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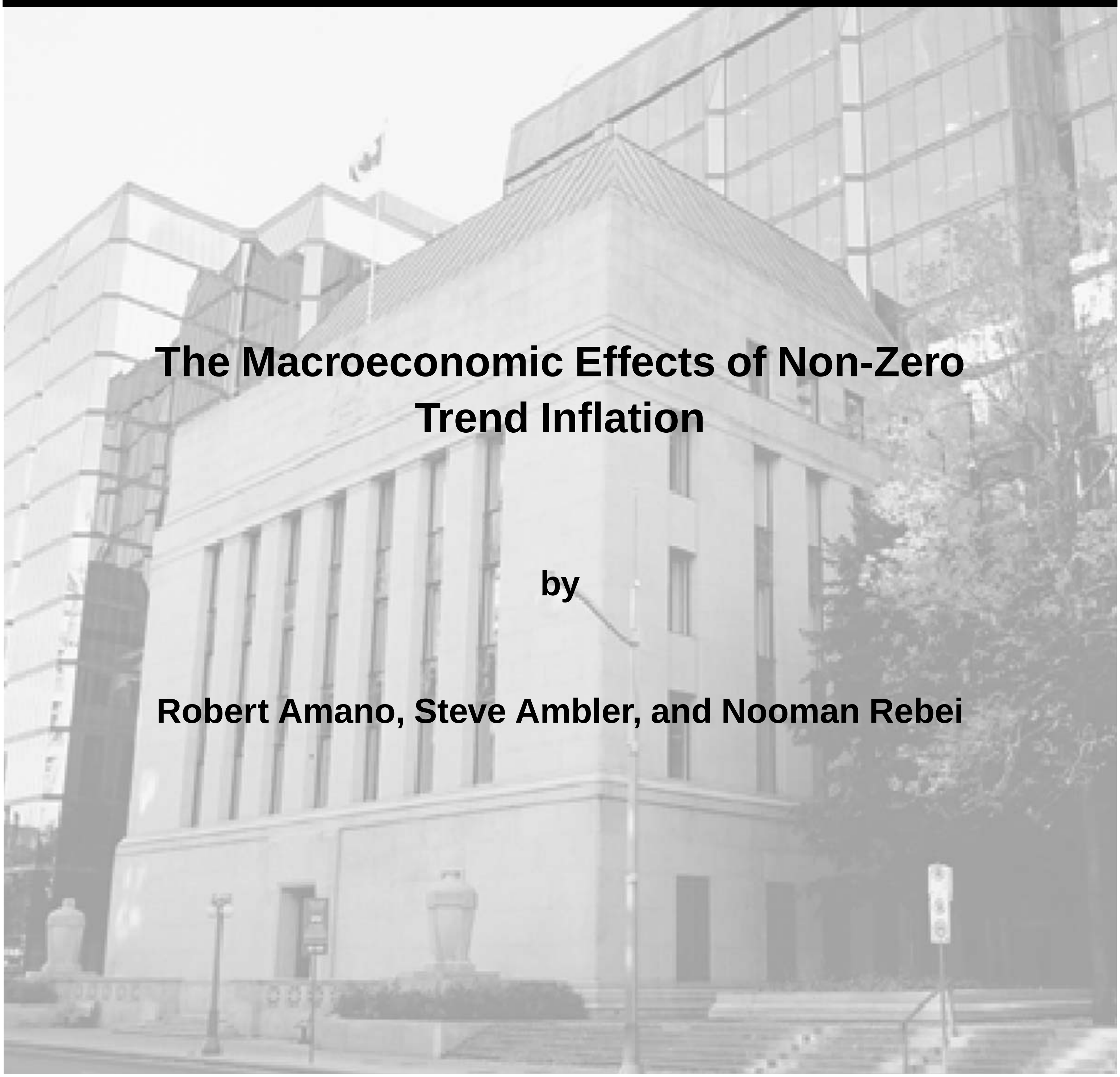

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The views expressed in this paper are those of the authors.
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Abstract

The authors study the macroeconomic effects of non-zero trend inflation in a simple dynamic stochastic general-equilibrium model with sticky prices. They show that trend inflation leads to a substantial reduction in the stochastic means of output, consumption, and employment. It also leads to an increase in the variability and persistence of most aggregates. Price dispersion across firms unambiguously increases the welfare costs of inflation. The effects hold qualitatively no matter how sticky prices are modelled, but they are quantitatively much stronger under Calvo pricing.

JEL classification: E24, E32
Bank classification: Business fluctuations and cycles; Economic models; Inflation and prices; Inflation targets

Résumé

Les auteurs étudient l’incidence macroéconomique d’un taux d’inflation tendanciel non nul à l’aide d’un modèle d’équilibre général stochastique et dynamique simple où les prix sont rigides. Ils montrent que la présence d’une inflation tendancielle réduit de beaucoup le niveau des moyennes stochastiques de la production, de la consommation et de l’emploi. Ils montrent aussi qu’elle accentue la variabilité de la plupart des agrégats et la persistance de leurs fluctuations. La dispersion des prix entre entreprises amplifie incontestablement l’effet négatif de l’inflation sur le bien-être. Ces résultats se vérifient sur le plan qualitatif, peu importe la manière dont l’hypothèse de rigidité des prix est modélisée, mais les effets sont plus accusés en termes quantitatifs dans un modèle de détermination des prix à la Calvo.

Classification JEL : E24, E32
Classification de la Banque : Cycles et fluctuations économiques; Modèles économiques; Inflation et prix; Cibles en matière d’inflation

1 Introduction

The New Keynesian Phillips curve (henceforth NKPC) is a workhorse of modern macroeconomics.[1] It has been used as a key element of dynamic stochastic general-equilibrium (henceforth DSGE) models for theoretical, empirical, and monetary policy analysis. The NKPC holds only under restrictive assumptions. Either trend inflation must be zero or firms must index their prices to past inflation, trend inflation, or target inflation.[2] Solving DSGE models without these assumptions is much more tedious. However, trend inflation rates of zero are exceedingly rare in real-world economies, and many prices are observed to remain fixed for long periods of time, suggesting less than full indexation. Moreover, the trend level of inflation tends to change over time. Levin and Piger (2003) and Levin, Natalucci, and Piger (2003) provide evidence on changing trend inflation rates for several developed countries.[3] Countries whose central banks have adopted official inflation targets have invariably opted for positive inflation targets.[4]

In the light of this evidence, it is not difficult to motivate the case for studying optimizing pricing behaviour without assuming zero trend inflation or complete indexation. In this paper, we study the macroeconomic effects of non-zero trend inflation in a simple DSGE model. We solve the model using a second-order approximation of its equilibrium conditions. In contrast to previous studies, we focus on the effects of trend inflation on the stochastic means of macroeconomic aggregates.

Our main findings can be summarized as follows. Price dispersion across different intermediate inputs increases with trend inflation. The stochastic mean of a summary measure of price dispersion increases by more than its deterministic steady state as trend inflation goes up. As a result, the

[1]See Clarida, Galí, and Gertler (1999) for a recent survey.

[2]That is to say, in periods when firms cannot reoptimize their prices, they can nevertheless adjust their prices according to the indexation rule. See Yun (1996) for a derivation in the case of indexation to trend inflation.

[3]In the case of Canada, one can isolate four periods with different levels of non-zero average inflation (see Demers 2003). The average rate of Canadian core inflation from 1961Q2 to 1972Q4 was 2.9 per cent, 9.3 per cent from 1973Q1 to 1981Q4, 4.5 per cent from 1982Q1 to 1990Q4, and 1.9 per cent from 1991Q1 to 2004Q1. Even though the Bank of Canada has had an inflation target range between 1 and 3 per cent for over a decade, it is possible that the target range (and corresponding midpoint) may be raised or lowered in the future.

[4]Ireland (2005) develops a model that allows inferences concerning the Federal Reserve's inflation target. His results indicate an increase from 1.25 per cent in 1959 to more than 8 per cent in the late 1970s, followed by a gradual reduction to below 2.5 per cent in 2004.

stochastic means of variables such as output, consumption, and employment decrease. The stochastic mean of inflation increases by much more than trend inflation when the latter is measured by its mean in a deterministic steady state. The variability and persistence of most aggregates increase with trend inflation, and the persistence of inflation itself is particularly sensitive to trend inflation. Monetary policy is less effective at higher levels of trend inflation, due to a flattening of the Phillips curve. This effect by itself leads to an inverse relationship between trend inflation and welfare. Price dispersion magnifies the welfare costs of trend inflation. Finally, our results hold qualitatively no matter how nominal-price rigidity is modelled, but the quantitative effects are much stronger under so-called Calvo pricing than under Taylor pricing or truncated-Calvo pricing.

Our paper is related to previous studies of the effects of trend inflation. Ascari (2004) and Bakhshi et al. (2003) set up dynamic general-equilibrium models with Calvo pricing. They show that because of price dispersion, the level of output declines in the deterministic steady state as trend inflation rises. Ascari analyzes the effects of trend inflation on output persistence by studying the impulse-response functions of output to a money-growth shock with different levels of trend inflation. Bakhshi et al. carefully examine the effects of non-zero trend inflation on the slope of the NKPC. They find that the curve is flatter at higher levels of trend inflation, so that inflation is less responsive to changes in either the output gap or a measure of firms' real marginal cost. Bakhshi et al. use linearized versions of first-order conditions to derive their New Keynesian Phillips curve. Ascari uses second-order approximations, but limits his analysis of the model's dynamic properties to impulse-response functions. Our paper innovates principally by using second-order approximations to uncover the effects of shocks on the stochastic means of variables, as well as their unconditional second moments (volatility, correlations, and persistence).

In the second section we outline our model. In the third section we discuss how to calibrate its structural parameters, and describe our numerical simulation methodology. We present results in the fourth section. The fifth section concludes.

