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# The Bank of Canada's New Quarterly Projection Model

# Part 3

# The Dynamic Model: QPM

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The views expressed in this report are solely those of the authors. No responsibility for them should be attributed to the Bank of Canada.

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## ABSTRACT

The Bank of Canada's new Quarterly Projection Model, QPM, combines the short-term dynamic properties necessary to support regular economic projections with the consistent behavioural structure necessary for policy analysis. The theoretical underpinnings of the model and the properties of its dynamically stable steady state are described in the first volume of this series. In this third volume, the authors review the history of macro modelling at the Bank and how that history has conditioned the nature of QPM and the methodology used in its construction. They then describe the model, focussing on the types of shocks it was designed to handle and the key elements of its dynamic structure. Two important features of that dynamic structure are forward-looking expectations and endogenous policy rules. Unlike previous Bank models, QPM is not estimated; rather, it is calibrated to reflect the Canadian data. The authors discuss the methodology of calibration and provide examples of how it was done for QPM. Finally, they illustrate QPM's properties in dynamic simulation by describing the results of numerous shocks to the model.

# RÉSUMÉ

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Le nouveau Modèle trimestriel de prévision (MTP) de la Banque du Canada combine les propriétés dynamiques à court terme nécessaires pour appuyer les projections économiques périodiques et la structure de comportement cohérente indispensable à l'analyse de la politique monétaire. Les bases théoriques du régime permanent du modèle et ses propriétés sont décrites dans la première étude de la série se rapportant à Dans la présente étude, la troisième de la série, les auteurs celui-ci. examinent l'évolution de la modélisation macroéconomique à la Banque dans une perspective historique et la manière dont cette évolution a conditionné la nature et la méthodologie du MTP. Les auteurs procèdent ensuite à une description du modèle en mettant l accent sur les types de chocs que celui-ci était sensé caractériser et les éléments clés de sa structure dynamique, qui est stable. Deux des caractéristiques importantes de cette dernière sont les anticipations prospectives et les règles de politique endogènes. Contrairement à certains modèles construits précédemment à la Banque, MTP n'est pas estimé; il est plutôt étalonné de manière à refléter les données canadiennes. Les auteurs commentent la méthode d'étalonnage et expliquent comment elle a été utilisée dans le cas du MTP. Puis, pour illustrer les propriétés du modèle obtenues en simulation dynamique, ils décrivent les résultats de nombreux chocs qui lui sont appliqués.

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# 1 INTRODUCTION

What are the implications for inflation and output of an increase in foreign demand for domestic goods and services? What must a monetary authority do when faced with a loss of confidence in the domestic currency? What role does wage-setting play in the inflation process? What determines the relative roles of interest rates and exchange rates in the transmission mechanism for monetary policy? Of what importance to monetary policy are improvements in production techniques? What are the implications for monetary policy of a change in fiscal policy? What are the implications of changing levels of public sector debt? What would be the cyclical costs of reducing the targeted rate of inflation? These are but a few examples of the difficult policy issues that central banks the world over must deal with on a regular basis.

In 1990, a small team of economists at the Bank of Canada set out to build a model that could address such questions in research on monetary policy issues and in the context of the Bank's regular quarterly projection exercise. In the autumn of 1993, their work came to fruition when the staff of the Bank began using MTP/QPM, or the *Modèle trimestriel de prévision*/ Quarterly Projection Model. In English, the model is referred to as QPM.<sup>1</sup>

Macroeconomic models have always played a central role at the Bank of Canada. Model-based quarterly projection exercises represent an important channel through which the Bank staff communicate views to senior management and vice versa.<sup>2</sup> Furthermore, questions regarding the implications of significant changes in macroeconomic policy or the structure of the economy are usually examined within the context of macroeconomic models. Models provide a language and a framework through which the staff at the Bank can bring together and synthesize what are seemingly disparate ideas. Models do this, in part, by enforcing

<sup>1.</sup> An overview of QPM, directed at a general audience, can be found in Poloz, Rose and Tetlow (1994).

<sup>2.</sup> A discussion of how economic projections figure in the Bank's policy formulation process is provided in Duguay and Poloz (1994) and Longworth and Freedman (1995).

accounting and behavioural constraints on the analysis and on the projection process, thereby helping the staff to separate the wheat from the chaff in the complex flow of data and other information about the economy and to create a coherent outlook. Moreover, since models embody a view of the functioning of the macro economy, they confront users with the aggregate and general equilibrium consequences of specific sectoral or partial equilibrium perceptions.

The Bank of Canada has devoted resources to understanding the economy through formal macroeconomic models for more than 25 years. During this time, several models have been constructed that have enjoyed varying degrees of success in addressing the needs of the projection environment or the requirements of policy analysis. None have been successful in addressing both on a systematic basis. The builders of QPM drew on this history to encompass in one framework those aspects of modelling that are necessary for policy analysis, while respecting the practical requirements of a projection environment.

QPM differs in significant ways from previous models built and maintained at the Bank. Dynamic macro models have often been built without a formal steady state (long-run equilibrium). Since the existence of a well-defined steady state was judged to be critical for dealing with the many contemporary policy questions that require a medium- to long-term perspective, the model builders included this feature in QPM. Indeed, not only is there a steady state within QPM, but it can be simulated as a wholly separate model, distinct from the dynamic structure of QPM itself. This distinction between dynamics and steady-state analysis is just one item, albeit a very important one, that makes QPM different from its predecessors.<sup>3</sup>

<sup>3.</sup> There are also some satellite models in the QPM system, which take the aggregate dynamic simulation results from QPM and break them down into more detailed components. For example, an investment satellite model takes total business fixed investment from QPM and splits it into its machinery-and-equipment and non-residential-construction components.

The structure and properties of the steady state of QPM are presented in the first report in this series documenting the model.<sup>4</sup> The second report describes a new algorithm developed at the Bank for simulating complex, non-linear models with forward-looking behaviour.<sup>5</sup> This, the third report in the series, focusses on the dynamics of QPM – how the model characterizes adjustment towards the steady state and its properties in dynamic simulation when perturbed by a variety of disturbances or "shocks."

Given the need for a model as a tool for organizing economic projections, QPM's short-term properties have been configured to reflect a number of key assessments as to how the economy functions and how monetary policy works. In the sense that there has been substantial focus on the data in this process, QPM is rooted in the historical tradition of the Bank. In something of a break from the past, however, the model is *calibrated* to reflect the empirical evidence, rather than directly estimated. The model builders contributed some of their own perspectives in the calibration, based on estimation and examination of the "stylized facts" contained in the postwar data, together with characterizations of particular historical episodes. Other research, both from within the Bank and from the literature generally, has also been used, wherever possible, to establish what properties are to be judged "reasonable." It is these properties that have been extracted from empirical studies for the calibration exercise, not specific parameters or structure.

QPM's medium- to long-term dynamics are derived using modern methods, which emphasize and impose an economic logic on the process.<sup>6</sup> The model has a steady state that will generally change in response to a shock in a manner that has a clear interpretation in terms of the effect of that shock on market equilibria. Moreover, the model has been calibrated to be dynamically stable – to generate a dynamic path that converges on

<sup>4.</sup> See Black, Laxton, Rose and Tetlow (1994).

<sup>5.</sup> See Armstrong, Black, Laxton and Rose (1995).

<sup>6.</sup> This aspect of QPM built on insights gained in developing the Small Annual Model, SAM, a decade earlier. See Rose and Selody (1985).

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the steady state – while retaining plausible properties in the short run. In this sense, QPM bridges the chasm that has often separated forecasting models from policy simulation models.

Naturally, to achieve this unification, certain compromises had to be made and new ground broken on making the logical structure of policy models consistent with the data. How this was done in characterizing the steady state is covered in the first volume of this series – Black, Laxton, Rose and Tetlow (1994). This third report describes the methodology used to characterize dynamics in QPM, but not the full array of tools developed to use the model in describing a practical control scenario within the quarterly projection exercise at the Bank. Some of that will be the subject of subsequent documentation. Here, the focus is on model properties, meaning how the model describes response to shocks, including changes of policy, around a control solution.

This report is not a typical documentation of a model. Unlike traditional macroeconometric models, QPM is much more than the sum of its individual equations; the model was built "from the top down" so as to bring aggregate macro behaviour to the forefront of the analysis. In keeping with this, our approach is to present the key ideas that lie behind the model's dynamic structure and to document the results through extensive discussion of its properties, rather than through the details of individual equations. Given the philosophy of modelling outlined below and the structure of the model, we submit that little is lost in taking this approach.

Following this introduction, we review, in Section 2, the history of modelling at the Bank of Canada, which provides important insight into the roots of the philosophy that underlies QPM and its construction. Section 2 also lays out the goals that were set for QPM and what the model has to offer in terms of providing a framework for economic analysis. Any discussion of this sort requires a benchmark; we compare QPM with the broad class of 1970s macroeconometric models, with a focus on contrasting the new model with RDXF, QPM's immediate predecessor in the projection function at the Bank of Canada.

The methodology used in building QPM reflects its tasks and their relative priorities as well as certain judgments made concerning how best to respond to resource constraints. Thus, while we think that our approach is the best for our purposes, we do not mean to suggest that our choices would be appropriate in all cases. For pure policy research, for example, QPM is perhaps too complicated and takes too long to solve, even given the tremendous advances in computer hardware and solution procedures, in part because of the extra structure that is necessary to make the model useful for economic projections. Moreover, institutions willing to devote substantial resources to model maintenance might wring more useful short-run dynamic structure from the data through estimation of more complex systems. We are not convinced of this, but do not deny the possibility. In any case, part of the decision to move to a calibration methodology for QPM reflected a judgment that the costs of building and maintaining the model would be lower, and that there would be added flexibility for customizing it in responding to new questions in the future. Our experience to date has firmed our views on these issues.

Section 3 describes the model and the general principles that underlie its formal dynamic structure. A number of particular issues are pursued in greater depth. These include the nature of nominal dynamics and how the monetary control process works, the nature of the model's policy reaction functions, and the determination of interest rates and the exchange rate.

Section 4 explains the logic and methodology of calibration – how parameters were chosen to establish the desired properties. A limited number of specific examples of calibration issues and our choices for QPM are reviewed.

In Section 5, we document QPM's properties by describing its response to an extensive list of shocks. It is by analysing a model's response to hypothetical shocks that economists study its properties and use it as an aid to thinking about policy issues. A shock can be an exogenous change that triggers a response in the economy, including a response by the central bank, or it can be a change in policy. Any model reflects an abstraction as to what will be explained and what will be left unexplained. A shock will normally enter a model through explicit exogenous variables – things that it *does not* attempt to explain. A good example of this is a change in foreign demand. In a small economy in a large world, domestic economic conditions will have little effect on world variables. Thus, QPM takes certain world variables as exogenously given; the model can then be used to study what happens in the Canadian economy when such variables change.

Modellers also use summary measures to stand for all omitted influences in the determination of the model's endogenous variables – things that it *does* attempt to explain. Shocks are often introduced through such terms, without an explicit identification of the precise source. The results are interpreted as changes relative to what otherwise would have happened, given the previous history and the model's dynamic predictions, when an endogenous variable is perturbed from an unknown source.

Finally, there are true policy shocks, where the disturbance to the system is a change in the underlying goals of some policy authority or in the way a policy is implemented.

We provide examples of all these kinds of shocks, and we use the discussion of the results to elaborate particular aspects of the model's dynamic structure.

In Section 6, we offer some concluding remarks and some reflections on possible directions for future development of the model.

# 2 THE ROOTS, SCOPE AND GOALS OF QPM

The primary objective of this project has been to create a relatively small, quantitative macroeconomic model capable of performing two main functions. The first function is preparation of the quarterly staff economic projection (SEP), as it stands at present at the Bank of Canada and as it is likely to evolve. The second is analysis of important changes to the structure of the economy, to the exogenous forces that influence the business cycle, and to the underlying macroeconomic policy goals (or the operating procedures used to achieve these goals); we refer to this as policy analysis.

Experience has taught us that there is potential for conflict in the simultaneous satisfaction of the needs of projection and policy analysis. Pure forecasting typically involves analysing the propagation of shocks into the immediate future, often with little explicit attention to the underlying forces of equilibration. The horizon of interest for a forecasting question is usually relatively short. Thus, the short-term dynamic properties of models tend to be of most importance in forecasting. These properties need not be fully understood, as long as they remain relatively stable.

Prediction can be based upon *recognition* of a regularity as well as upon *explanation* of a regularity. – Whittle (1963, 2)

Moreover, the shocks that arise in the context of a forecasting exercise may be idiosyncratic in nature and not amenable to direct interpretation in the context of policy analysis. In forecasting mode, models and their users must deal with the complexity of the real world.

Policy analysis, on the other hand, tends to require a longer-term perspective and a more fundamental understanding of equilibration forces. It is potentially misleading to base policy conclusions on results that are not "explained." In the context of policy analysis, the shock to which a model is subjected is well-defined and the channels through which the new equilibrium is attained and the nature of that new equilibrium must be central to the analysis.

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In the past, improved performance in terms of short-term forecasting properties has often been sought at the expense of long-term properties and vice versa. With QPM, we have moved a long way towards reconciling these twin goals in that the model produces what are judged to be realistic short- and medium-term dynamic profiles that converge to well-understood and theoretically consistent long-run solutions.<sup>7</sup>

The goals that were set for QPM were a reflection, in part, of the experiences with models and modelling at the Bank of Canada and elsewhere, and of the particular needs of a central bank. In recognition of this, and to provide the context in which the QPM modelling decisions were taken, we turn now to a brief history of macroeconomic modelling and projection at the Bank of Canada.

# 2.1 A brief history of modelling at the Bank of Canada

The history of formal macro modelling at the Bank began in the late 1960s with the model RDX1 (Research Department eXperimental, version 1), and progressed to the more fully articulated model RDX2.<sup>8</sup> Both versions were developed by teams consisting of Bank staff and academic economists, but with heavy reliance on the latter. These models were intended as research and learning tools that would allow medium-term simulations of alternative policy choices, although the modellers saw their main job as incorporating, through estimation, the short-run dynamics in the data.

These models contained a number of economic concepts that were not available from official data sources and were difficult to construct, and for which the underlying input data were difficult to maintain over time.

<sup>7.</sup> Strictly speaking, we should say "solution paths," since the steady state is not fixed or "stationary," but is rather an equilibrium growth path.

<sup>8.</sup> Helliwell et al. (1969a, 1969b, 1971). Bodkin, Klein and Marwah (1991) gives a comprehensive history of macroeconometric modelling in many countries, including Canada, and the RDX series' contribution to that history.

Also, the estimated dynamic structures in the model were often found to be unstable when new data were added to the estimation period; accordingly, RDX2 was modified repeatedly over the 1970s as evidence accumulated that certain sectors were deficient in explaining the data.<sup>9</sup>

The Bank's commitment to supporting a well-articulated macroeconomic model at the centre of policy discussions nevertheless strengthened over time, and by the late 1970s there was a growing demand within the Bank for model-based economic projections. RDX2 had not been intended as a forecasting tool. However, the model had been used to analyse economic shocks around a projected path for the economy based on eight-quarter-ahead judgmental forecasts developed by the Bank staff. Early experimentation with RDX2 to generate a model-based projection for the economy indicated that this model was not well-suited to the task; in particular, updating the data base was extremely cumbersome. Hence, RDXF was developed (F for "Forecasting").<sup>10</sup>

RDXF was intended to be more "practical" than RDX2. Although it had about the same number of equations as its predecessor, RDXF was designed to be simpler to maintain. The data requirements had been greatly simplified by decisions to give the model a short-term focus and to adopt the convention that the job was to explain the published data, whether or not these data reflected well the concepts of economic theory. Individual equations were developed with a focus on their ability to track the historical data, and less attention was paid to theoretical considerations than in previous models. Some of the properties of the model were compromised as a result. It was thought that this might not be terribly important, since the model was intended to be used for predicting economic behaviour over relatively short horizons.

<sup>9.</sup> See, for example, Bank of Canada Technical Reports 5 (1976) and 6 (1977).

<sup>10.</sup> This work spanned a number of years and involved a large number of people; the first complete version is documented in Robertson and McDougall (1982a, 1982b). Numerous updates were carried out in subsequent years.

Complementary to the primary goal of forecasting, much of the responsibility for maintenance and reconstruction of the model was handed to "sector specialists." This meant, for example, that the person responsible for monitoring the month-to-month movements in consumer expenditures was also given responsibility for the model's consumption block, and so on. The idea was that any deficiencies in the model's shortterm forecasting abilities could be addressed by reformulation and reestimation of the individual equations. This allocation of responsibilities reflected the philosophy of "bottom-up" modelling that dominated the era. This approach also led to model evaluation being highly decentralized; little thought was given to the analysis of the model as a complete entity; and because of the short horizon of the intended forecasting exercise, policy did not play a substantive role within the model.

Through the 1980s, however, the SEP evolved into something quite different from what had been expected. Notwithstanding the short-term orientation of RDXF, the focus of monetary policy was turning increasingly towards the medium term. Much of this new focus was a reflection of the lessons of the era: the stagflation of the 1970s had shown that short-term policies could have unforeseen consequences over longer horizons. Moreover, at the Bank of Canada, as elsewhere, policy makers were searching for an understanding of the failure to control inflation through the 1970s and ways to implement more forward-looking policy targets.

The experiences of the 1970s and early 1980s shook the foundations of the Keynesian consensus. The 1960s had seen economics accede to the cathedral of the social sciences, with macroeconomists as the high priests. All important debates seemed to boil down to disputes regarding elasticities that could be settled using scientific methods and the data. Many of the policy questions of the day centred around the distribution of the seemingly ever-increasing bounty that was flowing from the economy. When the economic downturns of the 1970s occurred in the presence of rising inflation and despite the application of the standard Keynesian prescriptions of expansionary policies, the consensus collapsed.

The stagflation of the 1970s gave resounding proof of the natural rate hypothesis (NRH) of Friedman and Phelps. Yet it was not on empirical grounds that the NRH gained its initial prominence; it was primarily the persuasiveness of the logic that expansionary monetary policy ought not to be able to produce permanently higher levels of output that carried the day.

From the NRH arose two other related notions: one regarding the importance of modelling the supply side of the economy and the second concerning the importance of expectations. Economic developments of the era, including the energy price shocks and other structural changes, some emanating from important changes in policy, gradually convinced economists that attention to the supply side of the economy was necessary to an understanding of cycles.<sup>11</sup> Meanwhile, closely associated with the NRH, the revolution of "rational" or model-consistent expectations was building momentum.

