payout behaviour, which has tended to change slowly over time.

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Appendix : A Modified Dividend-Discount Model[1]

We can write P, the price of equity, as the present value of the cash flows from equity (i.e., dividends), where D_t is dividends per period paid at time t, and r is the discount rate:

$$P = \int_0^{\infty} D_t e^{-rt} dt . \qquad \text{(A1)}$$

We introduce the concept of a normal, long-run dividend whose path is described by

$$D_t^* = D_0^* e^{gt}, \qquad \text{(A2)}$$

where D_0^* is the initial normal level of dividends, and g is the growth rate. As a result, the following equation holds:

$$\frac{dD_t^*}{dt}\frac{1}{D_t^*} = g . \qquad \text{(A3)}$$

If actual current dividends D_t are below their normal level D_t^*, and D_t converges towards D_t^* at a rate α per period, we can then specify an adjustment process as follows:

$$\frac{dD_t}{dt}\left(\frac{1}{D_t}\right) = g + \alpha\left(\frac{D_t^*}{D_t} - 1\right). \qquad \text{(A4)}$$

This equation states that the percentage rate of change of actual dividends, D_t, is equal to the growth rate of long-run normal dividends, g, plus an additional component that represents the convergence of actual dividends towards their long-term trend. The parameter α represents the speed of this adjustment.

Applying the quotient rule to differentiate (D_t/D_t^*) with respect to t, yields

$$\frac{d(D_t/D_t^*)}{dt}\frac{D_t^*}{D_t} = \frac{dD_t}{dt}\frac{1}{D_t} - \frac{dD_t^*}{dt}\frac{1}{D_t^*}, \qquad \text{(A5)}$$

and substituting equations (A3) and (A4) into (A5) gives:

$$\frac{d(D_t/D_t^*)}{dt}\frac{D_t^*}{D_t} = \alpha\left(\frac{D_t^*}{D_t} - 1\right). \qquad \text{(A6)}$$

Multiplying both sides of (A6) by D_t/D_t^* and solving the resulting first-order differential equation for D_t/D_t^* yields:

$$\frac{D_t}{D_t^*} = \frac{D_0}{D_0^*}e^{-\alpha t} + (1 - e^{-\alpha t}) . \qquad \text{(A7)}$$

Substituting for D_t using (A7) and (A2) into equation (A1) implies:

$$P = \int_0^{\infty} (1 - e^{-\alpha t}) D_0^* e^{(g-r)t} dt + \int_0^{\infty} e^{-\alpha t} D_0 e^{(g-r)t} dt . \qquad \text{(A8)}$$

Evaluating the integrals, we have:

$$P = \frac{D_0^*}{(r-g)} + \frac{(D_0 - D_0^*)}{(\alpha + r - g)}, \qquad \text{(A9)}$$

which is equation (5) (with $D_0^* = D^*$, and $D_0 = D$).

The first term of equation (A9) resembles the traditional constant-growth DDM, valuing stocks based on long-run normal dividends. The second term represents the contribution of the deviation of dividends from their long-run trend $(D - D^*)$.

1. Pierre Duguay and Shafiq Ebrahim, Bank of Canada, contributed to the development of this model.