2 The Model

The economy consists of a representative household with an infinite planning horizon, a representative final-good firm, a collection of monopolistically competitive firms that produce differentiated intermediate goods, and a monetary authority that sets the short-term nominal interest rate following a Taylor rule. It finances its issuance of cash balances with lump-sum taxation. The demand for money is motivated by real balances in the representative household's utility function.

2.1 Households

The representative household maximizes expected utility given by:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, H_t\right), \tag{1}$$

where C_t is consumption, M_t is nominal balances, P_t is the price level, H_t is hours worked, and $\beta \in (0,1)$ is a subjective discount factor. The functional form of period utility is given by:

$$U\left(C_t, \frac{M_t}{P_t}, H_t\right) = \log\left(\left[C_t^{\frac{\sigma-1}{\sigma}} + b_t^{\frac{1}{\sigma}}\left(\frac{M_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right) + \eta\log(1-H_t), \tag{2}$$

where b_t is a preference shock which can be interpreted as a money-demand shock. The parameter $\sigma > 0$ is the elasticity of substitution between consumption and real balances. This functional form leads to a conventional money-demand equation with consumption as the scale variable. The preference shock b_t follows a stationary AR(1) process in logs:

$$\log(b_t) = \rho_b \log(b_{t-1}) + (1-\rho_b)\log(b) + \varepsilon_{b,t}, \tag{3}$$

where $\rho_b \in (0,1)$, and where the stochastic shock term $\varepsilon_{b,t}$ is identically, independently distributed (i.i.d.) normal with a zero mean and a standard deviation of σ_{ε_A}. The representative household's budget constraint in period t is:

$$P_tC_t + P_tI_t + P_tCAC_t + M_t + \frac{B_t}{R_t} \leq P_tw_tH_t + P_tq_tK_t - T_t + D_t + M_{t-1} + B_{t-1}, \tag{4}$$

where w_t is the real wage, q_t is the real rental rate of capital, T_t is a lump-sum tax, D_t denotes nominal dividend payments received from monopolistically competitive firms, I_t is real investment, K_t is the stock of capital, CAC_t is a capital adjustment cost, and R_t is the gross nominal interest rate on debt between t and $t+1$.

Investment increases the household's stock of capital according to

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{5}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. Investment is subject to convex adjustment costs of the following form:

$$CAC_t = \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t, \tag{6}$$

where φ is a positive parameter. The first-order conditions associated with the optimal choice of C_t, M_t, B_t, H_t, and K_{t+1} are given by:

$$\frac{C_t^{\frac{1}{\sigma}}}{C_t^{\frac{\sigma-1}{\sigma}} + b_t^{\frac{1}{\sigma}}\left(\frac{M_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}}} = \lambda_t, \tag{7}$$

$$\frac{b_t^{\frac{1}{\sigma}}\left(\frac{M_t}{P_t}\right)^{-\frac{1}{\sigma}}}{C_t^{\frac{\sigma-1}{\sigma}} + b_t^{\frac{1}{\sigma}}\left(\frac{M_t}{P_t}\right)^{\frac{\sigma-1}{\sigma}}} = \lambda_t\left[1 - \frac{1}{R_t}\right], \tag{8}$$

$$\lambda_t = \beta E_t\left(\lambda_{t+1}\frac{R_t}{\pi_{t+1}}\right), \tag{9}$$

$$\eta\frac{1}{1-H_t} = \lambda_t w_t, \tag{10}$$

$$\lambda_t\left[1 + \varphi\left(\frac{I_t}{K_t} - \delta\right)\right]$$

$$= \beta E_t\left\{\lambda_{t+1}\left[1 + q_{t+1} - \delta + \varphi\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right) + \frac{\varphi}{2}\left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2\right]\right\}, \tag{11}$$

where λ_t is the Lagrange multiplier associated with the period-t budget constraint.

2.2 Firms

2.2.1 Representative final-good firm

The representative competitive final-good firm uses $Y_t(i)$ units of each type of intermediate good to produce Y_t units of the final good using the constant-returns-to-scale production function given by:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{12}$$

where $\theta > 1$ is a parameter denoting the elasticity of substitution between types of differentiated intermediate goods. The final-good firm sells its output at a nominal price, P_t, and chooses Y_t and $Y_t(i)$ for all $i \in [0,1]$ to maximize its profits, given by:

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di, \tag{13}$$

subject to (12) in each period. The first-order conditions for this problem are the constraint and:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\theta} Y_t. \tag{14}$$

Equation (14) expresses the conditional demand for intermediate good i as a decreasing function of its relative price and an increasing function of total output. The exact price index for final output is given by:

$$P_t = \left[\int_0^1 P_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}}. \tag{15}$$

2.2.2 Intermediate-goods firms

Each intermediate-good firm, indexed by i, uses $K_t(i)$ units of capital, $H_t(i)$ units of labour, and aggregate technology, A_t, to produce $Y_t(i)$ units of the intermediate good i. Its production function is:

$$Y_t(i) = A_t K_t(i)^{1-\alpha} H_t(i)^{\alpha}. \tag{16}$$

The level of technology, A_t, follows a stationary AR(1) process given by:

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{A,t}, \tag{17}$$

where $\rho_A \in (0,1)$, and where $\varepsilon_{A,t} \sim N(0, \sigma_{\varepsilon_A})$.

If allowed to reoptimize its price in period t, the firm maximizes the discounted sum of expected future profits:

$$\max E_t \sum_{l=0}^{\infty} d_{t+l} \left(\beta^l \frac{\lambda_{t+l}}{\lambda_t}\right) \left(\frac{D_{t+l}(i)}{P_{t+l}}\right), \tag{18}$$

where D_{t+1} represents dividends in period $t+l$, $(\beta^l \lambda_{t+l}/\lambda_t)$ is the stochastic discount factor used by shareholders to value profits at date $t+l$, and d_{t+l}

is the probability that the price set in time t will still be in force at time $t+l$. Different choices for d_{t+l} will lead to different pricing schemes, as shown below. Nominal dividends, $D_{t+l}(i)$, are given by:

$$D_{t+l}(i) = P_t^*(i)Y_{t+l}(i) - W_{t+l}H_{t+l}(i) - Q_{t+l}K_{t+l}(i), \tag{19}$$

where $P_t^*(i)$ is the price set by the firm in period t, W_t is the nominal-wage rate, and Q_t is the nominal rental rate of capital. The first-order conditions of the firm's problem with respect to $K_t(i)$, $H_t(i)$, and P_t^* are given by:

$$\frac{Q_t}{P_t^*(i)} = (1-\alpha)\psi_t(i)\frac{Y_t(i)}{K_t(i)}, \tag{20}$$

$$\frac{W_t}{P_t^*(i)} = \alpha\psi_t(i)\frac{Y_t(i)}{H_t(i)}, \tag{21}$$

$$P_t^*(i) = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{l=0}^{\infty}\beta^l d_{t+l}\frac{\lambda_{t+l}}{\lambda_t}\psi_{t+l}(i)Y_{t+l}(P_{t+l})^{\theta}}{E_t\sum_{l=0}^{\infty}\beta^l d_{t+l}\frac{\lambda_{t+l}}{\lambda_t}Y_{t+l}(P_{t+l})^{\theta-1}}, \tag{22}$$

where $\psi_t(i)$ denotes the real marginal cost at date t associated with firm i's maximization problem; it is also equal to the inverse of the markup. According to equations (20) and (21), the marginal products of labour and capital both exceed their respective marginal costs. Equation (22) is the firm's optimal-price equation, derived from the equalization of marginal cost with marginal revenue in a dynamic context. The term d_{t+l} gives the probability that the price set by the firm in period t will still apply in period $t+l$. The value of d_{t+l} depends on how we model nominal-price rigidity. We consider three different pricing schemes, as outlined in the following subsection.

2.2.3 Pricing schemes

The pricing schemes that we consider are all time-dependent in that the probability that a firm will readjust its price is either constant or depends only on the length of time since it last reoptimized its price, and does not depend on economic conditions at the firm or aggregate levels. Klenow and Kryvtsov (2005) show that microeconomic data are broadly consistent with time-dependent models of price adjustment.

Calvo (1983) develops a model in which each monopolistically competitive firm has a constant probability of being allowed to revise its price at the beginning of each period:

$$d_{t+l} = d^l, \quad 0 \le l < \infty.$$

Under Calvo pricing, there are an infinite number of cohorts of firms charging different prices. The Calvo pricing scheme has the well-known advantage that it facilitates aggregation and leads to simple laws of motion for the overall price level and for the price newly set by firms in the current period. Eichenbaum and Fisher (2005) find that Calvo pricing is generally well supported by the aggregate data.[5]

We present most of our results for the Calvo pricing scheme. In section 4.5, we compare the results with those obtained under two alternative pricing schemes. Under truncated Calvo pricing, price rigidity lasts, at the most, L periods. If a firm has not been allowed to reset its price after L periods, it resets its price the following period with probability one. This gives the following for d_{t+i}:

$$d_{t+l} = d^l, \quad 0 \leq l < L, \quad d_{t+l} = 0, \quad l \geq L.$$

There are L different cohorts of firms of differing sizes. Under Taylor (1979) pricing, all firms reset their prices after precisely L periods. This gives:

$$d_{t+l} = 1, \quad 0 \leq l < L, \quad d_{t+l} = 0, \quad l \geq L.$$

There are L different cohorts of firms, and firms remain in the same cohort. For simplicity, we assume that the cohorts are of identical size.