Imposing the NRH on RDXF proved to be an insurmountable task. The model had become so large and so decentralized in its construction and control that literally hundreds of restrictions would have had to be imposed. This made it clear that if a model was to encompass a core theory, it would have to be truly macro in character, not micro, and this implied a "top-down" modelling strategy.<sup>12</sup>

Part of this macro-oriented approach was insistence on the importance of a clear role for monetary policy within a model. Instead of the aggregate price level being "determined" by the sum of a myriad of prices from various sectors, each with its own idiosyncratic causal structure, aggregate price determination came instead to be viewed as

<sup>11.</sup> In Canada, for example, the major reform of unemployment insurance in the early 1970s seemed to have an important effect on the equilibrium level of unemployment.

<sup>12.</sup> Arguments similar to some of those described in this section – particularly in regard to the importance of top-down modelling – have been advanced by Persson and Svensson (1987) in a paper describing the deficiencies of a model used for fiscal policy planning by the Swedish National Institute for Economic Research. For an earlier discussion from the Bank, see Masson, Rose and Selody (1980).

principally a monetary-macro phenomenon. In particular, it became accepted that it was the role of the monetary authority to establish an anchor for expectations of inflation as a key part of establishing an overall nominal anchor for the system. Once this role was accepted, it was inevitable that policy would come to be expressed in terms of policy rules designed to establish such an anchor on expectations endogenously within the model.

It was the inclusion of policy reaction based on rules that blurred the formerly clear-cut distinction between forecasting and policy analysis, and led to the emergence of what the Bank of Canada calls *projection* exercises. Whereas a forecast is based on particular anticipated values of policy variables, treated as if they were exogenously fixed, a projection involves searching for settings of the policy variables to achieve specific objectives. Previously, this was done in an ad hoc, two-step fashion. In the first step, shocks were introduced to RDXF with "policy held constant," meaning, in essence, that real interest rates were held artificially at their control values. Once all interested parties were satisfied with the results, policy rounds were carried out involving the setting of nominal interest rates, with the exchange rate moving in response, to achieve some medium-term policy objective.<sup>13</sup>

During the 1980s, efforts at maintaining and reforming RDXF continued but were hobbled by that model's lack of a steady state and consequent inability to deal with medium-term policy and projection issues. A small annual model (SAM) with formal theoretical foundations was developed.<sup>14</sup> SAM had many of the features that were necessary to conduct reliable policy experiments, but it was relatively complex and difficult to maintain. SAM was also an estimated model; indeed, it was estimated initially using complex, systems econometrics, including some

<sup>13.</sup> This practice was fostered, in part, by computational limitations. It was not possible to solve models with the type of forward-looking control mechanisms required for a sensible characterization of monetary policy. That the strategy worked was also the result of extensive reliance on lags and quasi-recursive model structure.

<sup>14.</sup> See Rose and Selody (1985).

very large full-information maximum-likelihood exercises. Nevertheless, it suffered from some of the lack of robustness to new data that had plagued its predecessors. SAM, like RDX2, was used for a time to provide consistency checks on the baseline scenario from RDXF and for the analysis of certain shocks around that baseline, but the extent of the differences between the two models made such exercises difficult.<sup>15</sup> In the end, it was judged impossible for the Bank of Canada to support two full-scale modelling projects, and SAM was abandoned. On another front, some work was done using very simple models to help clarify the role of monetary policy in macro models.<sup>16</sup> Insights gained from these efforts served to heighten interest in developing a model that could be used both for assembling medium-term economic projections and for policy analysis.

## 2.2 Some general lessons learned

Several important conclusions were drawn from the Bank's experience with macroeconomic models over the period from 1970 to 1990.

First, despite their limitations, models can be very helpful in the analysis of both fundamental, long-term policy issues and shorter-term economic developments. It was concluded, moreover, that a model with strong economic content was required, even within the context of the projection exercise. The issues of the projection are too forward-looking to make the option of a pure forecasting model viable. Furthermore, many of the research issues that arise in policy analysis require Bank staff to consider the long-run equilibration processes in the economy and to provide the fundamental explanation of results that can only come from explicit economic structure. Moreover, part of this explicit structure must be a representation of the role and functioning of macro policy.

<sup>15.</sup> SAM was used primarily for policy analysis, especially for discussion of the effects of energy price shocks and things like the 1987 stock-market crash, where the model's complete stock and wealth accounting was invaluable. SAM, with annual structure, was never designed for forecasting, and only limited attention was paid to ensuring that it had defensible short-term properties.

<sup>16.</sup> See, for example, Longworth and Poloz (1986). The Longworth-Poloz model also introduced the idea of calibration to the Bank.

Second, the inability of relatively unstructured, estimated models to predict well for any length of time outside their estimation period seemed to indicate that small-sample econometric problems were perhaps more fundamental than had been appreciated and that too much attention had been paid to capturing the idiosyncrasies of particular samples. There had been a systematic tendency towards overfitting equations and too little attention to capturing the underlying economics. It was concluded that the model should focus on capturing the fundamental economics necessary to describe how the macro economy functions and, in particular, how policy works, and that it should be calibrated to reflect staff judgment on appropriate properties rather than estimated by econometric techniques.<sup>17</sup>

Third, it was concluded that an unavoidable tension exists between theoretical rigour and short-term forecasting ability. Rigour necessitates simplification in modelling that can compromise the ability to capture the complexity of short-term dynamics. Yet, full exploitation of the information in the data requires adding detail to a model that cannot be rationalized within a tractable analytical framework. Adding such detail can compromise a model's medium- to long-term properties. As noted above, it was concluded that the formal model must have the theoretical rigour to support policy analysis. The implication was that the projection role required additional structure to be built around the core model to capture some of the "regularities" that might be useful for short-term forecasting. The key difference from past practice was that this was to be done as an adjunct to the core modelling, not as the primary goal of the modelling. It was also concluded that very short-run forecasts (for the first couple of quarters) should be done entirely outside the model – by staff judgment.

Fourth, it was concluded that to meet the requirements of policy analysis, the model had to have systematic forward-looking behaviour and expectations. It was judged that computational difficulties were no longer a barrier to doing this in a working model. Nevertheless, at the time the

<sup>17.</sup> It was expected, of course, that such staff judgment would take into account available empirical evidence of all sorts.

project began, much work remained on how this would be done. Indeed, a substantial part of the development of the QPM system was the effort that went into developing the computing platform to support it.<sup>18</sup>

Fifth, on a more practical level, our history demonstrated the difficulty that an institution like the Bank of Canada faces in attempting to maintain intellectual momentum on more than one substantial macroeconomic model. This latter observation argued for a single model, despite the inevitable trade-offs in serving the forecasting and policy analysis roles.

# 2.3 The scope and goals of QPM

In setting out to create QPM, the model builders had a number of specific goals. In this subsection, we describe the scope of QPM by reviewing some questions it was designed to be able to handle. The discussion goes beyond a simple listing of these goals; we explain why particular features were desired, again setting this in the context of the evolution of models. The focus is on issues where QPM offers some advance in modelling or some particular feature worthy of note. Most of the issues discussed here are illustrated in the simulations reported in Section 5.

## 2.3.1 Dynamic stability around a well-defined steady state

We have already described the essential case for insisting that the model have a well-defined steady state. Many shocks that arise routinely in policy analysis imply changes in the long-term equilibrium values of important economic variables. Not accounting for these changes can lead to serious errors in policy evaluation and in medium-term forecasting.

Many macroeconometric models do not have a steady state; when subjected to shocks, their simulated responses can follow paths without economically defined limits or with no limits at all. In our view, this means that their simulation results are interpretable only for the first few quarters.

<sup>18.</sup> See the second volume in this series – Armstrong, Black, Laxton and Rose (1995) – for a description of the solution algorithm developed for QPM.

Moreover, the quantitative results for those first few quarters often come from estimates representing some weighted average of shocks in the past, a number where there was a substantive change in the steady state, and others where there was not. Since the outcome should be expected to be quite different in the two cases, this results in imprecise estimates and persistent short-term projection errors in particular episodes. The typical response has been to send modellers on fruitless searches for missing variables or better estimates. In the process, too much weight is given to the particular experience of the most recent episode. In the meantime, projections are held together by "judgment," and when circumstances change, the process starts all over again.

When a model has a well-defined steady state, it becomes clear when short-run dynamics must include accommodation of a change in the long-run solution and when the problem involves simply working off the effects of a temporary disturbance. One can ensure that when the model is shocked it will converge to the "correct" steady state. Exactly what steady state is the correct one is determined by the complete model and the nature of the shock. In the QPM system, the steady-state model, SSQPM, provides the answer as to how the steady state will change for a particular shock. QPM itself then ensures that the model solution will eventually reflect that answer.

The goal of dynamic stability around a well-defined steady state is, at least in part, based on the practical requirements of model use. Economists know that interesting hypothetical economies can be described where there are multiple equilibria or where dynamics are not stable.<sup>19</sup> It is possible that such features are inherent in market economies and therefore ought to be an essential part of policy analysis. Nevertheless, we must reject this in building and maintaining a core model for use in a policy institution, where there is no time to reopen fundamental issues every time a question is posed. The point of having a model is to provide a tool that does give clear answers to questions. Users must be aware that the answers

<sup>19.</sup> See Farmer (1993) for examples of this type of model, with a focus on multiple equilibria associated with expectations.

depend on the maintained hypotheses of the model and the particular experiment. These may be bogus, and the answers misleading, but the model at least provides a starting point for understanding the issues and evaluating the answers. For this to work well, the model must converge when shocked to an equilibrium that is clear and consistent. Understanding applied dynamic general equilibrium problems is difficult enough without requiring modellers and users to distinguish unstable solutions from mistakes.<sup>20</sup>

### 2.3.2 Complete wealth accounts and integrated stock-flow dynamics

An important part of a "well-defined steady state" is a set of behavioural decision rules specifying agents' choices in terms of asset *stocks*. Here, the key point is that simply enforcing budget constraints through time, albeit a crucial necessity, is not enough; desired levels for the stocks must emerge from the behaviour. This requires a complete set of wealth accounts and fully integrated stock-flow dynamics.

The requirements for supporting the stock equilibrium provide the foundation for determining the economy's flow equilibrium as well as essential conditions for the disequilibrium dynamics. Interpreting the medium-term (three to seven years ahead, say) results of a simulation requires a view as to where the economy is heading in the long run. That is the role of the steady state and its determinants. Part of the dynamics of QPM comes from resolving the flows so that they cumulate to support the desired levels of the stocks.

Only in this way, to cite an obvious example, can one model the cumulation of investment into a level of the capital stock (and, hence, potential output) in a way that is consistent with intertemporal budget constraints and relative prices. This is an important issue, since a temporary

<sup>20.</sup> If the world were truly characterized by multiple equilibria or dynamic instability, then the methodology used in model construction, which involves a search for structure that can be taken as proximately constant for the types of question that will be asked, would be questionable. That the Bank supports use of a core model reflects a judgment that such problems are not endemic to market economies and that it is possible to create a tool that will be reliable for a range of policy questions.

demand shock that induces an investment boom requires very different subsequent dynamics than does a permanent supply shock; the former implies a return of the capital stock to its original control level, while the latter does not. Moreover, in the former case, flow investment must not only decline back to its equilibrium level but must fall below that level for a time to eliminate any excess accumulation of capital during the boom period. In the case of a permanent supply shock, no such secondary cycle is necessary.

Another example comes from the intertemporal budget constraint of households and the role of their wealth in conditioning current flow spending decisions. The ratio of the net foreign asset (NFA) position of the economy to income is the principal measure of the national balance sheet used in the model, and the desired ratio of NFA to GDP represents the aggregation of private consumption-savings decisions. If the level of NFA is "too low," then consumers will choose to consume less in order to restore their savings balances.

The stock-flow dynamics of the NFA position have implications for inflation and monetary policy. For example, if some disturbance to foreign demand reduces exports from Canada, the monetary authority can offset, to some extent, the negative consequences of the disturbance by easing monetary policy and encouraging consumption – but only for a while. The higher levels of consumption (and lower export earnings from the original disturbance) will reduce the economy's NFA position, and individual households will see their savings fall. Consumers will not ignore the decline in their wealth indefinitely. Otherwise, they know they will suffer a reduction of consumption in the future. Eventually, they will have to reduce consumption to re-establish their savings balances and their desired intertemporal allocation of spending.

Such implications of stock-flow dynamics, driven by wealth considerations, are often lost in models that focus on flow equilibrium conditions. In such models, the monetary authority, if endowed with sufficient information and time to react, can offset completely the effects of such a foreign shock. The cumulative effect on wealth levels may be

computed, but if it does not feed back into behaviour, the analysis is incomplete.

#### 2.3.3 A fully articulated supply side

Most permanent shocks imply a reallocation of resources and with that a change in the steady-state equilibrium solution of the model. A well-defined steady state for a model means more than simply a set of numbers that satisfy the equilibrium conditions for the model under particular assumptions. If the model is to be useful for analysing the effects of permanent shocks, it must have sufficient structure to explain how the steady state changes under those shocks. This is the role of SSQPM. While permanent shocks include more than disturbances arising on the supply side of the economy, for a model to deal fully with any permanent disturbance, it must be capable of explaining supply response.

In addition, it has become clear that the economy is frequently buffeted by what can be characterized as supply shocks, whether temporary or permanent. Some, such as the oil price shocks of the 1970s and the mid-1980s, are large and easily identified, while others are small and identified as supply shocks only long after the effects have been observed.<sup>21</sup> Whatever their source, these supply disturbances have fundamentally different consequences for prices, wealth and potential output than do demand shocks. Treating supply disturbances seriously is a feature of modern macroeconomics, and this is reflected in QPM.

An important example of a permanent supply shock is a change in the level of productivity. A permanent productivity shock has a substantial impact on household income and wealth and hence on consumption and savings decisions. In spite of their now-acknowledged importance for understanding business cycles, many macroeconometric models cannot handle such productivity shocks. Those that do often produce a flow equilibrium only – the stocks are allowed to follow arbitrary paths without any level restrictions coming from economic theory. An important goal for

<sup>21.</sup> On the incidence of supply shocks and implications for the measurement of potential output, see Laxton and Tetlow (1992).

QPM was that there be a fully articulated supply structure that would give users complete answers to such questions.

#### 2.3.4 A clear role for monetary and fiscal policy

QPM is designed to answer questions about monetary and fiscal policy. A necessary condition for a model to be able to do this is that it embody clear roles for monetary and fiscal policy as well as descriptions of the way that policy choices affect private behaviour.

Fiscal policy in QPM is characterized in terms of choices of two ratios: the level of government spending on goods and services relative to output, and the level of public sector debt relative to output. The former describes what governments do that affects private agents through product and labour markets. The latter describes the role of the method chosen to finance government activity, which affects private agents through asset markets. Together, these choices also affect private agents through the government's intertemporal budget constraint and the implied profile of taxes. More detail on how this is done in QPM is provided in Section 3.

Surprisingly few macroeconometric models are capable of performing all but the most stylized of fiscal policy shocks. The main reason for this is that almost all fiscal policy shocks contain a stock-flow dimension, namely the cumulation of deficits into public debt in a way that impinges on future consumption possibilities. Most models either omit completely the link from debt to future consumption or provide for its inclusion only judgmentally. Because of this treatment, the responses to fiscal shocks of many models feature "free lunches," wherein, for example, expansionary fiscal policy is always beneficial, irrespective of the initial state of the economy or the financial position of the government sector. The government can play "Ponzi games" because of inadequate asset accounts to trace through the effects of government deficits on the national balance sheet, or because naive expectations assumptions mean that future liabilities are treated as if they can be deferred forever. In QPM, fiscal policy has a fundamental role through intertemporal debt dynamics.<sup>22</sup>

<sup>22.</sup> These issues are discussed at length in Laxton and Tetlow (1992) and in Macklem, Rose and Tetlow (1994, 1995).

For the role of the monetary authority, what is now the conventional view is adopted. It is the role of the monetary authority to put in place and communicate an underlying goal for policy (some fundamental nominal anchor) and a means of implementation (in the model this is done through a policy rule or reaction function) so as to create a regime of expectations that is consistent with achieving the goal over time.

Given this focus on expectations, the need for consistency of the policy rule and policy actions with the behaviour reflected in agents' decision rules, and vice versa, is emphasized. This overriding view – that policy, expectations and the functioning of the economy are intertwined – has been at the heart of the Bank's communications with the public for some time. As to the form of the nominal anchor, the fundamental objective of Canadian monetary policy has for some time been price stability.<sup>23</sup> Reflecting current practice, this is implemented in QPM as an inflation target. At present, Australia, Finland, New Zealand, Sweden and the United Kingdom also practice some form of inflation targeting, and the issue is under active discussion elsewhere.<sup>24</sup>

It is only recently that macro models have begun to have a clear role for policy. The joint Bank of Canada – Department of Finance *Seminar on Responses of Various Models to Selected Policy Shocks* illustrates this quite well.<sup>25</sup> The documentation of that exercise shows that, for the vast majority of 1970s vintage macro models, a permanent 1 percentage point reduction in the nominal interest rate in Canada was predicted to elicit, after 10 years, an increase in inflation of much less than a percentage point. Since the

<sup>23.</sup> See, for example, Bank of Canada (1991) and Crow (1988). Selody (1990) provides a review of the arguments for price stability. See Leiderman and Svensson (1995) for a recent general review of inflation targeting. See Bank of Canada (1994) for further discussion of the economics of price stability.

<sup>24.</sup> The requirements for European monetary union oblige countries to implement a form of inflation control, at least for a while. Lane, Prati and Griffiths (1995) outline a suggestion for Italy to target inflation following that country's departure from the exchange rate mechanism in 1992. The authors argue that the principal benefit of targeting inflation rather than some intermediate variable is one of transparency, which is associated with accountability and precommitment.

<sup>25.</sup> See O'Reilly, Paulin and Smith (1983).

nominal interest rate is an endogenous variable and cannot be controlled permanently by the monetary authority, an easing of monetary conditions of this nature ought to engender expectations of higher inflation. Then, as expected inflation works its way through the Fisher equation with the nominal interest rate capped, the ever-declining real interest rate should make inflation accelerate without limit. That this did not occur in a surprising majority of the models shows the small role played by monetary policy in macro models of this era.<sup>26</sup> This meagre role of policy arises from the fact that these models were not built with policy simulation in mind, or at least not with that as a primary objective. It also serves as a reminder that building a model with policy simulation capabilities without sacrificing short-term properties has proven to be quite difficult.

An important question that a policy model should be able to answer is what the cost would be, in terms of foregone output, to reduce inflation by, say, a percentage point. This is often called the "sacrifice ratio." We have already seen that this question was ill-posed for a large number of Canadian macroeconometric models in the 1980s; there seems to have been no method by which a monetary authority could bring inflation down in many of these models.<sup>27</sup> In some models where the question can be posed, the expected output cost of disinflation is precisely the same, regardless of the starting conditions and the pace of adjustment the monetary authority chooses. In effect, the answer comes from the properties of a particular equation, the Phillips curve, and there is no role for policy in determining the answer. In QPM, such issues must be addressed in the context of the complete model, and the explicit representation of policy within the model provides an important element of any answers.<sup>28</sup>

<sup>26.</sup> In contrast with these 1970s models, FOCUS (the model maintained at the University of Toronto's Institute for Policy Analysis) did appear to create an exploding path for inflation. The structure of SAM was such that the permanent decrease in the nominal interest rate was not technically feasible (that is, inflation would have exploded) and so the shock was reinterpreted as a permanent *real* interest rate shock. This latter shock is well-defined; however, interpreted this way, it is not a monetary policy shock.

<sup>27.</sup> Grant and Murphy (1994) provide evidence suggesting that not very much has changed.

<sup>28.</sup> See Section 4 for further discussion of this issue.

#### 2.3.5 Endogenous policy rules or reaction functions

Macroeconometric models of the 1970s typically treated policy as an exogenous add-on to a "policyless" economy. This approach began with specification and estimation, which rarely recognized any possible link between the policy environment and behaviour in the rest of the model. It continued in simulation exercises, where estimated structures were assumed invariant to changes in policy. The Lucas (1976) critique of this practice is now widely understood.

A more modern view holds that the dynamics of the economy and the data generated by it are both functions of the policy in place at the time. Policy is always at work; not to adjust policy instruments is just as much a policy as is an active feedback rule, and all dynamic responses of the model must be conditioned on this "non-policy." Incorporating this view within a model requires one to identify policy rules that encapsulate the endogenous response of policy authorities to the particular course of events. In other words, it is not enough to specify the goals of policy; the model must have explicit mechanisms for these goals to be achieved.

When discussion turns to policy analysis, the first thought that may come to mind is the case where fundamental policy choices are being considered or where policy implementation is being changed. An example might be a decision to lower the target rate of inflation. In the language used in the introduction, this is a case where a policy initiative is the shock that must be considered. However, the main job of monetary policy is to react to shocks coming from other sources in a manner that is consistent with the underlying goal of policy. To continue the example, with a fixed target for inflation, the basic role of the monetary authority is to react to economic developments in a manner that provides a nominal anchor for the system and keeps inflation from moving away from the target rate. This is the key role of the reaction function. It describes behaviour by the monetary authority that will do this job endogenously, automatically taking into account the particular circumstances arising from any economic disturbance. We have used the monetary reaction function here to make our points, but the same general argument applies to the specification of behaviour by the fiscal authority. With endogenous policy reaction functions, the *general equilibrium* role of monetary and fiscal policy is made precise. This plays an important role in overall model dynamics; the nature of the rules in place will have a significant effect on model properties.

## 2.3.6 Neutrality and superneutrality

The first version of QPM has been configured under the assumption that the economy exhibits both neutrality and superneutrality. That is, the real steady state will not depend on the general level of prices or on the rate of inflation.<sup>29</sup> SSQPM does have a very small effect from changes in seigniorage when the target rate of inflation is changed, but this is a minor qualification to the general point.

Superneutrality does not reflect the Bank's view of economic reality. However, it was agreed that the model should be built initially with these properties assumed. Achieving this in a simulation model is a notable accomplishment. So there will not be any long-term real effects from monetary policy choices. There will, however, be important short-term effects. An important part of this involves the consequences of expectations being slow to adapt to fundamental changes of monetary policy, such as a change in the target rate of inflation. We return to this issue later.

### 2.3.7 Nominal levels: not determined by policy choice

The "well-defined steady state" applies only to real variables in QPM. The model's monetary reaction function ensures the eventual achievement of a target *inflation* rate, but there is no restriction on the price level. The model's price and wage equations converge in the long run to satisfy, among other things, labour-market equilibrium as reflected in the steady-state real wage. The precise real wage that supports equilibrium is determined in SSQPM by real factors, such as the level of productivity.

<sup>29.</sup> See Black, Macklem and Poloz (1994) for an example of how QPM could be extended to include real effects from inflation.

While this condition determines what prices relative to nominal wages must be, there is no absolute price or wage level condition that the model must respect.

In the absence of such a level condition from the policy rule, the absolute levels of all nominal variables will tend to drift up or down, depending on the particular shock or sequence of shocks. There is no reason, technically, why the model could not be configured with some weight given to the level of prices in the monetary reaction function, but no such restriction has been included in the current inflation targeting regime in Canada, and the QPM's monetary reaction function reflects that fact.

#### 2.3.8 Forward-looking behaviour: dealing with the Lucas critique

Because QPM has an explicit representation of policy and of the interaction between policy choices and forward-looking expectations formation, it is relatively flexible for policy analysis. Users can select from a wide range of possible characterizations of policy, and the rest of the model can be assumed invariant to the choice, within limits. For example, most characterizations of monetary policy that provide a nominal anchor for expectations could, in principle, be simulated without compromising the dynamic structure in the rest of the model. This would include, for example, nominal income or money growth targets or even a fixed exchange rate, where the nominal anchor would come from the exogenous foreign inflation rate. Obviously, there are limits to this argument; the essence of the Lucas critique is that private decision rules will depend on the policy regime chosen, and this includes, in principle, more than the dynamics of expectations formation.

The use of forward-looking behaviour in models, once regarded as heretical, is enjoying increasing popularity in modelling at policy institutions.<sup>30</sup> The monetary authority can therefore bring about changes in expectations by changing its policy rule, and with that, there can be changes in the time-series properties of the model without changing the model itself.<sup>31</sup> Similarly, other changes in policy rules or in the fundamental exogenous forces that determine the properties of the underlying equilibrium or the dynamic structure of the economy will be endogenously incorporated in expectations formation.

This does not happen immediately in response to such changes. While behaviour in QPM is forward-looking, it is not specified to be "rational" in the strict sense of Muth (1961). Consistent with our interpretation of the stylized facts, expectations formation always has an element of predetermined, backward-looking behaviour. Consequently, none of the extreme and unrealistic predictions of New Classical business cycle theory arises from QPM. Moreover, with respect to policy changes, QPM does not allow any "free lunches" in the form of announcement effects or automatic acceptance of the new conditions as a requirement of steady state. Policy makers must act to ensure the adaptation of expectations.

### 2.3.9 Flexibility for handling special assumptions on information

One advantage of an explicit representation of forward-looking expectations in a model is that the structure is in place for posing questions

<sup>30.</sup> The use of forward-looking expectations in multicountry models in North America is common; examples include MULTIMOD at the International Monetary Fund (IMF), INTERMOD at the Canadian Department of Finance (built by a group headed by John Helliwell, in co-operation with the IMF), MX3 at the U.S. Board of Governors of the Federal Reserve System (Fed), and the McKibbin-Sachs (MSG2) model at the Brookings Institution and the Congressional Budget Office. Among single-nation open economy models, it has become the norm in the United Kingdom, where the City University (CUBS) model, the Liverpool model, the London Business School (LBS) model, the National Institute model and the U.K. treasury (HMT) model are all forward-looking. John Taylor also has built a forward-looking empirical macro model (Taylor 1981, 1993). Finally, the Fed has introduced model-consistent expectations in FRB/US, the successor to their venerable MPS model. See Bomfim et al. (1995) for some preliminary documentation.

<sup>31.</sup> This is, as we say, a "large first step," because to address this issue completely, one needs a theory of how agents come to learn about changes in the policy rule; and there is as yet no broadly accepted model of learning. See Laxton, Ricketts and Rose (1994) and Ricketts and Rose (1995) for one approach to modelling learning about policy regimes.
on the effects of different assumptions about the information that agents have in making decisions. Here, we review two examples of such issues: whether shocks are anticipated (or policy announcements are credible) and whether agents can identify the precise nature of a shock.

#### Anticipated versus unanticipated shocks

Senior officials of the Bank of Canada spend a considerable amount of time communicating the Bank's policy goals to the public. Among these goals are the announced target bands for inflation. An initial objective of announcing explicit targets was "to encourage Canadians to base their economic decisions on this [the announced] downward path for inflation so that the lower inflation will be more readily achieved." (Bank of Canada 1991, 6). If a disinflationary policy is clearly understood, the deleterious side effects should be smaller than would otherwise be the case. The construction of QPM provides a structure for such credibility effects to be considered and for the benefits from gaining credibility to be quantified. We hasten to add that this does not mean that credibility comes easily or quickly. Moreover, no gains from announcing disinflation are assumed in the base-case model. But the necessary structure is there for distinguishing between anticipated and unanticipated monetary shocks and for dealing with the issue of credibility effects.

The distinction between anticipated and unanticipated shocks is not limited to issues of monetary policy. Other relatively recent examples of policy changes announced in advance include the introduction of the Goods and Services Tax and the succeeding rounds of tariff reductions included in the major trade agreements. It is natural to expect rational agents to act in advance of known future changes of this sort. QPM can handle anticipated future shocks. This is an advantage not only for those rare but important shocks with explicit future structure, but also for the year-to-year handling of the expected future fiscal plans of governments. For example, analysis of Canada's current fiscal problems is facilitated, because the degree to which economic agents expect different future taxes as a consequence of current budgetary measures can be varied easily using the model's explicit representation of expectations. Note, though, that there is no requirement in QPM that future shocks actually be anticipated by private agents. Indeed, the default assumption is that shocks cannot be anticipated by either private agents or policy makers. The model user must determine whether particular future shocks are to be treated as anticipated or as partially anticipated (that is, anticipated by some proportion of economic agents) and introduce this assumption by modifying the default implementation.

#### Well-understood shocks versus confusing shocks

In a great many instances, projection participants face a situation where there is a shock in the sense of a surprise relative to the previous projection, but no obvious identification of its source. It seems natural to argue that the public also has incomplete knowledge as to the nature of the shock or shocks. At other times, the source and macroeconomic consequences of a shock are reasonably well known. One would normally expect somewhat different dynamic effects from a model when a shock is well-understood than when it is confusing to the public and policy makers. Again, the explicit role of expectations within a forward-looking framework in QPM provides a framework in which such distinctions can be given clear meaning.

#### 2.3.10 A model designed to work for a broad range of questions

That QPM can do the kind of things described above is not simply good fortune; the model was built from the start with these macro simulation issues in mind. The top-down modelling strategy allowed the model builders to keep a clear focus on what was needed to obtain macro answers to a wide range of policy questions. Notwithstanding the importance of the goals in this regard, it would not be satisfactory for the model to give plausible answers to policy questions for implausible reasons. In the next section, we provide more detail on the dynamic structure of QPM. Then, having discussed a number of issues concerning the model's calibration, we return to the topics discussed here, illustrating many of these points and the model's dynamic properties through extensive simulation exercises. One point we would like to stress, in closing this part of the discussion, is the breadth of the questions that QPM has been designed to handle. One can easily construct a model that provides an appealing path for a particular shock, if that is the sole objective. What is more difficult is to provide plausible solutions for many different shocks within a common framework.

# **3 THE DYNAMIC STRUCTURE OF QPM**

In this section, we describe the main elements of the dynamic structure of QPM. We begin with a brief review of the accounting structure of the model and the essence of the theory that describes behaviour. This part is quite brief; the reader is referred to the first volume in this series – Black, Laxton, Rose and Tetlow (1994) – for details. We then turn our attention to the dynamic structure of QPM, presenting a generic description of the way the ideas are reflected in model equations and giving some examples of equations from the model in stylized form.

## 3.1 An overview of the structure of QPM

QPM describes the behaviour of households, firms, foreigners, a government (consolidating all levels of the public sector), and a central bank. The decisions of these agents interact to determine the ultimate levels of four key stocks: household wealth (human and financial), fixed capital, government debt, and net foreign assets. These target stock levels in turn are key determinants of the associated flows, such as consumption spending, saving, investment spending, government spending and revenues, and the trade balance. The model's complete stock-flow accounting framework and forward-looking agent behaviour ensure full consistency among all variables, both in long-run equilibrium and along the dynamic adjustment path.

Households in QPM are modelled using a theoretical device known as "overlapping generations." Consumers live an uncertain length of time and must plan their consumption and savings over that unknown lifetime. In doing so, they must balance the desire for current consumption with the need to save to sustain consumption levels later in life. QPM provides solutions for both the desired long-run aggregate level of financial wealth of consumers and the consumption-savings path that will attain and sustain that level.

Firms in QPM take the long-run labour supply of households as given, and choose the optimal stock of capital to go with it, as well as the

path for investment spending that will take the economy to that equilibrium and maintain it.<sup>32</sup>

The government chooses a steady-state ratio of government debt to the size of the economy.

With these three steady-state decisions taken, aggregate net borrowing or lending for the economy as a whole (that is, with respect to non-residents) is determined, so that the net foreign asset position of the country emerges as a consequence. Associated with this equilibrium net foreign asset position will be a unique external balance supported by a specific solution for imports, exports and foreign debt service payments.

When this equilibrium is disturbed, a number of variables in the model adjust in order to generate a new steady-state solution. For example, real wages adjust to the level of labour productivity, which ensures that full employment will be re-established once the capital stock has reached its equilibrium level. In the government sector, the personal income tax rate is assumed to adjust to ensure that governments can finance their expenditures, while moving to their target debt-to-output ratio in the long run. The real exchange rate adjusts so as to generate export and import flows that will reconcile the level of national saving or dissaving with the country's equilibrium net foreign asset position. Not all prices are free to move, however. Some domestic prices are linked directly to world levels. For example, the steady-state real rate of interest is equal to the world real rate plus some constant that allows for the possibility of a risk or liquidity premium.

The formal structure of QPM is highly aggregated. For example, personal "consumption" is very broadly defined to include expenditures on housing and inventory investment as well as the usual consumer expenditures from the national accounts. A detailed identification of the components of QPM consumption is then carried out in a satellite model,

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<sup>32.</sup> The technology is assumed to be Cobb-Douglas, with constant returns to scale, and the rates of productivity and population growth are assumed to be exogenously given.

which has no feedback effects on the macro solution. The same level of accounting simplification is found in all parts of the formal core model. This simplification is what permits the high degree of rigour in the description of the general equilibrium. QPM has 27 equations that we would describe as behavioural. Yet even with this simplified framework, there are a total of 329 equations in the model, not counting the satellite structures. There are 155 equations describing expectations; most of the rest are identities. There are just 10 variables for which expectations are required. The large number of expectations equations is needed because we have to keep track of a number of leads for each of them. The number of leads varies, but for long bond rates, for example, which are represented in the model by 10-year rates, we must keep track of expectations for up to 40 quarters ahead.

### 3.2 Key structural features of QPM's dynamics

There are three main sources of generic dynamic structure in QPM. The first is what may be called intrinsic elements of dynamics. These are dynamics that arise from the technology and decision-making structure of the economy, encompassing all sources of gradual adjustment not related to perceptions or expectations. These include any sources of costly adjustment, things like constraints coming from contracts, irreversibilities associated with investment, and so on. Such features give rise to a gradual response to disturbances, regardless of how large the disequilibrium might be, even when the disturbances are fully understood and the consequences correctly anticipated. One can think of QPM dynamics as embodying a general phenomenon of costly adjustment that causes all agents in the economy to choose to adjust gradually to economic disturbances.

The second source is dynamics associated with expectations formation. Perceptions and predictions about the future play an important role in conditioning the current behaviour of economic agents. This is what makes the dynamics of economic systems fundamentally different from the dynamics of natural systems.<sup>33</sup> One can imagine a wide range of circumstances in which the behavioural response to a situation will differ considerably, depending on the perceptions and expectations of economic agents. Here the distinction between intrinsic and expectational dynamics can become blurred, since uncertainty about the nature or permanence of a shock may be an important source of reluctance to adjust fully, when there are irreversibilities or other costs of adjustment. Nevertheless, it is essential to separate the pure role of expectations from other sources of dynamics in order to deal with such issues within a model at all.

Finally, the endogenous reaction of monetary and fiscal policy adds an important contribution to the overall dynamics. In this regard, it is difficult to overstate the importance of policy reaction functions in models with forward-looking behaviour.

All the above topics are elaborated on below. There are also two features of dynamics in QPM that require particular attention. The first is an important asymmetry in nominal dynamics – in QPM, excess demand creates more inflationary pressure, and more rapidly, than excess supply creates disinflationary pressure. The second is not something that is at all unusual in QPM, but is nevertheless very important in any discussion of the dynamics of an open economy, namely, the role of the exchange rate.

#### 3.2.1 Intrinsic and expectational dynamics

Let us examine QPM's generic treatment of intrinsic and expectational dynamics by taking one of the model's price equations as an example. It is important to keep in mind that this is intended to be a generic presentation of the material. The actual equations in the model begin from a common framework, as described here, but each equation has specific features to reflect what was judged appropriate for the particular case. Usually, this means that extra elements are added, and an important example of that is provided. However, for this example, one element of the generic structure

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<sup>33.</sup> Learning by economic agents can introduce an additional source of non-linearity into economic dynamics. See, for example, Laxton, Ricketts and Rose (1994).

is deliberately not used, for reasons explained below. We chose the example of price dynamics because of its key importance in a model for monetary policy analysis.

Consider a representative firm facing monopolistic competition in the short run and trying to plan a path for its price, p, for the current and all future periods.<sup>34</sup> The firm has a desired (or profit maximizing) price,  $p^d$ , for all periods, based on its expectations of market conditions, costs and so on. However, it faces adjustment costs that prevent it from simply jumping to the optimal price. In QPM, such problems are represented as solutions of cost minimization problems with quadratic structure. That is, the firm in this case is presumed to choose its current price,  $p_t$ , and a planned path for this price over all future periods,  $p_{t+s}$ , all s > 0, to minimize the following expression:

$$E_t \left( \sum_{s=0}^{\infty} (\delta \rho)^s (1-\phi) (p_{t+s} - p_{t+s}^d)^2 + \phi [p_{t+s} - p_{t+s-1} - \bar{\pi}_t^e]^2 \right) , \qquad (1)$$

where  $E_t$  represents the expectation conditional on all information known when this plan is struck.<sup>35</sup>

This calls for the price-setting firm to balance the cost of being away from its desired price,  $p^d$ , as captured by the first quadratic term in (1), against the cost of changing prices, as captured in the second quadratic term. Future costs are discounted using factors  $\delta$  and  $\rho$ , which are, respectively, the subjective discount factor that is common to all behavioural decision-making by private agents, and a forecast weighting factor (Laxton and Tetlow 1992). The forecast weighting factor captures the idea that the confidence an agent would place on a forecast would, all else held equal, diminish as the horizon is extended. In essence, this factor is used to increase the effective rate of discounting of the future to limit the

<sup>34.</sup> Unless otherwise noted, all variables in all equations are measured in natural logarithms, except for rates of return and inflation.