2.3 Monetary authority

The monetary authority sets the short-term nominal interest rate in accordance with the following Taylor rule:

$$\begin{aligned}\log(R_t) &= (1-\rho_R)\log(R) + \rho_R \log(R_{t-1}) \\ &+ \rho_\pi \log\left(\frac{\pi_t}{\widetilde{\pi}}\right) + \rho_y \log\left(\frac{Y_t}{Y}\right) + \varepsilon_{R,t}. \end{aligned} \tag{23}$$

Variables without time subscripts denote deterministic steady-state values and $\varepsilon_{R,t}$ is a monetary policy shock with $\varepsilon_{R_t} \sim N(0, \sigma_{\varepsilon_R})$. The Taylor rule immediately implies that in the deterministic steady state the rate of inflation will be equal to $\widetilde{\pi}$. Therefore, it is natural to interpret $\widetilde{\pi}$ as the target level of inflation as well as its deterministic steady-state level.

[5]In their empirical model, Eichenbaum and Fisher allow for capital adjustment costs and variable demand elasticities. We retain the usual assumptions of factor mobility between firms and constant demand elasticity in order to simplify the exposition.

The money stock is determined by the demand for real balances, and lump-sum taxes are used to finance changes in the money supply. The monetary authority has the following simple budget constraint:

$$M_t - M_{t-1} = T_t. \tag{24}$$

2.4 Aggregation

Capital is perfectly mobile across firms. Therefore, all firms share the same capital-to-labour ratio and have identical real marginal costs, so we can drop the (i) argument in equation (22). All firms that optimize their price in a given period will choose the same price, so we can also drop the (i) argument after P_t^*. Firms setting prices at different dates will in general have different relative prices.

Each intermediate firm that sets its relative price accepts to supply demand at that price. Integrating over the conditional demand functions for firms' output given in (14) gives the following aggregate resource constraint:

$$Y_t^s = \left(C_t + K_{t+1} + (1-\delta)K_t + \frac{\varphi}{2}\left(\frac{I_t}{K_t} - \delta\right)^2 K_t \right) \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di. \tag{25}$$

We define

$$S_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\theta} di, \tag{26}$$

and we have:

$$Y_t^s = A_t {K_t}^{(1-\alpha)} {H_t}^{\alpha}, \tag{27}$$

with Y_t^s defined as aggregate supply and where K_t and H_t are, respectively, the aggregate capital stock and aggregate hours worked.

The aggregate resource constraint takes into account the inefficiency in resource allocation induced by price dispersion across firms. Because individual intermediate goods enter symmetrically and with equal weight in the production function for the final good given by equation (12), efficient resource allocation would dictate producing the same amount of each intermediate good. Price dispersion causes the macroeconomic equilibrium to deviate from this optimum. It can be shown that S_t is bounded from below by one.[6] Under Calvo pricing, S_t evolves according to a non-linear first-order difference equation, as shown in the appendix. Under Taylor pricing and truncated Calvo pricing, S_t can be expressed as a weighted average of

[6]See Schmitt-Grohe and Uribe (2005).

past optimal prices set by different cohorts of firms. Under Calvo pricing, the law of motion for S_t is given by:

$$S_t = (1-d)p_t^{*-\theta} + d{\pi_t}^{\theta} S_{t-1}. \tag{28}$$

The complete equation system used to simulate the model is given in the appendix.

3 Calibration and Simulation Methodology

The central bank's use of the short-term nominal interest rate as the instrument of monetary policy conveys a unit root to the price level. In order to solve the model, we normalize nominal variables by dividing by the level of nominal balances in the economy.

We solve the model by numerical simulation, using the Dynare program described in Juillard (2005). The program computes a second-order approximation of the model's equilibrium conditions around its deterministic steady state.[7] As shown by Schmitt-Grohe and Uribe (2004), going from a first-order approximation to a second-order approximation captures the impact of a model's stochastic shocks on the stochastic means of its endogenous variables. This is crucial for our results.

Calibrating the model's structural parameters is a prerequisite for numerical simulations. We calibrate the model to quarterly data using standard parameter values from the literature (Table 1). They are close to the values in Ambler, Dib, and Rebei (2004), who estimate a model similar to the one used here by maximum-likelihood techniques.

Under Calvo pricing, the d parameter captures the constant probability that an individual firm will not be allowed to reset its price at the beginning of a given period. A value of 0.75 implies that firms' prices remain fixed for four quarters on average. With Taylor and truncated-Calvo pricing, we assume that the maximum length of price rigidity is three periods. Under Taylor pricing, all firms reset their price after four periods. Under truncated-Calvo pricing, all firms that have not already been allowed to reset their prices after four periods do so. In principle, there are several different ways to equalize the average length of nominal rigidity across different types of pricing schemes. We could, for example, equalize the unconditional expected duration of price rigidity or the average length that prices have remained fixed for the cross-section of firms at a point in time. The unconditional

[7] The deterministic steady state is the long-run equilibrium of the economy when all stochastic shocks are set equal to zero, with constant levels of all normalized variables.

expected duration of contracts under the truncated-Calvo scheme is slightly shorter than under the other two schemes, but our results do not hinge on this.

4 Results

We present detailed results for Calvo pricing and discuss below how the results change with Taylor and truncated-Calvo pricing.

Tables 2 and 3 present a series of unconditional-moment statistics from our stochastic simulations. The parameter values are those given in Table 1. We compare the properties of our model economy for trend inflation rates of zero and 4 per cent on an annualized basis. Figures 1 and 2 illustrate, respectively, how the stochastic means and standard deviations of output, consumption, the nominal interest rate, and inflation vary with the trend rate of inflation. Figures 3 through 5 illustrate the impulse responses of various endogenous variables to one-standard-deviation shocks to (respectively) technology, monetary policy, and money demand. Figure 6 shows the time paths of the (gross) nominal interest rate from Monte Carlo simulations for different levels of trend inflation. The curves illustrate the mean response of the interest rate as well as 95 per cent confidence intervals around the mean.

4.1 The effect of price dispersion in the steady state

Table 2 shows clearly that the deterministic steady-state of the model is sensitive to trend inflation. The steady-state levels of output, consumption, and hours all fall as trend inflation increases. Steady-state output falls by 1.2 per cent and consumption falls by 1.7 per cent as trend inflation increases from zero to 4 per cent on an annualized basis.

These results confirm those found in similar models by Ascari (2004) and Bakhshi et al. (2003). The results are due to the effects of trend inflation on price dispersion across firms. With positive trend inflation, firms that are allowed to reset their price choose a price above the average price level, since they know that the relative price of their output will be eroded over time. Firms that have not been allowed to reset their price for many periods will have a low relative price. As trend inflation increases, the spread between firms' relative prices when they reset their price and the average price level increases, and the dispersion of prices across firms increases.

4.2 The effect of trend inflation on stochastic means

Table 2 illustrates another important result, which is new relative to the previous literature on the effects of trend inflation. For each of the model's endogenous variables, the spread between its value in the deterministic steady state and its stochastic mean is greater with a trend inflation rate of 4 per cent than with zero trend inflation. The left panels of Figure 1 confirm that the relationship between trend inflation and the spread is monotonic under Calvo pricing.

The mechanism that is responsible for increasing the spread between deterministic steady states and stochastic means operates via the price-dispersion variable, S_t. As equation (28) shows, shocks that modify the optimal relative price of firms that are allowed to adjust their prices have an impact effect on S_t, the size of which depends on the fraction of firms that adjust in any given period, given by $(1-d)$. S_t is also affected by its own first lag, and the size of this effect depends not only on the fraction of firms that do not adjust their prices, given by d, but also on the current value of inflation via the term in ${\pi_t}^{\theta}$. As trend inflation increases, inflation is higher on average and deviations of the dispersion variable are more persistent. The effects are non-linear: persistent increases in the spread have a greater impact on the model's endogenous variables than do decreases, leading to a spread between deterministic steady state and the stochastic mean.

The bottom row of Table 2 shows the impact of trend inflation on the price-dispersion variable itself. As shown by Schmitt-Grohe and Uribe (2005), under zero trend inflation the dispersion variable follows, up to first order, the univariate autoregressive process $S_t = dS_{t-1}$. To a first-order approximation, price dispersion has no real consequences. In addition, we know that going from a first-order to a second-order approximation affects only the stochastic means of variables and not their unconditional second moments. This is reflected by the very slight difference between the stochastic mean of the dispersion variable and its deterministic steady state. This translates into a very small impact on the stochastic means of other variables. With an annualized trend inflation rate of 4 per cent, the spread between the deterministic steady-state level of price dispersion and its stochastic mean goes up considerably. This increase in stochastic mean is transmitted to all of the other variables in the model.

Table 2 also illustrates a remarkable result concerning the average inflation rate. With an annualized trend inflation rate of 4 per cent, the stochastic mean of inflation is close to 7.4 per cent. This means that adopting a positive inflation target leads to an outcome where inflation systematically exceeds its target. When the inflation target is set to zero, inflation is still on average higher than the target, but only slightly. The gap between the inflation target and the stochastic mean of inflation grows to 3.4 per cent when the former increases to 4 per cent. Furthermore, the gap between the level of the short-term nominal interest rate in the deterministic steady state and its stochastic mean is 3.6 per cent. This implies that the stochastic mean of the real interest rate varies directly with trend inflation. This reinforces the negative impact of trend inflation on output and consumption by negatively affecting the average size of the capital stock.[8]

4.3 The effect of trend inflation on volatility and persistence

As trend inflation goes from zero to 4 per cent, the standard deviation of price dispersion increases considerably, as indicated by Table 2. The bottom row of Table 3 shows that, with positive trend inflation, fluctuations in price dispersion are highly persistent, with a first-order autocorrelation coefficient close to one. This translates into a large impact on the standard deviations and the persistence of all of the variables in the model.