<sup>35.</sup> This could include information about exogenous variables in period *t*, but would normally exclude the shocks and values of endogenous variables in period *t*.

effective horizon for forward-looking calculations. This is a common feature of forward-looking models. Economists have found that behaviour is better described in formulations where private agents discount the future at rates well above market interest rates. This idea is applied throughout QPM.

The transactions cost technology in equation (1) is quadratic around the trend inflation rate expected by the private sector,  $\bar{\pi}_t^{\ e}$ . What this characterization implies is that firms can increase their prices at a rate that their customers feel can be justified by general price inflation, without bearing adjustment costs. Increasing prices by more than that will induce customers to search for a better deal, while increasing prices by less will result in foregone profits at the margin.<sup>36</sup> The cost of adjustment, relative to the cost of being away from the desired price, is measured by  $\phi$ . The larger is  $\phi$ , all else equal, the slower will be the adjustment to a disequilibrium.

The solution to this problem yields a system of Euler equations that can be solved forward to obtain the firm's decision rule,

$$p_t = \bar{\pi}_t^e + \lambda p_{t-1} + [\lambda(1-\phi)/\phi] \sum_{s=0}^{\infty} (\delta\lambda)^s E_t p_{t+s}^d , \qquad (2)$$

where  $\lambda$  is the smaller of the two roots from solving the Euler equation and  $\bar{\pi}^e$  is the expected trend inflation rate, which is described below.

Equation (2) gives the current setting of prices as determined by lagged prices and the geometrically declining sum of future desired price levels. This is called a *target-seeking problem* with possibly nonstationary forcing variables (Pesaran 1987). The parameters  $\{\lambda, \phi\}$  embody the intrinsic dynamics of the model. As  $\phi$  approaches zero, the cost of being away from the target price becomes infinitely high relative to the cost of

<sup>36.</sup> It is not difficult to motivate the costliness of price increases and only slightly more difficult to justify costs of price decreases in terms of foregone (marginal) profits. What is somewhat more difficult to argue for is the symmetry of the adjustment cost technology. This is a simplification that we address in an indirect way later on through the introduction of asymmetry in the final price equations.

adjusting prices, and the forward terms in equation (2) drop out, reducing the problem to a standard partial adjustment problem. On the other hand, as  $\phi$  approaches 1, the cost of adjustment becomes relatively important,  $\lambda$ approaches 1, and convergence on the desired price becomes infinitesimally slow; in the limit, equation (2) becomes a first-order autoregressive process with a drift term given by expected trend inflation.<sup>37</sup>

The target-seeking problem can be specialized to handle particular sectors or issues. Additional variables are sometimes introduced to augment the dynamics from equations like (2). Sometimes, further lags of the endogenous variable are added, a practice that can be easily justified by appeal to higher order adjustment costs, as formally demonstrated in Pesaran (1991) and Tinsley (1993).<sup>38</sup> We shall discuss in some detail, below, what is added to the price equations. At this point, it is sufficient to note that lagged values of the output gap are generally included.

Making operational decision rules like equation (2) requires a finite maximum lead length. As noted above, this maximum lead is effectively determined by the choice of  $\rho$ , which determines a terminal lead length beyond which the coefficient values are so small as to be deemed inconsequential. For prices, we truncate at five years (20 quarters) ahead.

What remains is to specify the nature of desired future price levels. This is where the expectational dynamics come in. In QPM, expected prices are specified by starting with the firm's static price-setting problem,

$$p_{t+s}^{d} = \Psi_{t+s} + mc_{t+s} + \tau_{t+s}^{y},$$
(3)

**<sup>37.</sup>** Specifically,  $\lambda = \left\{ (\delta + \phi)/\phi - ([(\delta + \phi)/\phi]^2 - 4\delta)^{0.5} \right\}/2\delta$ . See Sargent (1981, 177-209). The parameter  $\lambda(\delta,\phi)$  is decreasing in each of its arguments.

<sup>38.</sup> Much the same strategy as described in this report is used for the Federal Reserve Board's new FRB/US model. The generalized polynomial-adjustment-cost technology of Tinsley (1993) is applied, with equations such as (2) written in an equivalent error-correction form. See French et al. (1995) for details.

where  $\Psi$  is the markup of price over marginal cost, *mc* is marginal cost, and  $\tau^{y}$  is the average gross rate of indirect tax.<sup>39</sup> Equation (3) is nothing more than the standard rule equating marginal revenue with marginal cost. The marginal revenue part is the price, net of the markup and the indirect tax rate. We assume that in the long run the markup converges on zero. That is, we assume that markets are perfectly competitive in the steady state, while some execution of market power is permitted in the short run.<sup>40</sup> In the short run, markups are linked to excess demand conditions in the goods market. Marginal cost is driven primarily by the wage rate, which is modelled similarly, in that measures of excess demand in the labour market and expectations of future prices are used.

If we were to assume that expectations are rational in the sense of Muth (1961), we would replace  $E_t p_{t+s}^d$  with  $p_{t+s}$ . We would proceed similarly for the right-hand side variables in equation (3), and the job of modelling expectational dynamics would be complete. Alternatively, expectations could be modelled as a fixed-coefficient, backward-looking process, as has traditionally been done in macroeconometric models. We do not use either of these extreme approaches.

The idea that some degree of forward-looking behaviour is needed is becoming standard. Still, some resistance remains. Some economists find it difficult to accept the notion of model consistency as representative of the actions of ordinary economic agents, arguing that it does not reflect the perceived level of sophistication of most economic agents or the information available to them. Other economists argue that most households and firms have access to the views of economic experts and that, in any case, models are intended to approximate the behaviour of the economy in aggregate, not the behaviour of particular individuals.

To balance these two views, and to produce a dynamic behaviour for the economy that seems to replicate the properties of time-series data

<sup>39. &</sup>quot;Gross" here means 1 plus the rate of tax. Recall also that all variables are in log form.

<sup>40.</sup> Provided that it were fixed in alternative steady states, a positive steady-state markup could be introduced with no important consequences.

reasonably well, expectations in QPM are modelled as a mixture of backward- and forward-looking components. The most general form of expectations used in the model would be written as follows:

$$p_{t+s}^{e} = \chi_{t}[\sigma^{s}A_{s}(L)p_{t} + (1-\sigma^{s})p_{t+s}^{SS}] + (1-\chi_{t})E_{t}p_{t+s}$$
(4)

$$\bar{\pi}_t^e = \chi_t B(L)\pi_t + (1-\chi_t)[(p_{t+20}^e - p_t^e)/5] \quad .$$
(5)

Equation (4) gives (the log of) the price expected at time *t*, for dates *s* periods into the future, as a flexible weighted average of a backward-looking component,  $A_s(L)p_t$ , the model-consistent solution for the variable in question,  $E_t p_{t+s}$ , and  $p_{t+s}^{SS}$ , the steady-state solution for period t + s. Equation (5) models core inflation along the same lines. In general, the parameters,  $\chi_t$  and  $\sigma$ , are bounded by 0 and 1.

Expectations formation of this sort can be thought of as reflecting the average behaviour of differently informed agents, some of whom are "rational" and use the model and others who operate using rules of thumb. Alternatively, the expectations can be considered as coming from identical agents who are all unsure of the true structure of the economy and of the shocks and the policies in place to respond to those shocks, and who therefore use a mixed rule.

Several points need to be made about this particular specification. First, assuming  $\sigma < 1$ , the weight applied to the steady-state solution for prices rises as the lead for expectations, *s*, increases. This represents the idea that while agents are typically uncertain of the duration of a particular shock, for those variables where there is a tendency to return to a predetermined path, agents place more weight on this path the further out in time they are projecting. That is, agents are assumed to understand that, although temporary shocks may last for a while, they do eventually disappear, and the solution returns to a control path. The rate at which the "average" disturbance can be expected to dissipate is reflected in  $\sigma$ .

Second, a number of special cases are permitted by this model of expectations formation, including fully model-consistent expectations and fully autoregressive expectations. The approach also allows a special case where no weight at all is placed on the steady-state solution for any date. This is important because, in some cases, it would be inappropriate to assume such automatic reversion to the steady state. For such variables – price levels and inflation are examples – the weight attached to the steady-state solution is, in fact, set to zero in QPM.

To allow a positive weight in expectations formation on the steadystate price level or inflation rate would create an error-correction mechanism for prices that would eventually ensure successful inflation targeting, irrespective of what monetary policy actions are taken. A monetary authority could, for example, announce a lower target inflation rate, and such a result would be achievable at relatively low cost, because it would be believed even in the absence of actions to carry out the policy, as reflected in the steady-state term, with a weight of  $\chi(1 - \sigma^s)$ . This would violate the principle underlying QPM that expectations of variables that are chosen by policy makers must avoid the possibility of "free lunches" in the form of pure announcement effects. The steady-state terms are used only where the expectations are about values dependent primarily on the structure of the real economy and never where using them would imply automatic credibility of a policy announcement.

Third, the backward-looking expectations component could, in principle, be anything from a univariate autoregressive process, to a vector autoregressive process, to numbers from some exogenous survey. To date, the modellers have used mainly autoregressive formulations with lag lengths ranging from very short, for financial variables, to very long, for capital stocks.<sup>41</sup> However, information from the Conference Board of Canada's survey of forecasters is used to condition historical measures of

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<sup>41.</sup> Note that, except for expectations for period zero, the backward-looking component involves, in principle, lags of future variables. These are substituted out so that only lags end up appearing in equations like (4), regardless of s. Effectively, this means that when equation (4) for s = 0 is updated over time, the backward-looking expectations process is updated independently of the model-consistent and steady-state components. (This is the reason for the s subscript on A.) Taking this literally favours the interpretation based on different classes of agents rather than a representative agent unsure of the true structure of the economy.

expected inflation and as starting values for expectations in projection scenarios.

This general specification for expectations is remarkably flexible. Judicious use of the component weights provides a rich variety of possible model dynamics in response to shocks.<sup>42</sup> This helps in calibrating the model to desired properties. More importantly, however, such choices can be given meaningful interpretation in terms of factors like the speed of learning, the degree of credibility of policy and so on. The interpretability also makes the incorporation of more complete endogenous learning rules within QPM a possibility for the future.

Working together, these two factors, expectational and intrinsic dynamics, can be configured in the calibration to propagate shocks and produce the kind of "Keynesian dynamics" seen in time-series data. The explicit identification of these two sources of dynamic properties allows the nature of a shock – whether it is permanent or temporary, anticipated or unanticipated, for example – to affect expectations and hence the overall model dynamics. This, in turn, provides more precision to the description of what the monetary authority must do in different circumstances to provide the anchor that keeps expectations consistent with the target rate of inflation.

We repeat that the theory sketched above is just the starting point for the QPM equations. The model builders have customized individual equations to suit the particular application. In some cases, more complicated lag structures were added; in others, terms that have no formal justification within the simple theory leading to equation (2) were added. Often, such elaborations were based on empirical evidence that a certain formulation helps explain the historical data; occasionally they were introduced to help achieve a desired property in the calibration.

<sup>42.</sup> The time-varying weights feature for  $\chi$  has not yet been used in the model when it is run for projections, although some use of it has been made in policy analysis experiments.

#### 3.2.2 Asymmetric price and wage equations

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Monetary policy works in QPM in a number of ways. Inflation cannot be set at its target level by fiat or controlled by direct intervention in the markets where prices are set. Rather, policy must work indirectly by influencing conditions in those markets. Of particular importance, in this regard, is the link between market excess demand conditions and the degree of inflationary pressure. The Bank's view of this process, which is embedded in QPM, is that inflation responds to the degree of excess demand in the economy. We can summarize this idea as the view that there is a Phillips curve. Policy is then seen as working by influencing the level of aggregate demand in the economy, and hence indirectly the rate of inflation. Staff at the Bank have published many papers discussing the empirical nature of these dynamic relationships in the Canadian context. It is important to note that this is not the only channel of monetary policy influence on the course of inflation. The impact of policy actions on the path of inflation expectations is important in its own right.<sup>43</sup> Moreover, to the extent that policy goals are credible and expectations are anchored to the underlying target inflation rate, the effective inflationary pressures arising from shocks to product markets will be less volatile.<sup>44</sup>

Equation (2) does not have the form of a Phillips curve, in that no explicit link to market conditions is evident, although it is evident that market conditions can play a role through both the forward-looking attention to expected costs and the markup term in equation (3).<sup>45</sup> We now describe the actual structure of the equations that provide the price and wage dynamics of QPM and do include such explicit links to market conditions.

<sup>43.</sup> For completeness, we must add that there is also an important channel that works through the effects of changes in the exchange rate on domestic prices.

<sup>44.</sup> See Laxton, Ricketts and Rose (1994).

<sup>45.</sup> Cozier (1989) describes how such a framework can be seen as providing the foundations for a more traditional Phillips curve, which relates the dynamics of inflation to the degree of aggregate excess demand in the economy.

There is growing evidence, for Canada and for other industrialized countries, of an asymmetry in price movements in response to demand conditions.<sup>46</sup> Specifically, the evidence for Canada suggests that prices rise more (and more quickly) in response to excess demand in the goods market than they fall in response to excess supply of the same magnitude (Laxton, Rose and Tetlow 1993b).<sup>47</sup> Also relevant are the findings of Laxton, Rose and Tetlow (1993c) to the effect that the costs, in terms of policy errors, of assuming symmetry in the effects of excess demand on prices when the true structure of the economy is asymmetry when the true structure is symmetric. Based on these findings, the model builders included within QPM's Phillips curve an asymmetry in the output gap so that equation (2) is augmented to add the terms shown below:

$$p_{t} = \bar{\pi}_{t}^{e} + \lambda p_{t-1} + [\lambda(1-\phi)/\phi] \sum_{s=0}^{N} (\delta\lambda)^{s} p_{t+s}^{d}$$

+ 
$$\sum_{j=1}^{4} \alpha_j ygap_{t-j} + \sum_{k=0}^{3} \beta_k ((ygap_{t-k} + |ygap_{t-k}|)/2),$$
 (2')

where *ygap* is the relative deviation of output from its sustainable or "potential" level. The second summation in the second line of (2') contains elements that are zero, if *ygap* is negative, and *ygap*, if it is positive. This function therefore has a kink at *ygap* = 0. Assuming that the  $\beta_k$  are positive, we have the desired form where excess demand is more inflationary than excess supply is disinflationary.

This modification from the more standard linear specification of a Phillips curve has important implications for model properties.

<sup>46.</sup> Theoretical models that predict such asymmetry and which can be seen as generalizations of the specification leading to equation (2) have also been developed. See, for example, Tsiddon (1991, 1993) and Ball and Mankiw (1994).

<sup>47.</sup> Other empirical evidence supporting the existence of asymmetry of this sort for the G-7 countries can be found in Laxton, Meredith and Rose (1995) and for the United States in Clark, Laxton and Rose (1996). See also Turner (1995).

Particularly noteworthy is the implication that the effects of a shock will depend in an important way on conditions in the economy when the shock arrives. An expansionary fiscal policy, for example, would have more impact on real output if introduced under conditions of excess supply than it would if introduced under conditions of full employment or excess demand, where the inflationary consequences would be relatively more important and would force offsetting action by the monetary authority. This plausible result is often missing in linear models.

#### 3.2.3 Interest rates and the monetary policy reaction function

Canada has a relatively small economy with financial markets that are highly integrated with world markets. As a result, over the longer term, real interest rates in Canada are largely determined in world markets. However, over the shorter term, domestic monetary policy exerts an important influence on real interest rates. Monetary actions affect shortterm interest rates most directly, and these effects reverberate up the term structure and over to the exchange rate, all of which impact on aggregate spending and ultimately inflation.

The determination of interest rates in QPM reflects this characterization of the transmission mechanism. Real interest rates in QPM are pinned to world real rates in the long run up to an exogenously specified risk premium. In the short run, however, monetary actions can affect real rates because prices are slow to adjust. The instrument of monetary policy in the model is the short-term nominal interest rate, and monetary actions are transmitted to real activity through the impact of changes in the short rate on the slope of the yield curve. Formally, the link between the yield curve and real activity in the model arises because consumption and investment spending are specified to be influenced by the yield spread – the short-term interest rate less a long-term rate.

The use of the yield spread as the principal channel through which monetary policy affects real activity in QPM reflects two main considerations. First, it reflects the view that the yield spread is a better indicator of the stance of monetary policy relative to the underlying momentum in the economy than are short-term real interest rates alone.

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Second, the use of the yield spread provides a parsimonious way to capture the effects of the full term structure of interest rates on aggregate spending.

An important challenge in the SEP is to interpret the underlying shocks in the economy that are influencing the incoming data. In this context, the yield spread has the attractive feature that it helps in isolating monetary influences on real interest rates. Movements in both long and short rates reflect fluctuations in the equilibrium real interest rate (as determined by productivity and thrift in the world economy). Changes in the short rate also reflect changes in the stance of monetary policy, while long rates are initially relatively immune to changes in monetary conditions; thus, to a large extent, the yield spread serves to isolate the monetary component of changes in real interest rates.<sup>48</sup>

For the monetary authority, long rates contain useful information on the credibility of monetary policy, information that can serve as a useful guide to the changes in short-term interest rates required to control inflation. In a typical interest rate cycle, long rates will initially rise with the short rate when the central bank tightens monetary conditions to combat inflation, since initially credibility will tend to be low. As the central bank continues to reveal its determination to reverse the rise in inflation, however, the long rate will begin to fall as inflation expectations respond. This serves as a signal to the monetary authority that it can ease off on short rates a little. Measuring monetary stance in terms of the spread is a convenient way to summarize this relationship between policy actions and their credibility.<sup>49</sup>

<sup>48.</sup> There are several stylized facts that are consistent with this interpretation: (1) most of the variability in the yield spread is at the short end; (2) the yield spread reverts relatively quickly to its mean, as one would expect if it primarily reflects the liquidity effects of monetary actions; (3) among the components of GDP, the spread is most closely correlated with consumer durables and housing. See Cozier and Tkacz (1994) and Clinton (1995).

<sup>49.</sup> Goodfriend (1993) provides an interesting narrative history of U.S. monetary policy since the 1970s, based on the relative movements of long- and short-term interest rates through several critical episodes.

The yield spread also captures an intertemporal aspect of consumers' expenditure decisions. In particular, the spread provides information on the expected path of interest rates, and this may influence the timing of expenditures and thus the dynamics of aggregate demand. For example, a consumer who is considering borrowing to purchase a car or a house may be enticed to do so sooner as opposed to later if the long rate is considerably above the short rate, indicating that short rates are expected to rise in the future. Conversely, faced with an inverted yield curve, the consumer is likely to postpone major expenditures on the expectation that the cost of financing is going to fall.<sup>50</sup>

For all these reasons, the monetary reaction function is expressed in terms of the yield spread. It is written:

$$R_{t}^{S} - R_{t}^{L} = \delta(R_{t-1}^{S} - R_{t-1}^{L}) + \theta\left[\sum_{i=6}^{T} (\pi_{t+i}^{e} - \pi_{t+i}^{T})\right], \qquad (6)$$

where  $R^{S}$  is the short-term nominal interest rate (specifically, the 90-day commercial paper rate),  $R^{L}$  is a long-term nominal rate (the 10-year-and-over Government of Canada bond rate),  $\pi^{e}$  is the rate of inflation expected by the monetary authority, and  $\pi^{T}$  is the target rate of inflation.

Note that this rule is explicitly forward-looking. The central bank reacts today to predicted deviations of inflation from the target rate six to seven quarters ahead. Any forces that create an expected inflation gap will be countered by an increase in short-term interest rates. It takes time for a change in the monetary instrument to influence aggregate demand and still more time before the full effects on inflation emerge. The six- to sevenquarter lead in the reaction function was chosen with this in mind; it reflects the horizon over which monetary action can be expected to have a meaningful effect on trend inflation. Note that, all else equal, the reaction

<sup>50.</sup> Evidence on the relationship between the spread and the major components of spending is broadly consistent with this interpretation. In particular, the relationship between the yield spread is strongest for consumer durables and housing, and these components are the ones for which timing considerations are likely to be most important. See Cozier and Tkacz (1994).

will not bring inflation to the target over this horizon. Actual achievement of the target will take a bit longer.<sup>51</sup>

The role of the term in the lagged value of the (inverted) yield spread,  $R^S - R^L$ , is to smooth the reaction of short-term rates to shocks. One can think of this as reflecting constraints facing the central bank in how fast it can move in responding to shocks. It also reflects an element that is missing from the formal model and the way it is normally used in responding to shocks – the uncertainty faced by a central bank in interpreting what is happening in the economy and what shocks will arrive in the next few periods. In any case, the weight placed on this term is relatively small and serves only to smooth the response over a couple of quarters.

In equation (6), the difference between short and long interest rates,  $R^{S} - R^{L}$ , can be thought of as the short-term *operational target* of monetary policy with  $R^{S}$  being the *policy instrument*.<sup>52</sup> The central bank acts to influence short-term interest rates, aiming to set a path for the slope of the yield curve that will eventually achieve its objective for inflation. In doing so, the authority takes into account that future short-term interest rates will affect current long-term rates in keeping with the expectations theory of

<sup>51.</sup> The reason is simply that the policy reaction to a shock with this rule must leave some inflation gap, relative to the baseline solution, over the six- to seven-quarter horizon. Otherwise, the rule would show no reaction, relative to that baseline – a logical contradiction. Thus, the model will solve such that inflation is controlled, with a stronger reaction the larger is  $\theta$ , but the effects of a shock on inflation will be removed over a horizon longer than six to seven quarters. For most shocks, the effective horizon in this sense is 8 to 12 quarters.

<sup>52.</sup> Distinguishing between instruments and operational targets is not always straightforward. The actual day-to-day operating procedure used at the Bank of Canada (see Clinton and Howard 1994) cannot be represented literally in a quarterly model. In principle, an instrument is exogenously controlled by the monetary authority. In both QPM and the real world, the 90-day commercial paper rate is not exogenous in this sense, but it is the closest reflection of such a control instrument in the model.

the term structure.<sup>53</sup> In this way, among others, it is the entire path of interest rates that matters, not just settings at particular dates.

The central bank cannot control inflation directly. Shocks that hit the economy will have an effect on inflation, regardless of how high the weight on the inflation gap is set in equation (6). The essential reason is that monetary policy influences the outcome with a lag. Hence, the monetary authority simply cannot keep inflation precisely at the target level. Deviations will occur, and may persist for some time, owing to the intrinsic and expectations dynamics of the system. Moreover, while a stronger response of policy to a particular shock may advance the timing of the return of inflation to the target level, if this is carried too far, the model's dynamics will generate overshooting and possibly even instability. The parameter  $\theta$  in equation (6) has been chosen with this in mind, such that the response to a wide variety of shocks yields effective control of inflation without generating excessive secondary cycling. Section 5 provides examples.

The rule in equation (6) has the advantage of capturing, directly or indirectly, those variables of concern to the monetary authority. It is also broadly representative of recent monetary policy. Moreover, as demonstrated in Section 4, the staff have been able to arrive at a calibration, using this rule, that seems to capture well at least part of the monetary policy transmission mechanism in Canada.<sup>54</sup> It should be noted that the absence of deviations of output from its sustainable level should not be taken as indicating a lack of concern for output or employment. Higher (lower) degrees of concern for output gaps would manifest

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<sup>53.</sup> The long-term Government of Canada bond rate in QPM is determined partly by the expectations theory of the term structure, partly by current short-term rates and partly by long-term rates in the rest of the world. The expectations theory states that, up to a fixed risk premium, the current long rate should be equal to the geometric sum of expected future short-term rates. Empirical evidence seems to suggest that long-term rates tend to overreact (from the perspective of this theory) to movements in short-term rates. In addition, there appears to be some arbitrage internationally across the rates of return of nominal long-term bonds.

<sup>54.</sup> For details on the monetary policy transmission mechanism in Canada see Duguay (1994), Longworth and Poloz (1995) and Hunt, O'Reilly and Tetlow (1995).

themselves in lower (higher) weights on the deviation of inflation from its target.<sup>55</sup> Finally, while the rule might not be "optimal" for the economy as represented by QPM, there are dangers in reading too much into "optimality" for a particular structure, given the evident uncertainties involved in modelling.<sup>56</sup> What is arguably a better alternative is to choose a simple, robust rule for base-case projections and to experiment with other rules as part of a broader research agenda. This is, in fact, a major thrust of the Bank's ongoing research, both with QPM and with smaller models designed to answer more stylized questions.

Equation (6) is a rule that is ultimately concerned with inflation targeting, consistent with the joint announcement by the Bank of Canada and the Department of Finance in 1991 of target bands for inflation reduction, and in December 1993 of their extension to the end of 1998. However, more complex rules could be entertained. These include, for example: rules that imply proportionally stronger policy response when inflation is expected to move outside the bands; rules that put some weight on limiting exchange rate volatility; and rules that consider price-level targeting instead of (or in conjunction with) inflation targeting.<sup>57</sup>

#### 3.2.4 Taxes, debt and the fiscal policy reaction function

The public sector in QPM reflects a consolidation of the activities of federal, provincial and local governments. This "government" sector spends on goods and services, makes transfers to the private sector, raises

<sup>55.</sup> If we leave aside the difficult issues of time consistency and precommitment, it is straightforward to show that the "optimal" value of  $\theta$  is a function not only of the "taste" parameter on  $\pi - \pi^T$  in a loss function, but also of the shadow price of output from the co-state equations, provided the usual stability and rationality constraints are satisfied. This means that output matters. See Hall and Henry (1988, ch. 7).

<sup>56.</sup> Using stochastic simulations of small, forward-looking models with endogenous policy rules, Laxton, Rose and Tetlow (1993c) and Coletti, Muir and Tetlow (1995) demonstrate some pitfalls for policy-making of erroneous assumptions regarding model structure.

<sup>57.</sup> Laxton, Ricketts and Rose (1994) use a small macro simulation model to explore the possibility of adding asymmetry (a different response depending on the sign of the deviation of inflation from target) as well as non-linearity (a different response depending on the size of this deviation) to such a rule.

revenue through direct taxation of income as well as indirect taxation of domestic transactions. It also issues debt denominated in domestic currency.<sup>58</sup>

The equivalent of the inflation target for monetary policy is a set of two target ratios that define the fundamental fiscal policy in QPM. These ratios are the level of debt relative to output and the level of spending relative to output. The rate of personal direct taxation and the government budget deficit adjust to validate these choices.

The expenditure target is a typical formulation of an exogenous fiscal choice. Governments must decide what they will do, and this is captured in the macro model as a decision on the relative level of government absorption of domestic output. One could also think of this as including a choice on the relative level of transfers to the private sector, with a net absorption rate. For QPM, however, transfers are lumped with taxes on the other side of the ledger, with the income taxes and rates expressed net of transfers. Technically, the spending ratio could be made endogenous and the tax rate set as the policy choice. Nevertheless, the modellers preferred to describe fiscal policy using the expenditure ratio because, whereas taxes are represented in the household sector's decision and therefore affect the real equilibrium, the role of government absorption is much less well-developed in the model.<sup>59</sup>

The choice of a debt ratio is not quite as conventional. It is essential that some closure rule be included that requires flow fiscal variables to respect the intertemporal government budget constraint in a way that permits the attainment of a steady state. This issue is generally discussed in the literature in terms of defining what "sustainable" fiscal policy

<sup>58.</sup> For further details, see Laxton and Tetlow (1992), Macklem, Rose and Tetlow (1994) and Black, Laxton, Rose and Tetlow (1994). Black et al. contains a complete description of the accounting structure of taxes in QPM.

<sup>59.</sup> There is no formal rationale for government within QPM. The goods and services absorbed by government disappear, essentially. More formally, we assume that whatever public good is produced affects neither the utility of private consumption nor the level of productivity.

means. There are a number of approaches used, all of which require the debt-to-output ratio to stabilize at some level in the steady state. In QPM, however, the relative level of government debt affects the real equilibrium; the model does not have the property of Ricardian equivalence. It therefore matters what level of the government debt-to-output ratio is attained. For this reason, the modellers chose to use this ratio as part of the definition of fundamental fiscal policy. This has the major advantage of ensuring that the steady state will not change owing to debt dynamics or any other aspects of the particular fiscal adjustment path followed in any shock – unless that shock contains an explicit change of fiscal policy.

Equation (7) shows a stylized version of the model's fiscal policy reaction function:

$$\Gamma_t = \eta \Gamma_{t-1} + (1 - \eta) (\Gamma^{ss} + \xi ((D/Y) - (D/Y)^{ss})) \quad . \tag{7}$$

The rate of personal direct tax, net of transfers,  $\Gamma$ , is adjusted to realize the target ratios. The steady-state model provides a measure of the tax rate, net of transfers, necessary to support the target level of government absorption plus the steady-state level of debt service (interest payments). This is the variable  $\Gamma^{ss}$  in equation (7). The actual tax rate will eventually converge on that level. However, the key element in the reaction function is that the tax rate adjusts to ensure that the debt target,  $(D/Y)^{ss}$ , is achieved. If the actual debt ratio, D/Y, is too high, then the tax rate will rise to raise revenues and bring the ratio down. Note that the fiscal policy reaction function is not explicitly forward-looking, in the sense that only contemporaneous measures of deviations from the steady-state ratios are considered. Tax rates are not typically adjusted rapidly in response to economic conditions.

The debt gap in equation (7) is an essential feature of the dynamic specification. A temporary increase in government spending, for example, will be financed, in the first instance, by a run-up of debt. If the tax rate were to remain constant, the temporary increase in spending would result in a permanent increase in the debt ratio. Equation (7) requires taxes to rise temporarily to prevent this before returning to their unchanged steady-state level. A debt shock also illustrates well the crucial role of the stock

gap in the dynamics. Suppose that the target debt ratio is reduced. The steady-state effect of this is to lower the tax rate, because there is less debt service. That is,  $\Gamma^{ss}$  falls. If the actual tax rate were to simply adjust gradually towards this new equilibrium, however, the new debt target would never be achieved. The tax rate has to rise first, to generate the revenue to pay off the desired portion of the debt, before it can fall to its new long-run equilibrium. The stock gap provides the mechanism in the model that generates that result.

The adjustment of the tax rate to a disequilibrium is configured to be gradual over a period of several years. As in the monetary reaction function, this is done by putting some weight on the lagged value of the tax rate. In contrast with the monetary reaction function, however, in the fiscal policy reaction function, this weight is set at a relatively high level so that tax-rate adjustment occurs at a relatively measured pace.

#### 3.2.5 The exchange rate in QPM

The exchange rate plays an integral role in the monetary transmission mechanism of an open economy. In the standard Mundell-Fleming model with static exchange rate expectations, the nominal interest rate is determined entirely by conditions in the rest of the world, and attempts by the domestic monetary authority to manipulate it manifest themselves entirely in movements of the exchange rate.

The historical record has shown that while open economies such as Canada's do face constraints on interest-rate setting, the Mundell-Fleming depiction of a small, open economy is too strong: the domestic monetary authority can influence the level of the domestic nominal interest rate, at least temporarily. Nevertheless, Mundell's uncovered interest parity (UIP) condition is the central principle guiding the determination of the nominal exchange rate.

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If *S* designates the log of the nominal price of foreign exchange (so that a rise in *S* represents a depreciation) and *R* and  $R^{f}$  are the domestic and foreign nominal short-term interest rates, UIP can be written as<sup>60</sup>

$$S_t = S_{t+1}^e + R_t^f - R_t.$$
 (8)

UIP says that an increase in the short-term domestic nominal interest rate above the world rate implies that there is an expectation of a depreciation of the dollar (a rise in the price of foreign exchange).<sup>61</sup> It is an arbitrage condition – it says that the domestic rate cannot rise above the world rate unless foreign lenders expect to lose the equivalent of the interest differential in the change in the value of the currency over the period.

Making this operational in the context of a model requires economic structure governing the determinants of expected future exchange rates and the short-term nominal interest rate. As a first step towards examining this issue, it is useful to solve equation (8) forward to observe that the current level of the nominal exchange rate is a function of expected short-term interest rates at all future dates, plus a terminal condition:<sup>62</sup>

$$S = \sum_{i=0}^{\infty} R_{t+i}^{f} - \sum_{i=0}^{\infty} R_{t+i}^{e} + lim_{i \to \infty} S_{t+i}^{e}.$$
 (9)

The terminal condition,  $\lim_{i \to \infty} S_{t+i}^e = \Psi$ , is often neglected in discussions of exchange-rate determination. There is, however, meaningful economic

<sup>60.</sup> For convenience, we have suppressed the risk premium that would normally appear in a UIP equation. In the data, this risk premium appears to be time-varying. In fact, as Cox, Ingersoll and Ross (1985a, 1985b) show, there is no a priori reason why the risk premium cannot vary over time if the variance-covariance matrix of real returns and inflation is not constant. To date, however, the risk premium remains an exogenous variable in QPM.

<sup>61.</sup> In simulation of such changes in interest rates, it will be a jump in the spot rate (in this case an appreciation) that will validate the expectation of a change in the other direction over the holding period of the transaction.

<sup>62.</sup> In a two-country model, foreign prices, output and interest rates would be taken as endogenous variables. However, these are taken as exogenous variables in QPM; for this reason, no expectations superscripts are attached.

content to the value of  $\Psi$ , and for our purposes it cannot simply be ignored. Expected future nominal exchange rates can be divided into their real and nominal (relative price level) parts,

$$S_{t+i}^{e} = Z_{t+i}^{e} + P_{t+i}^{e} - P_{t+i}^{f},$$
(10)

where  $Z^e$  is the log of the expected real exchange rate, and  $P^e$  and  $P^f$  are the logs of the expected domestic and foreign producer price levels, respectively. Equation (10) makes it clear that the terminal condition,  $\Psi$ , involves a real and a nominal component. The steady-state model SSQPM provides a solution for the steady-state real exchange rate:  $lim_{t\to\infty}Z^e_{t+i} = Z^{SS}.^{63}$ 

For prices, matters are not so clear cut. The dynamics of expected domestic producer prices are influenced by excess demand, by nominal costs, particularly wage costs, by foreign prices (through import prices) and, of course, by the conduct of monetary policy. These same prices are important and direct determinants of nominal interest rates, both through the Fisher equation and through the policy reaction function. So prices influence every endogenous variable in equation (8).

If the monetary authority acted to implement a price-level target, then the terminal condition for *Z* would have a precise counterpart in *P*:  $lim_{t\to\infty}P_{t+i}^e = P^T = P^{SS}$ .<sup>64</sup> Under inflation targeting, however, the steadystate price level is not determined uniquely. More precisely, the long-run solution for the path for prices will be a function not just of the policy rule but of the whole dynamic structure and the shocks that arrive. In short, there is no terminal condition for the price level (and the nominal exchange

<sup>63.</sup> The determination of the steady-state real exchange rate is discussed in some detail in Black, Laxton, Rose and Tetlow (1994).

<sup>64.</sup> That is, provided that the particular price-level targeting rule is in fact feasible. Such rules do work in QPM, but because level targeting is more demanding than is rate-of-change targeting, rules must be specified carefully to be feasible for all plausible shocks. It should also be noted that with  $z^{SS}$  predetermined, specifying a target in the level of the nominal exchange rate is equivalent, in the long run, to specifying a target for the price level.

rate) under these circumstances. This fact presents some minor numerical complications for simulating QPM, as it would for any forward-looking model. But no economic difficulties arise as long as the restrictions necessary for long-run equilibrium are built into the model's structure. The more economically interesting of these restrictions are those associated with the monetary and fiscal policy rules, where issues of controllability arise. As Holly and Hughes Hallett (1989) show, there are limits on the feasibility of policy rules. If these restrictions are not respected, then agents will rationally conclude that future prices, given policy instrument choices, are not consistent with the objectives of policy, thereby rendering the policy infeasible.<sup>65</sup>

More generally, the dynamics of the nominal exchange rate over the medium term will be governed partly by nominal factors that are inextricably tied to monetary policy and the reaction function and partly by real factors, since the real exchange rate acts as an equilibration mechanism. Inflationary nominal shocks oblige the monetary authority to tighten monetary conditions (raise the short interest rate). In doing so, they must take into account the induced appreciation of the dollar resulting from the UIP condition, since there is a direct link between the level of the nominal exchange rate and the domestic price level coming from the passthrough of import prices into consumption prices. Equation (9) makes it clear that the level of the nominal exchange rate at any point in time is a function not just of the contemporaneous interest differential, but also of agents' expectations of all future rates. These, in turn, will be functions of current and expected future economic conditions.

On the real side, the real exchange rate is set to generate the trade balance necessary to support the net foreign asset position chosen by consumers. The net asset balance determines a required payments flow, and the trade balance is set to provide for those payments. The desired stock of net foreign assets thus proximately determines the steady-state real exchange rate, and the market real exchange rate must adjust to

<sup>65.</sup> Arguably, this is what happened when financial market pressures brought about the departure of several countries from the European exchange rate mechanism in 1992.

establish and support the stock equilibrium. All else equal, increases in consumption lead to declines in the net foreign asset position and increases in net exports lead to increases. The need to resolve both stock and flow equilibrium conditions creates a tension for monetary policy over the medium term. Efforts to increase aggregate demand through an easing in interest rates, for example, bring about declines in the net foreign asset position that may eventually have to be reversed. The same easing elicits a depreciation in the dollar and an improvement in the trade balance: the expansion in aggregate demand brings about a substitution of net exports for consumption that at least partially alleviates the stock disequilibrium that would have otherwise occurred. But the timing of effects is not generally co-ordinated, and as a consequence, some secondary cycling often results.