Table 2 shows that the standard deviations of all variables except for real balances increase as trend inflation goes from zero to 4 per cent per year. The left panels of Figure 2 show that for output, consumption, nominal interest rates, and inflation, the relationship between trend inflation and the standard deviations of these variables is monotonic. The mechanism is the same as the one driving the spread between deterministic steady states and stochastic means. Since fluctuations in the spread variable S_t become much more persistent at higher rates of trend inflation, the unconditional variance of the spread is much higher at higher rates of trend inflation.

Table 3 shows that the persistence (measured by the first-, second-, and third-order autocorrelations) increases as trend inflation goes from zero to 4 per cent. The only exceptions to this rule are the first-order autocorrelation of the short-term nominal interest rate and the second- and third-order autocorrelations of the real rental rate of capital. Once again, the mechanism driving this increase in persistence operates via the macroeconomic impact of the spread variable S_t. Since fluctuations in S_t are more persistent at higher rates of trend inflation, this persistence spills over into more persistent

[8] If we take the Taylor rule seriously as a model of central bank behaviour, then it is clear that a sophisticated central bank could take the dispersion between the deterministic level and the stochastic mean into account when setting its inflation target. Insofar as this phenomenon is not widely recognized among academic economists, it is unlikely that central banks will be aware of it.

fluctuations of most macroeconomic aggregates. Interestingly, it is inflation itself whose persistence increases the most in response to an increase in trend inflation. The intuition for this effect is clear. Firms are restricted to fixing their nominal price for a number of periods. With low trend inflation, the most important determinant of profits is the expected evolution of real marginal cost. As trend inflation rises, the firm's profits are increasingly affected by the evolution of inflation over the life of the price contract. Firms' pricing decisions become more sensitive to fluctuations in inflation and relatively less sensitive to fluctuations in macroeconomic conditions. Fluctuations in inflation have a relatively larger impact on the optimal price set by firms that can revise their prices. The reduced sensitivity of pricing decisions to economic conditions means that, *ceteris paribus*, inflation takes longer to return to its long-term trend level after the economy experiences a structural shock.

Figures 3 through 5 present evidence concerning persistence in the form of impulse-response functions of endogenous variables to different structural shocks, comparing the impulse responses at zero trend inflation and with a trend inflation rate of 4 per cent. The impulse-response functions are compatible with the evidence provided by Ascari (2004), who shows that impulse responses are magnified and more persistent with higher trend inflation.

4.4 Trend inflation, the effectiveness of monetary policy, and welfare

Under Calvo pricing, the system of equations given in the appendix obscures the trade-off between output variability and inflation variability that is brought out by the NKPC. We show in a companion paper (Amano, Ambler, and Rebei 2005) that it is possible to derive the following extended NKPC using a linear approximation of firms' pricing decisions around the trend rate of inflation:

$$\pi_t = \beta \Pi E_t \pi_{t+1} + \gamma(\Pi) y_t + \varepsilon_t + v_t, \tag{29}$$

where y_t measures the output gap, $\gamma(\Pi)$ is a coefficient that depends inversely on the rate of trend inflation given by Π, ε_t is an ad hoc cost-push shock, and v_t is a term that obeys the following dynamic equation:

$$v_t = \frac{\gamma(\Pi)(\Pi - 1)}{(1 - \theta\Pi^{\lambda})} \left\{ \theta \Pi^{\lambda - 1} [(\lambda - 1) E_t \pi_{t+1} + E_t v_{t+1}] \right\}. \tag{30}$$

Equation (29) illustrates the result shown by Ascari (2004) and Bakhshi et al. (2003) that the Phillips curve becomes flatter at higher rates of trend inflation. At higher rates of trend inflation, firms put a larger weight on future inflation relative to future economic conditions (fluctuations in real marginal cost as proxied by fluctuations in the output gap) when choosing their optimal price.

One implication of a flatter Phillips curve at higher rates of trend inflation is that larger shifts in aggregate demand are required to effect the same change in current inflation. Monetary policy has effects in our model because of its impact on aggregate demand. This means that monetary policy is less effective in reducing short-term fluctuations in inflation at higher rates of trend inflation.[9] We show in our companion paper that this implies a negative relationship between trend inflation and welfare that results from the reduced effectiveness of monetary policy under trend inflation. This negative relationship abstracts completely from the effects of price dispersion on macroeconomic equilibrium and welfare. Once we account for the effects of price dispersion by solving the model using higher-order numerical approximation techniques, the argument for price stability (or at least for a very low rate of trend inflation) becomes even stronger. By reducing the stochastic means of output and consumption, increasing price dispersion at higher rates of trend inflation reinforces the negative impact of trend inflation on welfare. Table 2 illustrates this effect clearly. It shows the deterministic steady-state level of period utility under zero inflation and with inflation at an annualized rate of 4 per cent. Both decline as trend inflation increases, and as with other variables, the spread between the deterministic steady state and the mean increases with trend inflation.[10]

4.5 Taylor and truncated-Calvo pricing

Figures 1 and 2 show that the main results of our paper concerning the effects of trend inflation hold qualitatively under different pricing schemes. The spread between deterministic steady states and stochastic means and standard deviations are still positively related to trend inflation. However, the quantitative impact of trend inflation is very much smaller. These results confirm and extend previous results in the literature. Ascari (2004) shows in a similar model that when Taylor pricing is used instead of Calvo contracts,

[9] For some empirical evidence that this in fact has been the case in the United States, see Boivin and Giannoni (2003).

[10] Period utility is directly related to unconditional expected welfare. It ignores any transitional welfare costs during the transition from high trend inflation to a lower level. Wolman (2001) and Ambler and Entekhabi (2005) show that low positive rates of trend inflation can be beneficial for welfare because they reduce the average size of firms' markups, but this effect is quantitatively not very important.

the level of output in the deterministic steady state is much less sensitive to variations in the level of trend inflation. Bakhshi et al. (2003) show that steady-state inflation has a much smaller influence on the deterministic steady-state level of output in a model with Calvo pricing if the probability that firms revise their prices is made to depend on the level of trend inflation. Our results are new in showing the quantitative importance of different pricing schemes on the spread between deterministic steady states and stochastic means and on the unconditional variances of macroeconomic aggregates.

The reasons for the quantitative importance of price dispersion under Calvo pricing are clear. With Calvo pricing, a small fraction of firms have not been allowed to adjust their nominal prices for many periods. With positive trend inflation, their relative prices are substantially below those of their competitors. As trend inflation increases, they come to capture an increasingly large share of the total market for intermediate goods. Ascari (2004) and Bakhshi et al. (2003) show that for even moderate rates of inflation, the steady-state level of output can fall to zero. The inflation rate at which this occurs depends inversely on the elasticity of substitution across different types of intermediate goods. One interpretation of their results is that at higher rates of inflation the firms with low relative prices capture the entire market, leaving no demand for firms with higher relative prices.

The strong quantitative differences between the stochastic properties of the model under Calvo pricing and other pricing schemes can explain some important differences in results in the literature concerning the macroeconomic effects of trend inflation. For example, two recent papers compare optimal monetary policy and the optimal choice of the inflation target in models with price rigidity.

Schmitt-Grohe and Uribe (2005) analyze optimal monetary policy in a model with Calvo pricing. Their model includes government transfers to households that are calibrated to match their average value in the U.S. data as a fraction of GDP. In this context, inflation acts as a non-distortionary way of taxing back a fraction of government transfers. By itself, this effect would lead to an optimal inflation target that is significantly greater than zero. However, Schmitt-Grohe and Uribe find that the optimal inflation rate is very close to zero. In their model, there are three fundamental forces that affect the optimal inflation rate. The first is the public finance effect that would lead to positive inflation. The second is the Friedman rule, which is operative in their model and would lead to negative inflation. The third is the impact of price dispersion both in the deterministic steady states and in response to shocks. Schmitt-Grohe and Uribe's results suggest that, with Calvo pricing, it is the effect of price dispersion on macroeconomic equilibrium and welfare that dominates quantitatively. They also find that the variability of inflation under optimal monetary policy is very low.

Laforte (2003) studies optimal monetary policy in a model with firms that adjust their prices subject to quadratic adjustment costs. In such a model, the distribution of relative prices across firms is degenerate as long as firms initially charge the same price and are identical. This assumption completely eliminates the macroeconomic effects of price dispersion from the model. Laforte finds an optimal inflation rate that is not as close to zero, and the variability of inflation under optimal monetary policy is much higher than in Schmitt-Grohe and Uribe (2005). The relative importance of the effects of price dispersion depending on different pricing schemes can be used to interpret other results in the literature on optimal monetary policy.

5 Conclusions

We have shown that the macroeconomic properties of a simple dynamic general-equilibrium model with nominal-price rigidities are quite sensitive to the assumed level of trend inflation due to the impact on resource allocation of price dispersion across monopolistically competitive firms. We go beyond previous results by showing that the effects of shocks on the stochastic means and unconditional variances of variables including output and consumption depend on the level of trend inflation. We confirm and extend previous results by showing that the quantitative effects of price dispersion on macroeconomic equilibrium are much more important under Calvo pricing than under alternative pricing schemes.

To the extent that issues such as the optimal rate of target inflation and the optimal variability of inflation relative to output hinge on the importance of price dispersion on macroeconomic equilibrium, the path for future research is clear. We need more detailed comparisons of the quantitative effects of trend inflation on price dispersion in dynamic general-equilibrium models, and we also need to uncover empirical evidence on the relationship between average inflation and price dispersion, when the latter is measured in ways that are comparable with the price-dispersion variables in our theoretical models.

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Table 1: Model Calibration

Parameter	Value
Preferences	
β	0.99
σ	0.25
η	1.5
Technology	
α	0.34
θ	8.00
δ	0.025
φ	10.0
Price adjustment	
d	0.75
Taylor rule coefficients	
ρ_R	0.80
ρ_π	1.50
ρ_y	0.20
Stochastic processes	
ρ_b	0.80
σ_{ε_b}	0.01
ρ_A	0.80
σ_{ε_A}	0.01
σ_{ε_R}	0.01

Table 2: Steady States, Average Values, and Standard Deviations

	$\pi = 1.00^{\frac{1}{4}}$			$\pi = 1.04^{\frac{1}{4}}$		
Variable	Steady state	Average	Std	Steady state	Average	Std
y_t	0.7877	0.7820	0.0262	0.7728	0.7323	0.0345
c_t	0.6405	0.6368	0.0084	0.6290	0.5994	0.0151
m_t	0.5063	0.4983	0.0490	0.4199	0.3624	0.0373
i_t	0.1473	0.1451	0.0212	0.1438	0.1328	0.0252
h_t	0.3326	0.3317	0.0124	0.3301	0.3245	0.0154
w_t	1.4508	1.4420	0.0341	1.4270	1.3543	0.0493
r_t^k	0.0351	0.0354	0.0021	0.0351	0.0355	0.0026
π_t	1.0000	1.0012	0.0044	1.0098	1.0181	0.0054
ψ_t	0.8750	0.8738	0.0289	0.8708	0.8575	0.0357
R_t	1.0101	1.0113	0.0034	1.0201	1.0284	0.0052
S_t	1.0000	1.0009	0.0000	1.0068	1.0246	0.0075

Table 3: Autocorrelations: Order 1, 2, and 3

	$\pi = 1.00^{\frac{1}{4}}$			$\pi = 1.04^{\frac{1}{4}}$		
Variable	Autoc. 1	Autoc. 2	Autoc. 3	Autoc. 1	Autoc. 2	Autoc. 3
y_t	0.4104	0.3293	0.2832	0.5216	0.4596	0.4290
c_t	0.9278	0.8743	0.8278	0.9678	0.9421	0.9185
m_t	0.8489	0.7441	0.6591	0.9258	0.8724	0.8270
i_t	0.2246	0.1479	0.1138	0.2232	0.1510	0.1279
h_t	0.0495	0.0009	-0.0024	0.0708	0.0106	0.0066
w_t	0.3129	0.2600	0.2415	0.5238	0.4794	0.4642
r_t^k	0.0924	0.0420	0.0353	0.0956	0.0341	0.0276
π_t	0.4854	0.3935	0.3354	0.7903	0.7230	0.6727
ξ_t	0.0880	0.0349	0.0262	0.0989	0.0355	0.0271
r_t	0.8384	0.7274	0.6378	0.9140	0.8542	0.8035
S_t	—	—	—	0.9889	0.9688	0.9434