The extent to which conflicts exist is largely determined by the adjustment of the expected real exchange rate. This is a subject about which economists know very little. In reflection of this, and as befits such a key variable in the model, QPM has been constructed to provide a great deal of flexibility in the modelling of expected future real exchange rates. In the base-case model, the staff use the following specification:

$$Z_{t+1}^{e} = \mu_1 Z_{t-1} + \mu_2 Z_{t+1} + (1 - \mu_1 - \mu_2) Z_{t+1}^{SS}.$$
(11)

The weights on the backward-looking, model-consistent and steady-state components are set at 0.60, 0.25 and 0.15, respectively. The relatively substantial weight on the model-consistent solution for Z means that there is ample room for the exchange rate to jump in response to shocks and overshoot its fundamental value – given by  $Z^{SS}$  – as in the influential Dornbusch (1976) model. Note also that the backward-looking portion of expectations formation is a random walk; that is, there is no memory in the determination of the real exchange rate (abstracting from price expectations errors) other than that given by the determinants of the steady-state real exchange rate. Finally, while there is a modest weight of 0.15 applied to the steady-state solution so as to ensure that Z does not range too far from "fundamentals" for too long, it is important to note that

no weight is strictly necessary on  $Z^{SS}$  to obtain convergence. That is, the weight applied is a calibration decision, not a stability requirement.<sup>66</sup>

Taking equation (11) as given, the amplitude and duration of cycling is a function of the policy rules and of the particular shock. Attempts to quickly bring the (flow) economy and inflation back to their desired paths will generate more cycling than more gradual approaches. Shocks that imply a different steady-state ratio of net foreign assets to GDP will, in general, involve greater conflicts at some point along the adjustment path.

The preceding makes clear that the exchange rate in QPM is a function of almost everything: it is perhaps the most endogenous variable in the model, which reflects its central role in the macro economy. Its dynamics will therefore reflect virtually all aspects of disequilibrium adjustment in the model. Sometimes this means that the exchange rate's path defies easy explanation. In this, the model reflects the real world rather well.

<sup>66.</sup> Putting a small weight on the steady-state value speeds up the resolution of stock disequilibrium considerably. Otherwise, it takes implausibly long for QPM to get to its steady state.

# 4 CALIBRATION

One feature of QPM that represents a departure from past practice is that the model is not directly estimated. Rather, the model is calibrated to reflect aspects of the data and to have desired simulation properties. In this section, we discuss the reasons for this approach and some of the specific methodology used. We also provide some examples of calibration decisions taken for QPM. The reader is referred to the first volume in this series, Black, Laxton, Rose and Tetlow (1994), for an extensive discussion of the calibration of the steady-state model. Here, the focus is on dynamic issues.

# 4.1 Why calibrate?

The traditional method of fixing a model's parameters is through econometric techniques. This has always been problematic, however, in a number of ways. We review two related issues here. The first difficulty is that the estimation has rarely reflected the nature of the economic problem. For example, single-equation techniques have often been used despite obvious problems of simultaneity bias. The second concerns the logic of macro models. We interpret macro models as being necessarily false; heroic abstraction from detail is necessary for economists to formulate workable, understandable systems for analysis of policy issues. With this premise, it is not clear what properties to expect from even fully articulated systems estimators, such as full-information maximum likelihood (FIML).

Macroeconomic models are, in principle, descriptions of highly simultaneous, general equilibrium systems. Yet most estimation exercises have been limited to single-equation techniques or small subsystems. In small models, estimation by FIML may be feasible, but once a model is of a size that makes it useful for examining the broad range of questions that a monetary authority might ask, full simultaneous estimation becomes impossible. One is obliged to turn to other methods, such as instrumental variable techniques.

Instrumental variable techniques, however, often have poor or unknown small-sample properties that bring into question their usefulness.<sup>67</sup> This is just one reason why there have been conflicts between what fits the data and what is needed for macro models to obey any reasonable theory. In the past, this conflict was usually settled in favour of estimation and a search for extra structure to solve problems. Many modellers continue to rely on estimation. They are tending, however, to put greater emphasis on the essential macroeconomics in their modelling, and they are less reluctant to impose theoretically consistent parameters (French et al. 1995). Sometimes this is defended as part of good econometric practice, indicating that the distinction between estimation and calibration is not altogether clear. However, we would stress the related point that macroeconomic models necessarily abstract from much real-world detail that matters from time to time in an idiosyncratic manner. This makes the small-sample econometric problem particularly difficult in this context.

The identification problems caused by the high simultaneity of a macro system is of the greatest importance in this discussion. Take the example of the difficulty econometricians have had identifying a link between monetary policy and business investment spending through a variable such as the user cost of capital, on which Blanchard (1986, 153) has remarked: "The discrepancy between theory and empirical work is perhaps nowhere in macroeconomics so obvious as in the case of the aggregate investment function."<sup>68</sup> Estimating any long-run relationship,

<sup>67.</sup> One early example of the rejection of overidentifying restrictions, common in limitedinformation (LI) estimations of quadratic adjustment cost models under rational expectations, is Sargent (1981). West (1986) shows that using LI techniques produces a "moderate" deterioration from full-information techniques, provided the model specification is literally correct. Once it is accepted that the quadratic adjustment cost model is only approximately true, however, LI techniques do considerably less well. West and Wilcox (1994) show that neither FI nor LI techniques do very well in finite samples.

<sup>68.</sup> Another vivid reflection of the profession's unsuccessful battle with the data on investment is found in the title of an article written many years ago by Robert Eisner "Investment and the Frustrations of Econometricians" (Eisner 1969). Even modern and careful estimation techniques seem to go unrewarded, as testified by Abel and Blanchard (1988) and by Ford and Poret (1990). However, see also Cabellero (1994).

much less the negative one that neo-classical theory suggests, between investment and some measure of the cost of capital has proven difficult.

The desire is to identify the slope, in a particular direction, of a demand curve. In this case, there is an added complication of a stock/flow issue, but let us leave that aside. If it were true that the world featured repeated monetary innovations that changed the cost measure which, in turn, affected investment spending, then the econometrician would readily recover a reasonable estimate of the relevant elasticity. The predominant correlation in the data would be the negative association of interest rates and spending, associated with movements along a demand curve. If the shocks were primarily from other sources, however, the identification might not be so easy. Suppose, for example, that the dominant shock is to aggregate demand and operates on investment through the traditional multiplier/accelerator mechanism. If a monetary authority is pursuing any policy that implies a nominal anchor, then it will typically be *responding* to stronger spending, including stronger investment spending, by putting upward pressure on interest rates. In this case, the data may show a positive correlation between interest rates and investment spending. This does not mean that there is no underlying demand function with the opposite slope, but it does mean that it may be very difficult to identify it with econometric techniques in a small sample.

This is but one example of many that arise in macroeconometrics. Our experience has led us to the view that this problem is severe enough to seriously impair the power of econometric techniques to provide good parameterizations of macroeconomic models. Indeed, calibration to properties or features of the data that are robust may provide the best possible estimates of the model's fundamental structure.

The goal in building QPM was to achieve a workable marriage between short-term forecasting properties and the kind of medium-term dynamics, disciplined by a steady state, that enable and enrich policy analysis. The simultaneous achievement of these goals places special demands on the model and restrictions on its parameters that cannot be readily achieved through the kinds of estimation techniques which in the past have been judged to be appropriate. Moreover, since QPM is the model used for the Bank's SEP, its properties had to encompass the staff's beliefs. These beliefs are based on broader knowledge and experience than the disinterested econometrician would bring to bear in a particular estimation.<sup>69</sup> This is especially true when the staff consider a counterfactual question – where they consider what would have happened had the Bank followed a different policy during some particular time period. What this means is that the metric by which one might measure "goodness of fit" may be broader in a policy institution than traditional econometrics permits.

## 4.2 Calibration in context

Calibration in dynamic macroeconomics began in the early 1980s with real business cycle (RBC) models; in fact, it was in the seminal RBC article, Kydland and Prescott (1982), that the term calibration was first applied in this sense, as far as we know, although many of the same ideas had been used earlier in computable general equilibrium models.<sup>70</sup> From RBC models, the idea has spread to Keynesian macro models (McKibbin and Sachs 1989) and multisector macro models (Macklem 1993). An early example of a calibrated model at the Bank of Canada is Longworth and Poloz (1986). The methodological foundations of this approach can be traced back to Frisch (1933) and Simon (1969), and are explained in some detail by Kydland and Prescott (1991) and by Hoover (1995).

<sup>69.</sup> It is interesting, in this regard, to note the implications for this question of the structure in which projections are conducted at the Bank of Canada. Since the late 1970s, the staff projection has always been built around the predictions of a model of the economy. The properties of the model are properties that the staff have, by and large, agreed upon. Changing the model requires a consensus of the various parties involved in the projection. This is one reason why the parameterization of QPM draws from so many different sources. At most other institutions, no one model plays such a central role. At the Federal Reserve Board, for example, many forecasts are produced using many models at the staff level. More senior people then use these forecasts as inputs for the official Green Book judgmental forecast. The more widespread use of calibration at the Bank of Canada thus reflects, in part, a projection process that moves much of the debate concerning any projection from questions about outputs to questions about inputs.

<sup>70.</sup> For a recent application to a computable general equilibrium model, see Shoven and Whalley (1992).
In the RBC literature, calibration is applied to simplified, tightly structured models, where the theory is given primary importance, even when it obviously could not be literally correct. The following quote makes the essential point:

The interesting question ... is surely not whether it [a real business cycle model] can be accepted as "true" when nested within some broader class of models. Of course the model is not "true": this much is evident from the axioms on which it is constructed. We know from the outset in an enterprise like this (I would say, in *any* effort in positive economics) that what will emerge – *at best* – is a workable approximation that is useful in answering a limited set of questions.

- Lucas (1987, 45)

Eschewing direct estimation of the model does not imply that model assessment is downgraded. Nor, for that matter, does it mean that econometrics is forsaken. Rather, the criteria by which one judges the model's appropriate coefficients change. QPM is calibrated according to two broad goals: *matching properties* and *matching moments*.

Matching properties means choosing parameters to replicate certain global features that are judged to be desirable. At present, these features run the gamut from the merely convenient, such as ensuring that stocks are seen to approach an equilibrium over the length of the projection horizon, to the obligatory, such as ensuring that real variables attain the correct steady-state values. Other properties are taken from the impulse responses of econometrically estimated reduced-form models, vector autoregressive models and so forth.

Matching moments refers to replication of certain "stylized facts." While desirable properties contain an important judgmental element, the stylized facts are, to a first approximation, judgment-free and help to maintain the objectivity of the calibration. Among the stylized facts are the typical lead-lag relationships between variables, their relative standard deviations and autocorrelations.

The matching-moments criterion is one shared with the RBC literature. In RBC models, tight specification and sparse parameterization

allow researchers to use estimates from independent sources for some parameters, leaving the rest to be chosen to match moments. In the case of QPM, the variety of intended uses meant that the model could not be so sparsely parameterized. There are more parameters that, while not "free," have no obvious correspondence with parameters that are commonly estimated in empirical work. These parameters gave the model builders sufficient freedom to calibrate the model to simulation properties. Owing to this focus on replicating properties, the model serves its projection goals more closely than would have been the case had calibration been conducted as with RBC models. In this sense, then, it is clear that QPM is very much an empirical model, albeit not in the sense of classical econometrics.

A major benefit of parameterization by calibration is that it works. That is, if the model is constructed skilfully, a calibration can be found that will permit a fairly wide range of simulations to be carried out with satisfactory results. The difficulties should not be ignored, however. The principal disadvantage is that with calibration it is not clear how one can reject the underlying paradigm of the model. There is no formal metric for testing, and no textbook of calibration techniques exists, as yet.

It is the dominance of theory in the choices that modellers make that lies at the heart of the difference between calibrators and estimators.<sup>71</sup> Notionally, econometrics is a competitive methodological strategy, where theory proposes and testing by estimation disposes (Hoover 1995). In principle, a whole range of models compete for support from the data. However, as we have already indicated, the methodology of macroeconometrics does not completely follow this ideal; elements of calibration methodology, if not calibration per se, can be easily seen. On the other hand, calibration is an adaptive strategy. With calibration, one starts with a model of the economy and examines it in detail to see what might be explained by its use. Thereafter, features that are needed to better

<sup>71.</sup> By estimators, we mean, in particular, those who adhere fairly rigidly to classical econometric techniques and "let the data speak," even if the message violates theoretical arguments.

explain the data, or to explain a broader set of phenomena, are added. This is precisely what a macroeconometrician would do as well: seek small changes to the specification to "solve" the empirical problem. Thus, calibration and estimation need not be all that different in this respect.

The complementarity of calibration and (direct) estimation is recognized by some of the former's proponents. Manuelli and Sargent (1988), Gregory and Smith (1990) and Canova (1991), among others, interpret calibration as estimation by simulation. To these authors, calibration merely expands the set of criteria by which one might judge "goodness of fit." What remains is to derive a reliable, agreed-upon metric to measure closeness. This need not be a metric drawn from econometrics. For example, at policy institutions, the balancing of type I versus type II errors of econometric inference matters less than the balancing of type I versus type II errors in policy advice. Thus, it may be better for a policy model to assume that a particular economic structure exists, even when the evidence is not overwhelming, if the costs of incorrectly assuming otherwise would be relatively high.<sup>72</sup>

The literature on calibration in general and on calibration as testing in particular is expanding rapidly.<sup>73</sup> It is likely that new techniques will emerge, including some that will be applicable to the sorts of problems that arise for modellers in policy-making institutions. The main need we see, in this respect, is the development of criteria for model evaluation.

## 4.3 Some examples of calibration issues in QPM

As one might expect, the Bank of Canada has conducted a substantial amount of research on various aspects of the monetary transmission

<sup>72.</sup> Laxton, Rose and Tetlow (1993c) take this approach in considering the *policy* implications of incorrectly assuming that inflation responds symmetrically to excess demand and excess supply versus incorrectly assuming asymmetry.

<sup>73.</sup> There is, however, another camp, where it is argued, following Lucas (1987), that calibration will always be distinct from estimation and should be so in order to keep theory ahead of measurement. See, in particular, Prescott (1986). Kydland and Prescott (1991) go so far as to argue that calibration *is* econometrics.

mechanism in Canada. While some of this involves structural estimation, there has also been a lot of atheoretical econometrics as well as institutional analysis.<sup>74</sup> In this subsection, we offer a few examples of calibration issues that pertain to the monetary policy mechanism and how the staff have dealt with them in QPM.

### 4.3.1 Interest rates and aggregate demand

One of the key links in the monetary transmission mechanism of the model is the effect of interest rates on aggregate demand. The model has a Phillips curve, which links inflation to excess demand in the goods market; the influence of the monetary authority on short-term interest rates, working through the effect on the level of aggregate demand, is an important part of the process of controlling inflation. Staff at the Bank of Canada spend a significant amount of time studying various aspects of this mechanism, from a variety of perspectives and with a host of different measurement tools. Given the uncertainties associated with any macroeconomic relationship, the staff feel more comfortable when the model contains an empirical relationship that is confirmed by estimates from more than one source or methodology. This minimizes the chance of imposing a spurious result. To identify the effects of interest rates on aggregate demand, the staff have tried estimated reduced-form equations of various types (for example, Duguay 1994), have used structural vector-autoregressive models (VARs), and have attempted to infer the total macroeconomic effect from estimates of the linkages for the individual expenditure components.

Fortunately, this work has tended to produce a cluster of results: an increase of 100 basis points in the 90-day commercial paper rate for two years produces a decline in output that "peaks" at a bit more than 1 per cent, on average, in the third year. There are significant effects in the second year as well, but not much effect during the first year. The results of an experiment of this nature on QPM can be compared, for example, with an impulse response to a structural VAR (SVAR). Figure 1 shows just such

<sup>74.</sup> See Longworth and Poloz (1995) for a summary.

a comparison, prepared from work done under the aegis of the Bank for International Settlements (BIS 1995, Gerlach and Smets 1995).<sup>75</sup>

The experiment here is a temporary (eight-quarter) increase in interest rates, starting from an equilibrium with inflation at the target rate. Note how closely QPM mimics the impulse response function from the estimated SVAR over the shock period and the next few quarters. This shows that the model's calibration has successfully captured the properties

## Figure 1: Real GDP response to a 100 basis point increase in short-term nominal interest rates for eight quarters



(Per cent, shock minus control)

<sup>75.</sup> This chart is not included in the published summary of the work (BIS 1995), but is taken from the working documents of the exercise. Part of this picture is published in Gerlach and Smets (1995).

of the data as reflected in the very short-run predictions of an estimated SVAR. Thereafter, the model tells a different story, and here the economics of the exercise become important. From the model's perspective, the decline in output puts downward pressure on inflation. When the shock is removed and the model's reaction function begins to operate normally, it indicates that to keep inflation at the original target level, the monetary authority must reverse some of the previous tightening. A secondary cycle ensues, wherein output rises above control for a time. This is clearly the correct answer for this experiment under the assumed starting conditions. The SVAR has little theory content and can only reflect average historical experience, which does not exemplify effective inflation control. Thus, while it does show a return towards control after the shock is removed, the SVAR is incapable of reflecting the logical implications of the policy rule acting to keep inflation at a particular target level.

### 4.3.2 The sacrifice ratio

We noted above that one key question for a monetary authority is the cost in terms of foregone output of permanently reducing inflation. We also argued that a sensible macroeconomic model should be able to offer some insight as to how policy might be carried out so as to limit that cost. The Bank of Canada has carried out a substantial amount of research on the sacrifice ratio. In recent years, much of the effort in this area has centred around the estimation of "accelerationist" Phillips curves (Cozier and Wilkinson 1991, Dupasquier and Girouard 1992). Phillips curves of this sort posit a linear relationship between inflation and the sum of current and past output gaps. Thus, notionally, one can invert the Phillips curve to find the cumulative output gap necessary to reduce inflation by 1 percentage point. This calculation is commonly referred to as the sacrifice ratio.<sup>76</sup> Cozier and Wilkinson compute a sacrifice ratio of about 2, using inflation measured by the GDP deflator, while Dupasquier and Girouard

<sup>76.</sup> Models in which it is possible to compute a unique sacrifice ratio represent expected inflation by lagged inflation with the sum of lags restricted to unity. This is often called the "integral gap" model. It is questionable on both theoretical and empirical grounds; see Laxton, Rose and Tetlow (1993b). Given their reduced-form nature, it is not surprising that Phillips curves have tended to be unstable over time or to exhibit unstable sacrifice ratios; see Hostland (1995) and Lipset and James (1995). See also Ball (1994).

arrive at a lower figure working with a measure based on the consumer price index. These results are at the low end of the estimates in the literature.