Figure 1: Stochastic Means

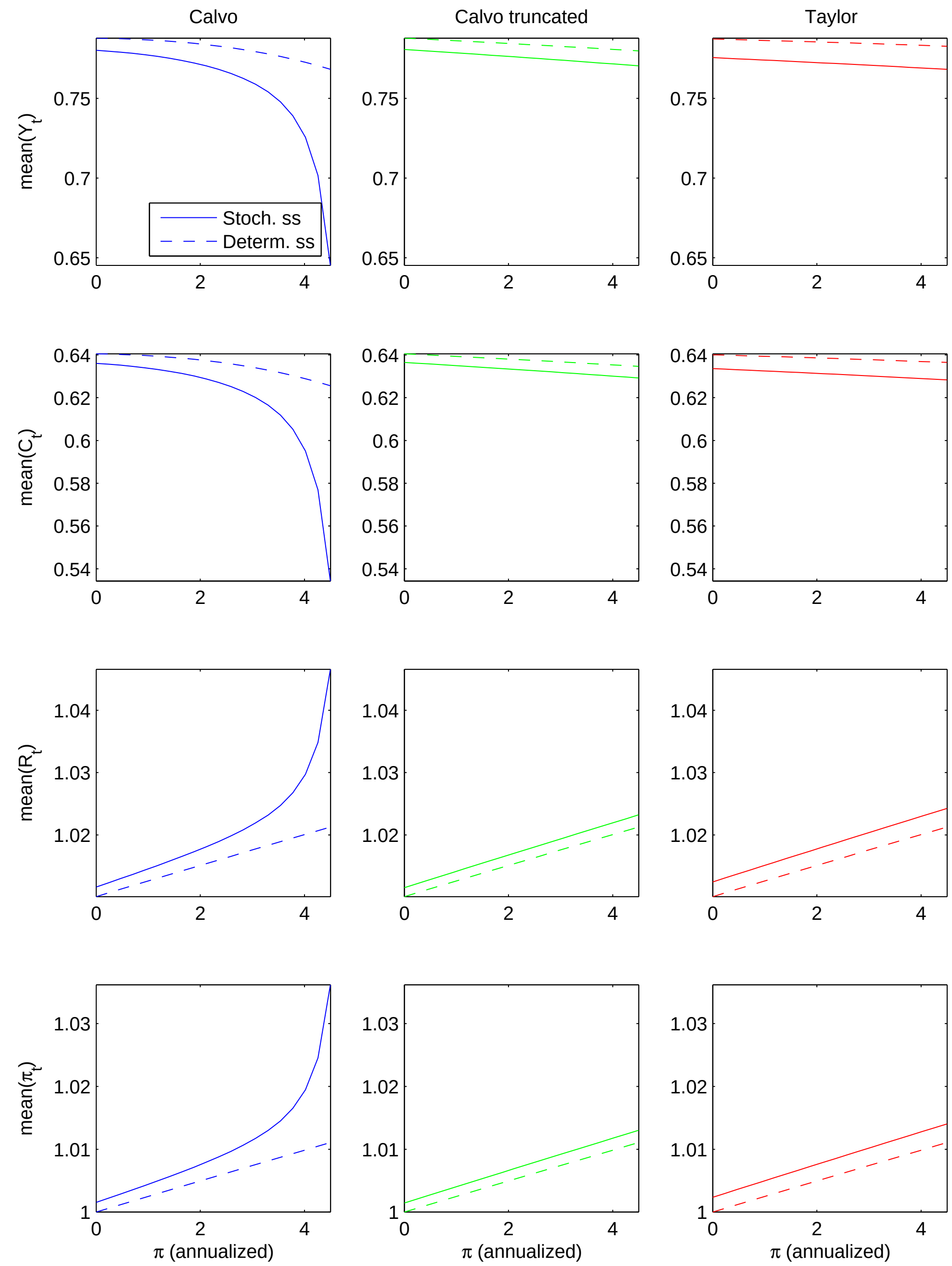


Figure 2: Standard Deviations

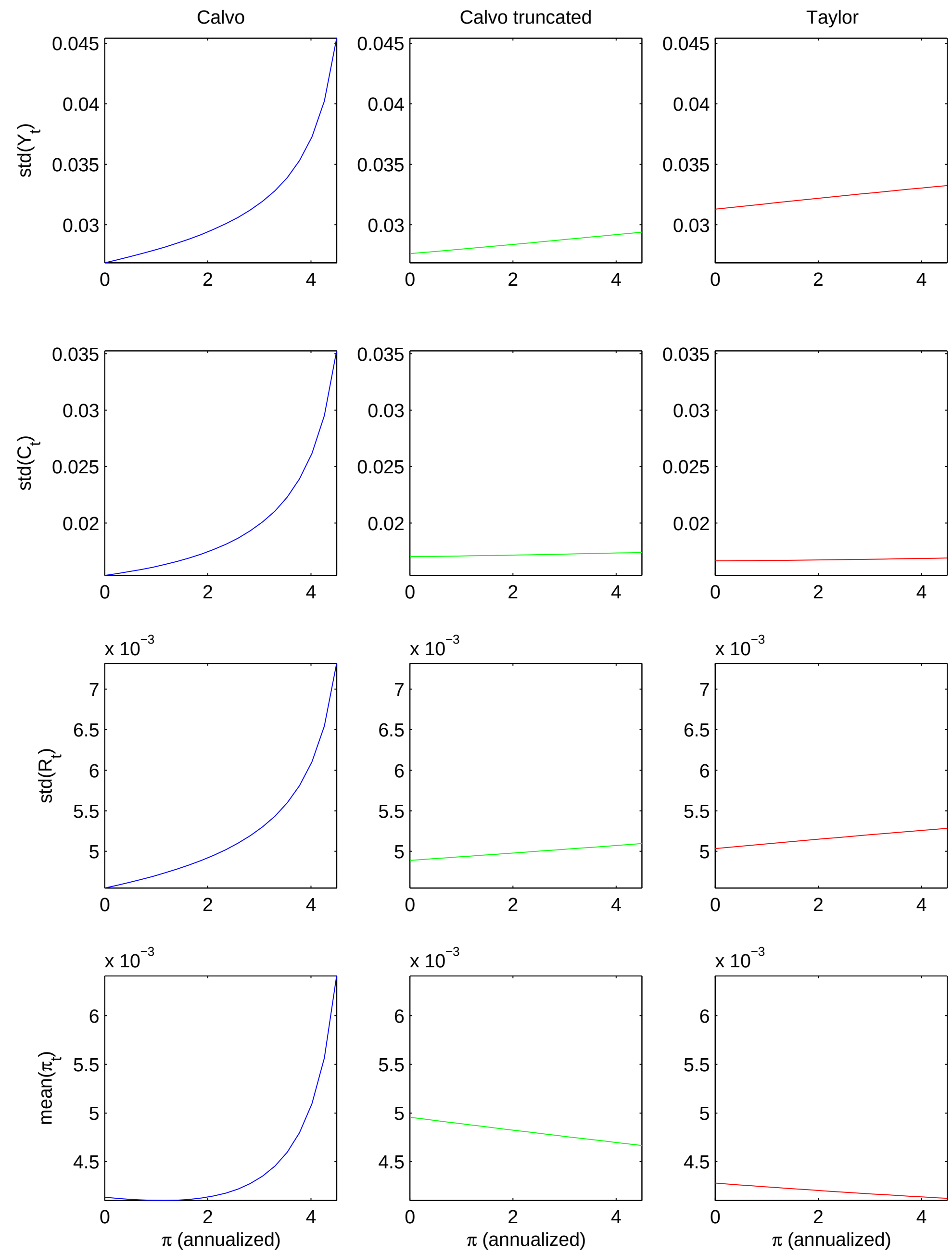


Figure 3: Technology Shock

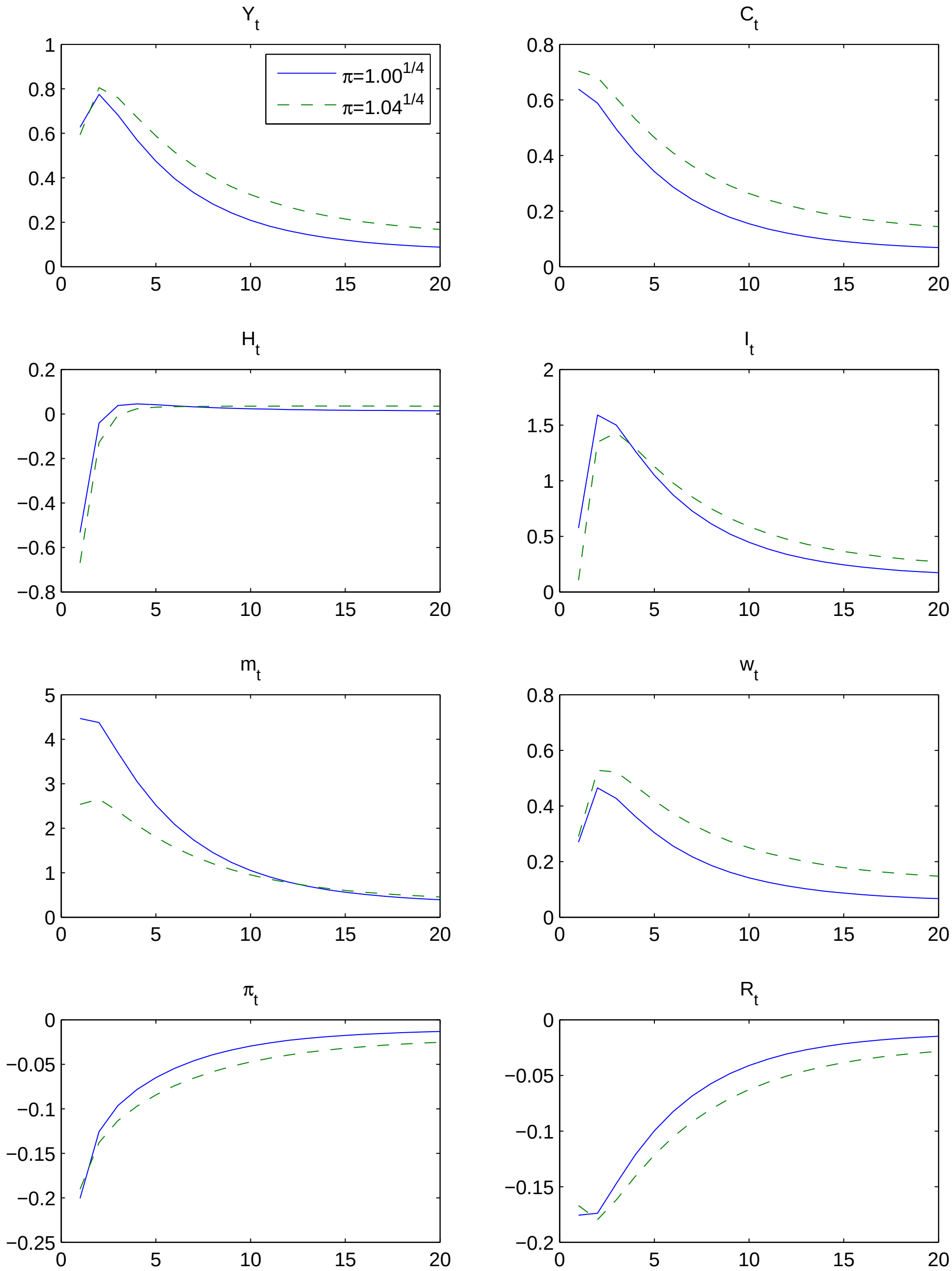


Figure 4: Monetary Policy Shock

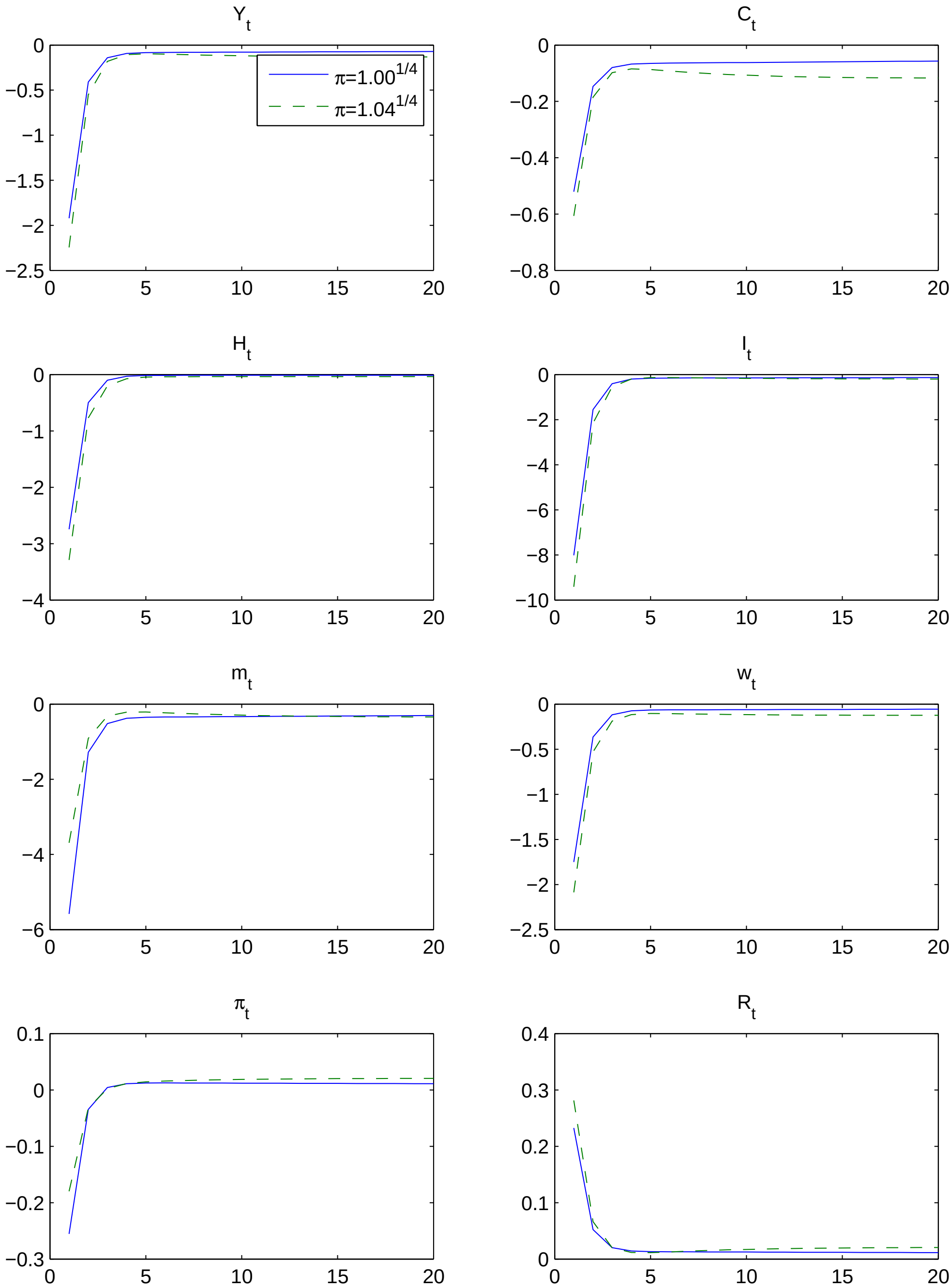


Figure 5: Money-Demand Shock

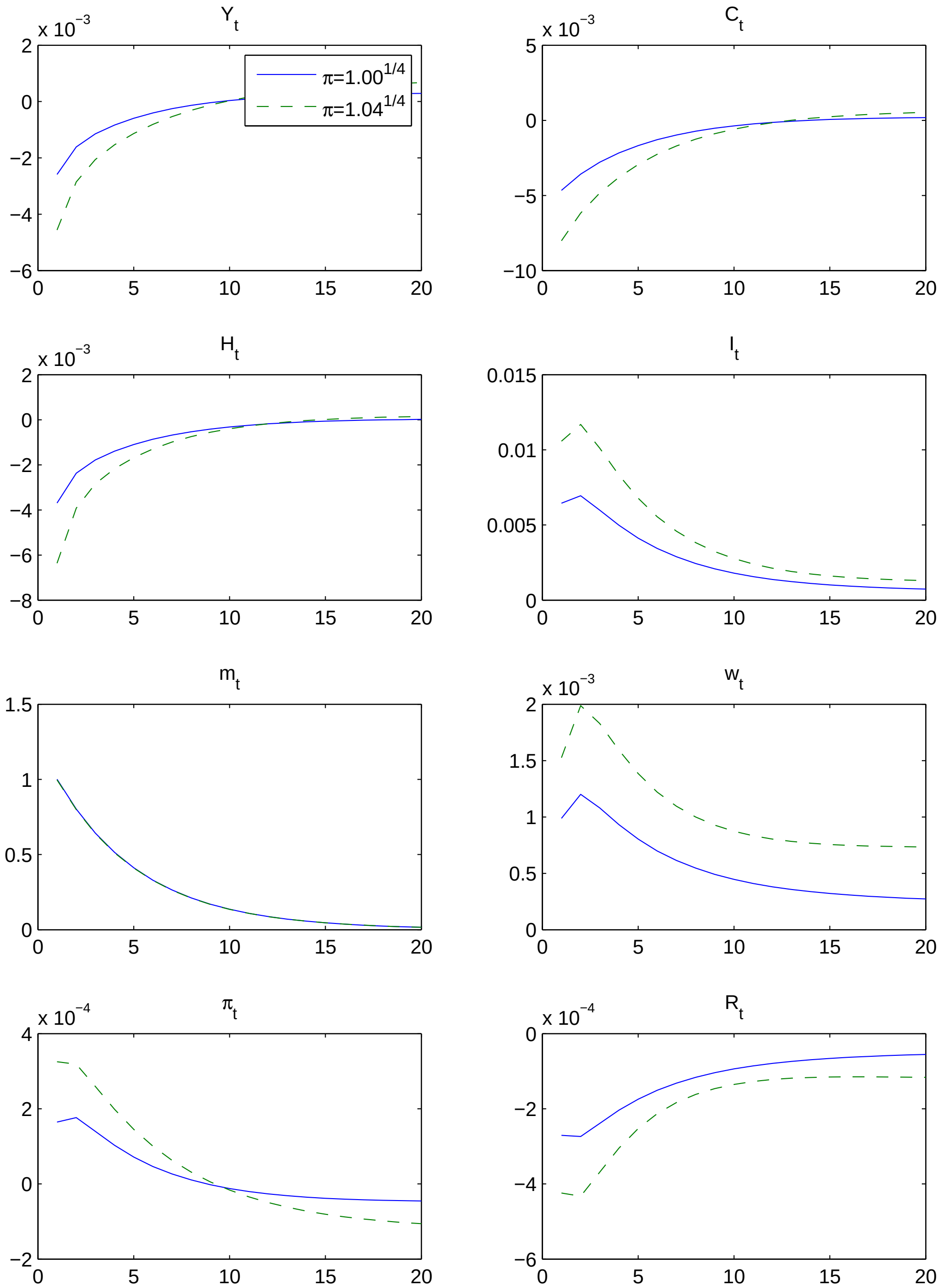
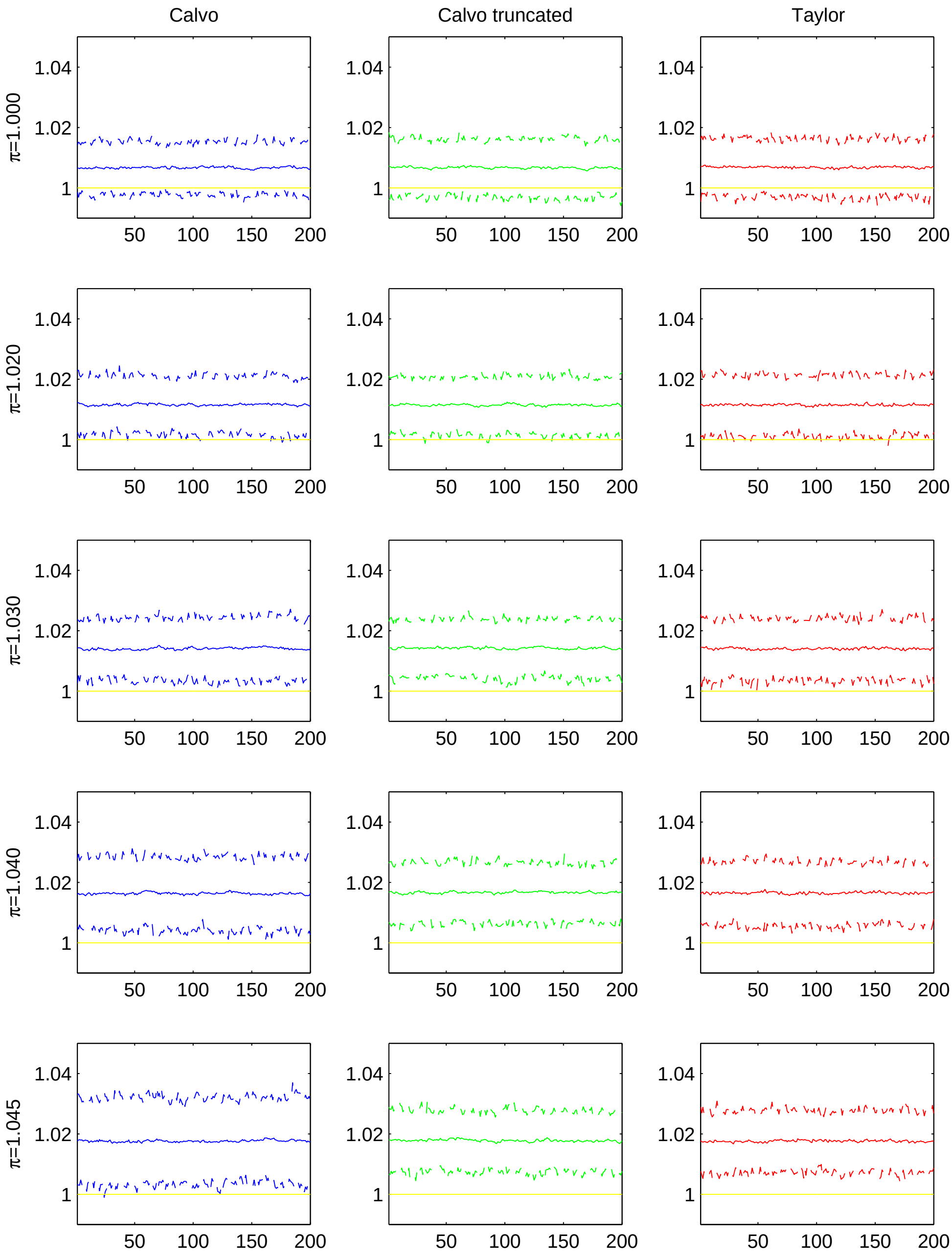


Figure 6: Model Simulated Data



Appendix

Here, we summarize the equation system used to simulate the model. There is one unit root in the model, induced by the monetary policy rule, which specifies that the nominal interest rate should react to deviations of inflation from its target. As a result, the price level, the price set by firms adjusting their optimal price in period t, nominal money balances, and the nominal wage rate have a unit root and are cointegrated. We normalize all nominal variables by dividing through by the price level.

After normalizing, we get the following equations that are common to the three different pricing schemes that we consider:

$$\frac{C_t^{\frac{1}{\sigma}}}{C_t^{\frac{\sigma-1}{\sigma}} + b_t^{\frac{1}{\sigma}} m_t^{\frac{\sigma-1}{\sigma}}} = \lambda_t;$$

$$\frac{b_t^{\frac{1}{\sigma}} m_t^{-\frac{1}{\sigma}}}{C_t^{\frac{\sigma-1}{\sigma}} + b_t^{\frac{1}{\sigma}} m_t^{\frac{\sigma-1}{\sigma}}} = \lambda_t \left[1 - \frac{1}{R_t}\right];$$

$$\lambda_t = \beta E_t \left(\lambda_{t+1} \frac{R_t}{\pi_{t+1}}\right);$$

$$\eta \frac{1}{1 - H_t} = \lambda_t w_t;$$

$$\lambda_t \left[1 + \varphi \left(\frac{I_t}{K_t} - \delta\right)\right]$$

$$= \beta E_t \left\{\lambda_{t+1} \left[1 + q_{t+1} - \delta + \varphi \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right) + \frac{\varphi}{2} \left(\frac{I_{t+1}}{K_{t+1}} - \delta\right)^2\right]\right\};$$

$$K_{t+1} = (1 - \delta) K_t + I_t;$$

$$q_t = (1 - \alpha) \psi_t \frac{Y_t^s}{K_t};$$

$$w_t = \alpha \psi_t \frac{Y_t^s}{H_t};$$

$$Y_t^s = A_t K_t^{(1-\alpha)} H_t^{\alpha};$$

$$Y_t^s = Y_t S_t;$$

$$Y_t = C_t + K_{t+1} + (1 - \delta) K_t + \frac{\varphi}{2} \left(\frac{I_t}{K_t} - \delta\right)^2 K_t;$$

$$\log(R_t) = (1-\rho_R)\log(R) + \rho_R \log(R_{t-1}) + \rho_\pi \left(\frac{\pi_t}{\widetilde{\pi}}\right) + \rho_y \left(\frac{Y_t}{Y}\right) + \varepsilon_{R,t};$$

$$\log(b_t) = \rho_b \log(b_{t-1}) + (1-\rho_b)\log(b) + \varepsilon_{b,t};$$

$$\log(A_t) = (1-\rho_A)\log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{A,t}.$$

The model is completed by a set of pricing equations for firms. These equations are different under Calvo pricing, Taylor pricing, and truncated-Calvo pricing.