Sacrifice ratios calculated from Phillips curves alone do have their purposes, but some caution is in order. Inflation is, after all, an endogenous phenomenon, determined by a myriad of factors, many of which interact in complex ways. Single-equation analyses of the costs of inflation may be interesting as exercises in economic history, but they are otherwise devoid of policy content, since they fail to acknowledge the importance of the way in which each disinflation was carried out (as well as whether such outcomes were intended in the first place).<sup>77</sup>

Work by Laxton, Rose and Tetlow (1993b) tries to deal with the problem of computing a sacrifice ratio through an eclectic mixture of estimation and simulation. They argue that for the question to be interesting, the sacrifice ratio must depend on the way in which a disinflation is carried out. That is, it would have to depend on things like the economic conditions extant when the new policy is introduced, the policy rule in place prior to the change, the state of expectations (and the credibility of the monetary authority) as well as the reactions of other domestic and foreign policy authorities. Laxton, Rose and Tetlow include two important features in their Phillips curve: forward-looking expectations and asymmetric effects of imbalance of supply and demand (where excess demand creates more inflationary pressure faster than excess supply of the same magnitude creates disinflationary pressure). These hypotheses are not rejected by their data.

Next, Laxton, Rose and Tetlow embed their Phillips curve in a small macro model and conduct simulation experiments. Two interesting results emerge. First, the cyclical cost of reducing inflation (measured as the cumulative loss of output during the transition) is considerably larger than

<sup>77. &</sup>quot;Partial equilibrium or single-structural-equation 'costs of inflation' analyses are void of policy implications." (Buiter and Miller 1985, 11)

the corresponding gain from raising inflation.<sup>78</sup> Second, the magnitude of this cost is quite sensitive to the weight placed on the forward-looking component of expectations. Since this parameter is not precisely measured, this imprecision must be taken seriously.

For calibration of QPM, the staff elected to assume only a modest degree of forward-looking behaviour, about 20 per cent, which results in a sacrifice ratio of about three and a "benefit ratio" of about unity.<sup>79</sup> These properties are demonstrated in simulations reported in the next section.

There is considerable uncertainty regarding this aspect of the model's calibration. The important point, however, is the establishment of an ordered way of thinking about the determinants of the sacrifice ratio as well as a method for incorporating new thinking into the model with a minimum of disruption. This allows research on inflation to be carried out independently of the model per se, with the results being brought into QPM through adjustments to the calibration or other judgmental changes.

## 4.3.3 Exchange rate pass-through

An issue of ongoing concern for the SEP has been the "view" as to the passthrough of changes in the exchange rate into domestic prices. More precisely, the question has often been posed in terms of whether level changes in the nominal exchange rate result in changes in the price level or changes in the inflation rate. Put this way, the answer seems quite important, since a monetary authority targeting inflation can safely ignore the direct price implications of an exchange rate disturbance, if there are

<sup>78.</sup> We remind the reader that these results are for the cyclical aspect of the question. This version of QPM has been configured to embody superneutrality – inflation has no real effects in steady state. This analysis is therefore only part of a complete discussion of the choice of a level of inflation. See, for example, Black, Macklem and Poloz (1994) for an analysis of how real costs of inflation can arise and can be reflected in models like QPM.

<sup>79.</sup> These results also depend on the degree of forward-looking behaviour in wage determination. In accordance with empirical evidence, QPM is configured with greater weight on forward elements in price determination than in wage determination.

only price-level effects, but must respond if there are inflation-rate effects.<sup>80</sup> This impression is quite misleading.

Of late, the data on this question have tended to support the pricelevel effect conclusion (Duguay 1994), but this has not always been so. Ambiguity in results is what should be expected in this instance, since the pass-through of exchange-rate disturbances into prices (or inflation) depends crucially on the monetary policy reaction to those disturbances.<sup>81</sup> If the monetary authority has accommodated exchange rate disturbances by allowing the inflation rate to drift in response, then an inflation-rate effect will be found in the data. On the other hand, a regime of strict pricelevel targeting would produce data that show no long-run effect of exchange rate disturbances on inflation rates or price levels. Other rules will produce other empirical regularities.

Given a policy rule, the extent of exchange rate pass-through in QPM is influenced by the answers to two questions. What are the import components of consumption and investment? And for what proportion of imported goods might we expect the so-called law of one price to hold? The answer to the first question came from an examination of the inputoutput tables and consultations with specialists. It was concluded that it would be reasonable to assume that roughly 25 per cent of consumption goods and 65 per cent of capital goods are imported. The answer to the second question came from diverse sources; it was concluded that it would be reasonable to assume that 90 per cent of imported consumption goods and 80 per cent of investment goods are subject to the law of one price. These figures were then used to determine what proportion of each index reflects direct exchange rate influences. Thus, they provide a benchmark for the steady-state pass-through of exchange rates to prices, *holding otherwise constant the aggregate price level.* This is not the end of the

<sup>80.</sup> Movements in the exchange rate may also have implications for aggregate demand through the terms of trade and the real trade balance. We abstract from these channels for the purposes of this discussion.

<sup>81.</sup> See section 3.2.5 for further details.

story, of course, because other prices will be influenced by the effects of the shock on the economy.

One must use some criterion or criteria to pick a policy rule and narrow the range of feasible model responses to a shock. The modellers chose to calibrate QPM to retain the essence of the short-run properties in RDXF with respect to pass-through. This means that they endeavoured to choose a base-case policy rule and price-equation parameters that would result in a significant increase in measured inflation in the short run, following a depreciation of the Canadian dollar, but only a price level effect in the long run. This property is shown in the simulations of the next section. It is worth emphasizing, however, that the absence of lasting inflation effects is due to action by the monetary authority to ensure this result; it is not imposed in any way as a structural feature of the model.

#### 4.3.4 Investment and the cost of capital

In a standard, backward-looking macroeconometric model, an important part of the way monetary policy works – the monetary transmission mechanism – is through real interest rates in the structural demand equations. In spite of the theoretical importance of this link, however, it has proven quite difficult to obtain acceptable properties from econometric estimates of the relevant elasticities. Nowhere has this proven more difficult than in the estimation of the effects of interest rates and the cost of capital on business fixed investment. We reviewed above the essence of our explanation of why econometric estimators have great difficulty picking out the true causal relationship between monetary policy and investment demand. Let us now turn to how the difficulty is handled in the calibration of QPM.

Recognition of the importance of supply shocks is one step in reconciling the stylized facts of investment with the predictions of economic theory. Besides being procyclical and trailing the business cycle by a few quarters, investment is about three times more volatile than aggregate output.<sup>82</sup> This can occur only if changes in the desired capital stock have been frequent, which in turn implies a noteworthy incidence of supply shocks. Empirical work does appear to support this interpretation. Dea and Ng (1990) find that about 70 per cent of output fluctuations in Canada could be attributed to (permanent) supply shocks.

The precise incidence of supply shocks is difficult to pin down. There is no consensus in the literature, and the range of estimates that various economists defend spans virtually the whole range of possible answers. Nevertheless, it has become generally accepted that it is necessary to recognize a role for supply shocks in macroeconomic modelling and forecasting, and that there is a strong case for thinking about potential output as a stochastic concept. This range of territory is covered in detail in the forthcoming fourth volume in this series, which describes the extended multivariate filter used by Bank staff in estimating and updating estimates of potential output.

On the specific issue of calibrating the effect of the policy instrument on aggregate investment spending in QPM, we used the following approach. We set the parameters such that if we combine the response of investment to a pure supply shock, using a weight of 30 per cent, with the response to a demand shock, using a weight of 70 per cent, we produce something like the stylized fact concerning the relative variability of investment and output. At the same time, while the average properties of the combination would show little correlation between investment and interest rates, as in the data, the properties for specific shocks are preserved. Thus, in calibrating this relationship, we do far better than could ever be hoped for in attempting to estimate an investment demand function.

<sup>82.</sup> Based on quarterly data from 1959Q2 to 1994Q4, the standard deviation of investment growth is 2.9 versus 1.0 for GDP. The precise number depends on a few small measurement issues. The salient point always remains that investment is relatively volatile.

## 5 MODEL PROPERTIES

To illustrate the properties of QPM, we report the results of simulations that reveal the model's response to a wide variety of shocks. In most cases, the analysis begins from an artificial control in which all variables are in a steady-state equilibrium. Projection problems are more complicated; staff must take into account why there is a disequilibrium – what shocks have come before and are now being felt - so that all aspects of the disequilibrium can be properly identified and taken into account. As we have emphasized, the properties of QPM will depend on the economic conditions when a shock arrives. In the real world, shocks do not arrive conveniently, one at a time; nor do they wait for the implications of previous shocks to have been fully worked out before presenting a new twist. It is impossible to discuss this in its infinite variety, but we do illustrate the point with examples where we overlay a shock on a dynamic disequilibrium created for the purpose from a single prior shock. In general, however, it is clearer to discuss model properties starting from an artificial steady-state control solution and to deal with one issue at a time.

We begin with a somewhat unusual shock – a natural disaster that eliminates a portion of the existing capital stock. This shock, which fortunately does not arise often as a projection issue, is nevertheless ideal as a device to illustrate that QPM does, indeed, have a fully articulated supply side and a steady state to which it will return. In this case, which is an example of a temporary supply shock, there is no change in the steady state, but the shock disturbs an important part of stock equilibrium. The process of restoring that equilibrium shows a great deal about the dynamic structure of the model and illustrates well the important role played by the equilibrium structure of the model in conditioning dynamic adjustment paths.

We then continue with examples of shocks that do have permanent real effects. These shocks allow us to expand our demonstration that QPM has a fully articulated steady-state component that provides the new equilibrium as well as stable dynamic structure that brings about that solution. We begin with a permanent supply shock coming from an increase in productivity. We then introduce the notion of a policy shock, showing how fiscal policy choices can have permanent real consequences in QPM. We begin with the effect of a change in the relative level of government debt and then consider the effect of a switch to lower income taxation and higher indirect taxation, holding other aspects of government behaviour unchanged. Finally, we consider a shock to the rate of time preference, the degree to which households discount the utility from future consumption. This provides an example of a fundamental and permanent shock to "demand" coming from household consumption/savings decisions.

We report only key macro variables, but the description of these shocks is comparatively detailed. Having established the broad picture of how macro adjustment works in QPM with the above shocks, we pare down the descriptions of the results for some of the subsequent examples to focus on output, prices and the monetary policy instrument. The examples considered include monetary policy shocks and a variety of other temporary shocks.

A comparison of the results described here with those in the first volume in this series on the steady-state model, SSQPM, will reveal some small differences in the steady-state effects of some shocks. There have been a number of small adjustments to the calibration described in the previous report. None of these changes is important enough to warrant special attention here.

## 5.1 A well-defined and dynamically stable steady state

A key feature of QPM is its clearly defined steady state, with a fully consistent stock equilibrium that determines the levels of the three key stocks in the model: physical capital, government bonds and net foreign assets. For example, in the case of capital, the desired level comes from the level of available labour and the relative price of capital services, based on a standard marginal analysis of maximization of the present value of the firm along a neoclassical growth path.

In all three cases, the model has mechanisms that make flow spending respect the steady-state restrictions on the stocks. The required steady-state flows are brought about and sustained by relative prices. In the case of fixed capital, the real interest rate that underlies the user cost is tied closely to conditions in world markets, but the real wage adjusts such that there is full employment of labour, given the desired level of capital. Investment spending adapts to ensure that there is the right level of capital in the long run, relative to output, and that this level is sustained along the equilibrium growth path. The household budget constraint also reflects the necessity to provide the resources to sustain this capital equilibrium, with households seen as owner-workers.

#### 5.1.1 A natural disaster that reduces the level of the stock of capital

To illustrate how the model handles stock disequilibrium we conduct a shock to the stock of capital – characterized as a natural disaster like a hurricane or an earthquake that wipes out part of the capital stock. Bank staff do not have to deal with many earthquakes or hurricanes in the Canadian part of the SEP, but they do often have to deal with initial conditions that, for a variety of reasons, imply a gap between the actual and the desired stock of capital.

The shock we analyse is a large one by normal standards – we assume that about 2 per cent of the capital stock is wiped out at a stroke. Miraculously, there are no deaths or other short-term complications; the only disruption is to the stock of capital. Moreover, the disaster does not affect any other economy.

The results are shown in Figure 2. In this and all other figures that follow, the plot comes from quarterly simulations, but the horizontal axis is labelled in years from the start of the simulation, which is usually the period when the shock is introduced. QPM is intended to represent underlying movements in the economy, and one can think of the output as reflecting seasonally adjusted measures.

Capital is not a fixed endowment; its equilibrium level is chosen according to optimization rules, and is put in place by obtaining real resources from somewhere that could otherwise have been consumed. Nothing in the optimization problem is disturbed by this shock and so the steady state is not changed. Therefore, the essence of the scenario is that there must be a period when consumption is foregone as the capital stock is rebuilt, but the solution must eventually return to the control values.

The first row in Figure 2 shows that this process is well-captured by QPM. It takes about 10 years to rebuild the stock of capital, although most of the adjustment is complete within five years. The left panel in the second row illustrates what happens to output and the output gap. Note that the adjustment is hindered in the short term by the fact that potential output has been reduced by the loss of capital (shown by the fact that there is excess demand initially, despite the fall in output).<sup>83</sup>

Consider next the role of the external sector in the adjustment process. Note that the economy uses the facility of external buffering – imports rise sharply as the economy attempts to replace the capital in the face of reduced domestic potential to produce, and exports fall as more of the available output is diverted to domestic use. This is financed by foreign borrowing (the net-foreign-asset ratio falls). There are no long-term effects on the real exchange rate, but in the adjustment period it appreciates, which helps facilitate the switch of resources to domestic reinvestment.<sup>84</sup>

The short-term (90-day) and long-term (10-year) interest rates also reflect the effects of the excess demand. The short-term rate is up about 140 basis points in the first two quarters. As shown in the final row, prices come under considerable upward pressure from the initial excess demand. Inflation, measured here using the CPI excluding food and energy, peaks at about 1.0 percentage points above control, despite the exchange rate

<sup>83.</sup> Nevertheless, the excess demand keeps employment from falling initially. Despite lowered marginal productivity, labour is needed to keep output from falling further. In this sense, the "Luddite" view is corroborated – destroying capital does lead to higher reliance on labour input. But this effect is very short-lived. For most of the adjustment period, employment is below control.

<sup>84.</sup> Exchange rates in QPM are expressed as Canadian dollar prices of foreign exchange, which implies that negative movements in the exchange rate are appreciations of the currency and positive movements, depreciations.



appreciation and strong, immediate monetary response. The effect of the rise in interest rates on aggregate demand is sufficient to bring inflation back to control. It also helps in the adjustment process by giving households an incentive to increase saving (reduce consumption), which frees resources for capital formation.

This simulation illustrates well the automatic functioning of the monetary policy rule in QPM. This is a shock to supply that creates excess demand both from a drop in potential output and a rise in demand coming from the need to rebuild the stock of capital. Without the response of interest rates, inflation would escalate rapidly. This is not a case where the monetary authority is acting to put upward pressure on rates because of some new monetary initiative. All the market price variables, including interest rates, move in response to the rationing requirements of the economic conditions generated by the shock.

Moreover, during the adjustment period, the aggregate price level is allowed to drift up (this amounts to about 1.5 per cent in the end). This is an important feature of the monetary rule in QPM. The concern is to bring inflation to the target level. There is no explicit concern over the level of prices. Thus during a period of inflation, such as arises in the short run from this shock, there will be upward drift in the level of prices that will not be reversed as the rate of inflation is brought back to the target level. Similarly, in periods with disinflationary pressures, there will be a downward drift in prices. Several examples with important price level drift are included in the sections that follow.

# 5.2 The steady-state and dynamic effects of permanent shocks

An important feature of the QPM system is that the steady-state model, SSQPM, identifies the new long-run equilibrium implied by permanent shocks. Moreover, the nature of the dynamic adjustment path shown by QPM is very much influenced by the nature of the changes to the equilibrium, as well as by the explicit dynamic structure of QPM itself.

#### 5.2.1 A permanent increase in domestic productivity

Our first permanent shock is an increase in the level of productivity in the Canadian economy, with no parallel increase in productivity in the rest of the world. This is interpreted as an exogenous shock to the supply side of the economy.<sup>85</sup> The particular shock is a 1 per cent increase in the level of total factor productivity. With the Cobb-Douglas production function of QPM, this is equivalent to an increase of about 1.4 per cent in labour productivity. This shock allows us to show clearly that permanent supply shocks are handled well by QPM and have important effects on the steady state. The results are shown in Figure 3.

An increase in productivity raises the desired level of the capital stock. To realize this new level, firms must increase the rate of investment; in the short run, investment must rise well above the rate needed to sustain the new permanently higher level of capital to bring about the change in the stock.

The investment boom, with its large import component, is the most striking feature of the short-term dynamics. The adjustment of capital is virtually complete by the fifth year, and the flows settle down on their new higher levels, all with some very minor tertiary cycling. The real producer wage rises, relative to control, from the start, overshoots its new equilibrium level slightly at about year six and then settles into its steady state at about year 10. The new steady state is 1.4 per cent above control, as predicted by the neoclassical theory of income distribution embodied in QPM.

Output rises by about 1 per cent in the first year, with demand coming from several sources, including the start of the investment boom, but with an important contribution from exports. Nevertheless, owing to the higher level of productivity and potential output, the shock creates excess supply, initially. This excess supply and the reduction in expected unit production costs, arising from the improved labour productivity, place

<sup>85.</sup> The nuance here is that there is no link to any government activity, as there might be in a model with endogenous productivity.

## Figure 3: A permanent increase in domestic productivity

Per cent (%) or percentage point (%p), shock minus control



downward pressure on prices. The monetary authority works against these pressures by easing monetary conditions: interest rates decline and the dollar depreciates. The depreciation acts directly to remove some of the downward pressure on domestic prices by raising import prices. It also has an important effect on the trade balance. The depreciation allows an outlet for some of the new productive capacity through exports and import substitution. The decline in interest rates simultaneously encourages an increase in domestic spending, which also helps create the demand needed to absorb the new potential supply. The disinflationary conditions do not last long; indeed, at the peak of the investment boom there is a degree of excess demand. However, the subsequent effects on interest rates and inflation are relatively small and short-lived.

The model's forward-looking monetary reaction function works automatically, ensuring that the inflation rate returns promptly to control with little disruption to markets. There is a permanent decline in the price level that comes from the initial period of excess supply. The decline in the price level is an important part of the mechanism whereby the higher productivity is reflected in a higher real wage – some of this comes in the form of a higher nominal wage, but some of it comes from the lower price level.