Calvo pricing

Under Calvo pricing, we can write the optimal pricing equation as well as the measure of price dispersion in recursive form. See Schmitt-Grohe and Uribe (2005) for a detailed derivation. Price dispersion evolves according to a non-linear first-order difference equation:

$$S_t = (1-d)p_t^{*-\theta} + d{\pi_t}^{\theta} S_{t-1},$$

where we define $p_t^* \equiv P_t^*/P_t$, the relative price of firms that reset their price in the current period. The overall price level is just a weighted average of the last period's price level and the price set by firms adjusting in the current period. This gives:

$$1 = d{\pi_t}^{(\theta-1)} + (1-d)p_t^{*(1-\theta)}.$$

It is somewhat more complicated to derive the equations for the evolution of the price set by firms adjusting in the current period. It is possible to express each of the two infinite sums in equation (22) recursively, in terms of two artificial variables, labelled x_t and z_t. This gives:

$$x_t = Y_t \psi_t {p_{t-1}^*}^{(-\theta-1)} + d\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^*}{p_{t+1}^*}\right)^{(-\theta-1)} {\pi_{t+1}}^{\theta} x_{t+1} \right\},$$

$$z_t = Y_t {p_{t-1}^*}^{-\theta} + d\beta E_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{p_t^*}{p_{t+1}^*}\right)^{-\theta} {\pi_{t+1}}^{(\theta-1)} z_{t+1} \right\},$$

$$x_t = \frac{\theta}{\theta-1} z_t.$$

Under Calvo pricing, we have a system of 19 equations for the following endogenous variables: C, b, m, λ, R, π, H, I, K, Y, Y^s, S, q, w, ψ, A, p^*, x, z.

Taylor pricing

Under four-period Taylor pricing, this becomes:

$$\left(\frac{\theta}{\theta-1}\right)E_t\Bigg\{\psi_t Y_t\left(\frac{P_t^*}{P_t}\right)^{-\theta}+\beta\frac{\lambda_{t+1}}{\lambda_t}\psi_{t+1}Y_{t+1}\left(\frac{P_t^*}{P_t}\frac{P_t}{P_{t+1}}\right)^{-\theta}$$

$$+\beta^2\frac{\lambda_{t+2}}{\lambda_t}\psi_{t+2}Y_{t+2}\left(\frac{P_t^*}{P_t}\frac{P_t}{P_{t+1}}\frac{P_{t+1}}{P_{t+2}}\right)^{-\theta}+\beta^3\frac{\lambda_{t+3}}{\lambda_t}\psi_{t+3}Y_{t+3}\left(\frac{P_t^*}{P_t}\frac{P_t}{P_{t+1}}\frac{P_{t+1}}{P_{t+2}}\frac{P_{t+2}}{P_{t+3}}\right)^{-\theta}\Bigg\}$$

$$=E_t\Bigg\{Y_t\left(\frac{P_t^*}{P_t}\right)^{(1-\theta)}+\beta\frac{\lambda_{t+1}}{\lambda_t}Y_{t+1}\left(\frac{P_t^*}{P_t}\frac{P_t}{P_{t+1}}\right)^{(1-\theta)}$$

$$+\beta^2\frac{\lambda_{t+2}}{\lambda_t}Y_{t+2}\left(\frac{P_t^*}{P_t}\frac{P_t}{P_{t+1}}\frac{P_{t+1}}{P_{t+2}}\right)^{(1-\theta)}+\beta^3\frac{\lambda_{t+3}}{\lambda_t}Y_{t+3}\left(\frac{P_t^*}{P_t}\frac{P_t}{P_{t+1}}\frac{P_{t+1}}{P_{t+2}}\frac{P_{t+2}}{P_{t+3}}\right)^{(1-\theta)}\Bigg\}.$$

This can be rewritten as follows:

$$\left(\frac{\theta}{\theta-1}\right)E_t\Bigg\{\psi_t Y_t+\beta\frac{\lambda_{t+1}}{\lambda_t}\psi_{t+1}Y_{t+1}{\pi_{t+1}}^{\theta}$$

$$+\beta^2\frac{\lambda_{t+2}}{\lambda_t}\psi_{t+2}Y_{t+2}(\pi_{t+1}\pi_{t+2})^{\theta}+\beta^3\frac{\lambda_{t+3}}{\lambda_t}\psi_{t+3}Y_{t+3}(\pi_{t+1}\pi_{t+2}\pi_{t+3})^{\theta}\Bigg\}$$

$$=$$

$$p_t^*E_t\Bigg\{Y_t+\beta Y_{t+1}\frac{\lambda_{t+1}}{\lambda_t}{\pi_{t+1}}^{(\theta-1)}$$

$$+\beta^2Y_{t+2}\frac{\lambda_{t+2}}{\lambda_t}(\pi_{t+1}\pi_{t+2})^{(\theta-1)}+\beta^3Y_{t+3}\frac{\lambda_{t+3}}{\lambda_t}(\pi_{t+1}\pi_{t+2}\pi_{t+3})^{(\theta-1)}\Bigg\}.$$

With four-period Taylor pricing, there are four cohorts of equal size. The price level is given very simply by:

$${P_t}^{(1-\theta)}=0.25\Bigg\{P_t^{*(1-\theta)}+{P_{t-1}^*}^{(1-\theta)}+{P_{t-2}^*}^{(1-\theta)}+{P_{t-3}^*}^{(1-\theta)}\Bigg\}.$$

This can be written as follows:

$$1=0.25\Bigg\{p_t^{*(1-\theta)}+{p_{t-1}^*}^{(1-\theta)}{\pi_t}^{(\theta-1)}+{p_{t-2}^*}^{(1-\theta)}(\pi_t\pi_{t-1})^{(\theta-1)}+{p_{t-3}^*}^{(1-\theta)}(\pi_t\pi_{t-1}\pi_{t-2})^{(\theta-1)}\Bigg\}.$$

The price dispersion index also has a simple representation under four-period Taylor pricing. We have:

$$S_t = 0.25\left\{ \left(\frac{P_t^*}{P_t}\right)^{-\theta} + \left(\frac{P_{t-1}^*}{P_t}\right)^{-\theta} + \left(\frac{P_{t-2}^*}{P_t}\right)^{-\theta} + \left(\frac{P_{t-3}^*}{P_t}\right)^{-\theta} \right\}.$$

This can be rewritten as follows:

$$S_t = 0.25\left\{ {p_t^*}^{-\theta} + {p_{t-1}^*}^{-\theta}{\pi_t}^{\theta} + {p_{t-2}^*}^{-\theta}(\pi_t\pi_{t-1})^{\theta} + {p_{t-3}^*}^{-\theta}(\pi_t\pi_{t-1}\pi_{t-2})^{\theta} \right\}.$$

This gives three equations (the equation for the optimal revised price, the price level definition, and the law of motion for the price-dispersion variable). We have a system of 17 equations in 17 unknowns.

Truncated-Calvo pricing

The probability that the firm's price contract will still be in force is equal to d^i as long as $d < L$. The equation determining the evolution of p_t^* under truncated-Calvo pricing becomes:

$$\left(\frac{\theta}{\theta-1}\right) E_t\left\{ \psi_t Y_t + \beta d \frac{\lambda_{t+1}}{\lambda_t}\psi_{t+1}Y_{t+1}{\pi_{t+1}}^{\theta} \right.$$

$$\left. +(\beta d)^2\frac{\lambda_{t+2}}{\lambda_t}\psi_{t+2}Y_{t+2}(\pi_{t+1}\pi_{t+2})^{\theta} + (\beta d)^3\frac{\lambda_{t+3}}{\lambda_t}\psi_{t+3}Y_{t+3}(\pi_{t+1}\pi_{t+2}\pi_{t+3})^{\theta} \right\}$$

$$=$$

$$p_t^* E_t\left\{ Y_t + \beta d Y_{t+1}\frac{\lambda_{t+1}}{\lambda_t}{\pi_{t+1}}^{(\theta-1)} \right.$$

$$\left. +(\beta d)^2 Y_{t+2}\frac{\lambda_{t+2}}{\lambda_t}(\pi_{t+1}\pi_{t+2})^{(\theta-1)} + (\beta d)^3 Y_{t+3}\frac{\lambda_{t+3}}{\lambda_t}(\pi_{t+1}\pi_{t+2}\pi_{t+3})^{(\theta-1)} \right\}.$$

Under truncated-Calvo pricing, firms that are currently resetting their prices are doing so either because they have randomly drawn the right to do so or because they have not reset their price for L periods. The cohort of firms that set their price i periods ago is a fraction d^i as big as the cohort currently setting their price. So, the size x_i of cohort i (where $0 \leq i < L$) is given by:

$$x_i = \frac{(1-d)}{(1-d^L)}d^i,$$

so that the sum of the x_i is equal to one.

The price index with four-period truncated-Calvo pricing can be written as follows:

$$1 = \left\{ \frac{(1-d)}{1-d^4)} p_t^{*(1-\theta)} + \frac{(1-d)}{(1-d^4)} d p_{t-1}^{*\,(1-\theta)} \pi_t^{\,(\theta-1)} \right.$$

$$\left. + \frac{(1-d)}{(1-d^4)} d^2 p_{t-2}^{*\,(1-\theta)} (\pi_t \pi_{t-1})^{(\theta-1)} + \frac{(1-d)}{(1-d^4)} d^3 p_{t-3}^{*\,(1-\theta)} (\pi_t \pi_{t-1} \pi_{t-2})^{(\theta-1)} \right\}.$$

The price-dispersion index can be written as follows:

$$S_t = \left\{ \frac{(1-d)}{(1-d^4)} p_t^{*-\theta} + \frac{(1-d)}{(1-d^4)} d p_{t-1}^{*\,-\theta} \pi_t^{\,\theta} \right.$$

$$\left. + \frac{(1-d)}{(1-d^4)} d^2 p_{t-2}^{*\,-\theta} (\pi_t \pi_{t-1})^{\theta} + \frac{(1-d)}{(1-d^4)} d^3 p_{t-3}^{*\,-\theta} (\pi_t \pi_{t-1} \pi_{t-2})^{\theta} \right\}.$$

With truncated-Calvo pricing, we have a system of 17 equations in 17 unknowns.

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