In the new steady state, the dollar has depreciated in real terms by about 0.4 percentage points, reflecting another feature of QPM. The model describes an *almost* small open economy. The essence of the small, open economy model is retained, but we allow for small influences of domestic conditions on world prices of Canadian exports. In this case, to increase sales to foreigners in the face of a rise in potential, there must be a small decline in the world price of Canadian exports. The depreciation raises the cost of capital, through the import component, which explains why the capital-output ratio falls slightly in the new steady state.

#### 5.2.2 A decrease in the ratio of government debt to output

The next shock we consider is a permanent, 10 percentage point decrease in the target ratio of government debt to output, implemented gradually through temporary increases in the rate of personal direct tax net of transfers. This is our first example of a policy shock – one where the disturbance comes from a decision by policy makers.

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In QPM's overlapping-generations theory, households care about the level and dynamic path of government debt. In this case, they perceive themselves to be less wealthy when government debt is being reduced. Essentially, they put more weight on the temporary increase in taxes required to create the budgetary surplus needed to reduce the level of debt than they do on the decline in taxes that will come later with the reduction in interest payments. Thus, the level of debt matters, because current consumers see that they will bear a disproportionate share of the adjustment costs.<sup>86</sup>

The government determines the supply of government bonds, and these must be held by someone (for simplicity, we assume that all the new domestic debt is taken by residents). But the household sector determines its overall level of assets, and in the open economy the free margin is the level of net foreign assets (NFA). The lower stock of domestic government debt in this shock induces households to switch to hold more foreign assets. Since Canadians are net debtors to the rest of the world, this shows up as a reduction in net foreign liabilities. In effect, domestic residents buy foreign assets with funds that otherwise would have gone into holdings of domestic government debt. This requires an initial *depreciation* of the currency to generate a larger current account surplus to provide the funds necessary to acquire those foreign assets.

In our scenario, this initial real depreciation peaks at just over 3 percentage points, in the third year of the simulation. However, the permanent effect is a small *appreciation* of the real exchange rate, because a smaller trade balance is required to service the lower equilibrium foreign debt. All else held equal, Canadians will eventually export less and consume more because of the reduction in the country's foreign

<sup>86.</sup> This is symmetric, of course. Current generations reap the gains of a run-up in debt, because they do not bear their full share of the increased tax burden to come. For a complete discussion of the welfare effects of this shock, therefore, one would have to ask who gained from the previous increase in the level of debt.

indebtedness. Moreover, the long-run real appreciation lowers the cost of imported capital and with it, the user cost of capital. The stock of capital desired by firms is consequently larger, and as the actual stock rises to reflect this, potential output increases. In the long run, therefore, there is a small, positive effect on output from the decrease in government debt, and a significantly larger positive effect on the level of consumption.<sup>87</sup>

Figure 4 shows all these features at work. As noted above, the reduction in debt is achieved through a temporary increase in net personal direct taxes.<sup>88</sup> The peak increase is just over 2 percentage points. For reasons already discussed, this results in a fall in consumption in the short run. In the second year of the simulation, consumption is down about 5.5 per cent. This creates aggregate excess supply and disinflationary pressures, leading to downward pressure on interest rates. The monetary authority plays a role in this, but the fall in long-term interest rates illustrates that private agents also anticipate the disinflationary consequences of the shock. The decrease in interest rates results in a depreciation in the nominal value of the dollar. For a short while, the falling dollar offsets the underlying disinflationary pressures as increases in the price of imports get passed through to consumers; the CPI excluding food and energy prices actually rises briefly. With domestically determined prices being sticky in the short run, the nominal depreciation in the dollar is also a (temporary) real one, and this has the obvious implications for the trade balance: imports fall and exports rise, the current account balance improves and creates the capital account deficit called for by the long-run desired decrease in net foreign liabilities.

Eventually, the tax rate falls to reach its new steady-state level, which is slightly *below* the control value. Payments to foreigners are also

<sup>87.</sup> The steady-state implications of this shock are described in more detail in the first volume of this series – Black, Laxton, Rose and Tetlow (1994). See Macklem, Rose and Tetlow (1994, 1995) for a detailed discussion of debt shocks using QPM.

**<sup>88</sup>**. In this version of QPM, the equilibrium labour supply does not respond to changes in the after-tax wage rate, and it does not matter whether the extra net revenue is raised through higher income taxes or lower transfers. In the extended version of QPM described in Macklem, Rose and Tetlow (1994, 1995) this is not the case.



Per cent (%) or percentage point (%p), shock minus control



permanently lower, reflecting lower foreign indebtedness. This results in a steady-state increase in consumption of 0.5 per cent versus a rise in investment and output of only 0.4 and 0.1 per cent respectively.

Note the tension between the long-term result (higher consumption in every period) and the short-term result (lower consumption in the transition period with higher taxes). Macklem, Rose and Tetlow (1994, 1995) conduct a similar shock on a slightly different version of QPM and show that the social planner's net present value of shock minus control changes in consumption from a shock like this one is strongly positive. That is, the long-run gains in consumption outweigh the transition costs. However, since the benefits accrue to later generations and the costs accrue to current generations, welfare comparisons are not straightforward.

In the long run, inflation returns to its targeted level, as do interest rates, and the output gap closes. While there are no permanent effects on inflation, the price level drifts down by about 3 per cent through the adjustment period.

#### 5.2.3 A change in the tax structure

Our next shock is also a permanent one that arises from a change in policy, albeit a change of a different nature. Here we consider a substitution of one type of tax financing, indirect taxes, for another, personal direct taxes net of transfers. It may appear on the surface that there is little to be expected from a shock such as this one, since the budgetary stance of fiscal policy is unchanged. This expectation would be reinforced by the observation that labour supply is inelastic in the version of QPM being discussed here, and so there are no tax distortions of labour supply choices. We shall see, however, that the model generates some interesting results.

The change in tax policy is implemented through a 1 percentage point increase in the indirect tax on consumption goods, with the net direct tax rate falling to leave total revenues constant.

Let us focus, first, on the results for consumption, where we see a decline in the short run but an increase in the long run (Figure 5). In QPM,



Per cent (%) or percentage point (%p), shock minus control



there is a continuum of agents by age – in every period some agents die and others are born, with net positive population growth. At all points in time, the probability of death and the productivity of each living agent is the same. This means that their expected future lifetime labour earnings are the same and their human wealth levels are the same. Agents differ only in the amount of financial wealth that they have accumulated. Older agents have more financial wealth than do younger agents. The older, wealthier consumers suffer a loss in this shock, because they saved out of relatively highly taxed income and now must pay again, in the form of relatively high prices, when they consume. The older and wealthier the agent, the larger is this effect.

Newly born agents with no financial wealth have higher human wealth owing to the lower rate of income tax, and this leads them to consume more from the outset. The same is true for recent entrants who have not accumulated much financial wealth. In aggregate, however, these agents are not as numerous as those who have more wealth and who suffer a transition loss in what amounts to a tax on financial wealth. Therefore, aggregate consumption falls initially. As time passes, however, and the older agents are replaced by new agents, the longer-term increase in aggregate consumption emerges.

Note that output is also higher in the new steady state. This reflects a higher equilibrium stock of capital, the result of a lower cost of imported capital. This, in turn, reflects a decision of households to hold more financial wealth, which leads to a rise in NFA and an appreciation of the real exchange rate.

How, one might well ask, could a tax on consumption end up increasing consumption when the alternative tax is effectively lump-sum and hence non-distortionary in the usual sense of not affecting the supply of labour. The answer is to be found in the phenomenon of "overdiscounting" by individuals in this model. This introduces another form of distortion. The introduction of the possibility of death leads individual consumers to discount the future at a rate higher than the market rate. From a social planner's perspective, this is distortionary, because it induces undersaving and overconsumption. Taxing consumption then moves the economy towards the social optimum by compensating for overdiscounting by individual consumers. In one sense, this is why households choose to hold more financial assets, reinforcing the human-wealth effect on consumption and rationalizing the changes to capital and output described above.

Despite the opening of a negative output gap in the short term, the rise in the rate of indirect tax results in an initial increase in prices. The direct effect of the higher tax rate outweighs any effect from weaker economic conditions. The monetary authority reacts to prevent this from triggering higher inflation expectations, pushing up short-term rates briefly.<sup>89</sup> As there is no fundamental underlying inflationary pressure created by the shock, this tightening is soon reversed and policy settings ease in response to the excess supply.

### 5.2.4 Canadians become less patient

This shock is a change in the rate of time preference of Canadian households. It is useful for illustrating the theoretical underpinnings of household behaviour in QPM, and provides another example of a shock that changes the steady state. Specifically, we suppose that consumers increase the rate at which they discount the utility from future consumption by 0.1 percentage points. In this type of model, such a change is a very large shock.

Because its household behaviour comes from an explicit intertemporal optimization framework, the SSQPM model automatically provides us with the impact of the shock on the desired level of wealth in the steady state. With households presumed to be less patient, the model predicts that they will choose to take on more debt to facilitate a short-term binge of consumption and accept the higher debt service and lower

<sup>89.</sup> This experiment uses the standard reaction function, with no special accommodation of the price level increase associated with higher indirect taxes. SSQPM is configured to be neutral under nominal level changes, but the dynamic paths generated by QPM would be different under different monetary reaction functions. In fact, the Bank of Canada now focusses operationally on the CPI excluding food, energy and indirect taxes.

permanent level of consumption that this implies. Given relatively fixed conditions for the supply of government bonds and the stock of physical capital, most of the action on the asset side from a fundamental demand shock like this one comes in the form of changes in NFA. In this case, the desired ratio of NFA to output falls by about 12.5 percentage points (Figure 6). The rundown in foreign assets must be accompanied by a temporary swing in the trade balance, and we see the consumption binge reflected in imports.

This shock provides a good illustration of how the external adjustment process works in QPM. An appreciation of the real exchange rate facilitates the large swing in the trade balance as Canadians borrow from foreigners to finance the extra consumption, which peaks at about 6 per cent above control. In percentage terms, more of the trade swing comes in the form of higher imports, but the appreciation also reduces exports, freeing resources for domestic consumption. Eventually, however, there must be a real depreciation to earn the extra debt service payments implied by the lower level of NFA. This effect is relatively small, as are all the other permanent effects, compared with the short-run effects.

The immediate demand effects of the shock push up output and employment, and the monetary authority has to react vigorously to contain inflation. The short-term interest rate peaks at about 4 percentage points above control. This contributes to a cyclical decline in output, primarily in the form of a drop in investment that is more than sufficient to reduce the capital stock by the desired amount. The desired capital stock is lower, in part owing to the permanent depreciation and the resulting higher cost of imported capital.

It takes a long time (nine years), however, to get inflation back to the target level. The consumption binge is long-lasting and the economy remains in excess demand for four years. Indeed, this shock illustrates an important point about the characterization of monetary policy in the model. The reaction function works for this shock, in the sense of not losing control. However, it has been calibrated to bring inflation back to control over an acceptable horizon, three years say, for standard demand



and supply shocks. This shock is not one that would be confronted routinely in dealing with projection issues. One can explain virtually anything by saying that tastes have changed, and a model would not be very useful if we could not assume that such fundamental structure was relatively constant. Thus, the difficulty in controlling inflation here is instructive in reminding us that a fixed parameter reaction function suffers from some of the problems of any proximate quasi reduced-form representation of behaviour. Since this shock is used primarily to help understand the model as opposed to a real world policy issue, the results do not raise a concern regarding the calibration.

## 5.3 Changes of monetary policy

In this section, we demonstrate the model's ability to analyse shocks to monetary policy. In particular, we consider what happens if the monetary authority chooses to lower (or to raise) the target rate of inflation by 1 percentage point, phasing in the transition relatively quickly. Aside from a very small seigniorage effect, there are no permanent effects on real variables; QPM has not been configured to include the permanent benefits of lower inflation. It is important, therefore, to establish that this analysis should be viewed as limited to dynamic business-cycle effects.

In addition, we show that QPM is able to handle a distinction between anticipated and unanticipated shocks by repeating the disinflation experiment, but this time assuming that the reduction in the target rate of inflation is announced six quarters in advance.

### 5.3.1 Changing the target rate of inflation

As noted earlier, QPM exhibits asymmetric price and wage response to excess demand. Inflation responds both more strongly and more quickly to excess demand than it does to excess supply. This asymmetry implies that the temporary gain in output stemming from a 1 percentage point increase in the target inflation rate would be considerably smaller than the lost output in the equivalent disinflationary experiment. These results are confirmed in the simulations shown in Figure 7.

## Figure 7: Higher and lower inflation targets

Per cent (%) or percentage point (%p), shock minus control (Solid is higher inflation; dashed is lower inflation)



To bring about a reduction in the rate of inflation, the monetary authority tightens monetary conditions, inducing higher short-term interest rates, which rise by about 100 basis points on average in the first year. However, long-term rates increase just slightly in the first year and fall, relative to control, beyond the first year as expectations of lower inflation begin to develop. The rise in short-term interest rates results in a modest appreciation of the dollar, and together these variables act to depress aggregate demand. It takes until the third year before the maximum effect on aggregate demand is felt, at which point a (negative) output gap of over 1 per cent has built up. It takes until the end of the fifth year for inflation to reach its new target level. By that time, real short-term interest rates have actually overshot their long-run level, having fallen slightly relative to control in order to curtail the building disinflationary momentum.

That the output costs of reducing inflation far exceed temporary gains from letting it rise can be seen in both the relative size of the output cycle (top-left panel) and directly in the cumulative effects (top-right panel). In the end, the total output forgone in reducing inflation by 1 percentage point from a steady state is about 3 per cent of one year's output compared with a gain of 1 per cent of one year's output from the opposite shock.<sup>90</sup> The reader is again warned that this reflects only the cyclical part of the analysis. The permanent costs of a higher level of inflation have not been included. This analysis does show, however, that any plan to let inflation rise temporarily and to reverse this later will lead to negative overall consequences for output. We return to this point in a later example.

The asymmetry is also shown in the relative profiles of interest rates. While short-term rates must rise by about 100 basis points, initially, to trigger the disinflation, a drop of only 60 basis points triggers the equivalent increase in inflation. Note, finally, that QPM predicts that nominal long-term interest rates will rise, albeit just a bit, in the first

<sup>90.</sup> Different figures would have been generated had the disinflation been conducted more gradually or from different initial conditions.

quarter of the transition to higher inflation. This shows the effect of private agents' forward-looking expectations of inflation in the model.

5.3.2 Anticipated versus unanticipated shocks: an anticipated disinflation shock We illustrate that QPM can handle a distinction between anticipated and unanticipated shocks by repeating our disinflation shock – a reduction of the target inflation rate by 1 percentage point – but this time the reduction in the target is announced six quarters in advance. The results are shown in Figure 8, with the corresponding results from Figure 7 repeated for comparison.

We see that QPM allows no free lunch for the monetary authority. The extra lead time allows a small reduction in the cumulative output loss, but it is a very minor change. To achieve a new target inflation rate, the monetary authority has to change expectations, and this experiment shows clearly that allowing some lead time does not make that much easier in QPM, despite the forward-looking component in the expectations of private agents.

This reflects a deliberate choice made in modelling and calibrating expectations formation. It would be technically trivial to add some announcement effects or other explicit representation of credibility effects. However, we chose to assume as our base case that credibility must be earned based on performance and that no direct effects of announced policy changes on expectations should appear. This shock shows that we have implemented such a view.

# 5.4 Temporary demand shocks and the role of monetary policy

In the previous subsection, we looked at fundamental monetary policy shocks, where the disturbance to the system is a change in policy. This is an important dimension of policy analysis, but the day-to-day job of the monetary authority is not to introduce disturbances into the economy. Rather, the main job, and the one that is featured in projection analysis, is to *react* to shocks arising from elsewhere to preserve a policy goal – here,

## Figure 8: An announced future reduction in target inflation

Per cent (%) or percentage point (%p), shock minus control (Solid is the future disinflation; dashed is the contemporaneous disinflation)



the inflation target. In this subsection, we analyse a number of temporary shocks coming from the economy and focus on what the monetary authority must do to ensure that inflation returns to an unchanged target level. This is the quintessential day-to-day role of monetary policy.

In this subsection, the model's responses to two shocks that affect the level of aggregate demand on a temporary basis are presented. The first shock is an autonomous increase in domestic demand, coming from both consumption and investment. The second is an increase in aggregate demand originating in the rest of the world.

These demand disturbances are also used to illustrate some other features of QPM, including its flexibility, which allows it to be tailored to the requirements of a particular experiment. The implications of the speed with which the monetary authority recognizes and responds to disturbances is demonstrated using the shock to domestic demand. The flexibility of the endogenous monetary policy reaction function embodied in QPM is illustrated using the foreign demand disturbance.

#### 5.4.1 An autonomous increase in domestic demand

This shock consists of an autonomous increase in consumption of 1.0 per cent for two quarters, with an investment shock that builds up to the same magnitude over two quarters. Both shocks are gradually reduced to zero over an additional one-year period. The solid lines in Figure 9 show the implications of this temporary disturbance to aggregate demand.

Since the economy begins in equilibrium, the shock pushes activity above capacity and generates upward pressure on inflation. The monetary authority in QPM sees that inflation is heading above the target and responds with upward pressure on short-term interest rates, which peak at about 130 basis points above control. The dollar appreciates by 0.7 percentage points, before reversing its course. Inflation creeps up despite this tightening in monetary conditions and peaks at just less than 0.6 percentage points above control. The tightening in policy has a contractionary effect on the economy and a depressing effect on prices, but the stickiness of prices and the persistence of expectations prolong the
# Figure 9: A temporary shock to consumption and investment

Per cent (%) or percentage point (%p), shock minus control (Solid is immediate monetary response; dashed is delayed response)



inflationary consequences of the shock. While output moves below control during the second year and stays below control for roughly six years, inflation remains above its target level until year six. In the end, the cumulative price-level drift amounts to about 1 per cent.

With the shock acting to raise consumption and investment for about six quarters, all told, the trade balance plays a major role in bringing aggregate demand down to non-inflationary levels; exports are down and imports up from the outset. Eventually, all real variables and inflation return to their original steady-state levels.

## 5.4.2 What if the monetary authority delays its response?

Next, and also shown in Figure 9 (dashed lines), we illustrate what happens when the monetary authority delays its response by two quarters.

Two points come through clearly. First, the main result of delaying monetary response to inflationary pressures is a rapidly growing inflation problem and a consequently more difficult correction when inflation is eventually reined back to the target. The secondary contraction in output is substantially larger and longer if the monetary authority delays its response. Even with no delay of policy response, to keep inflation at the target level there must be an output response that more than offsets the initial effects of the shock. The cumulative effect is always negative. But it is much more severe if the problem is allowed to build. In QPM, containing inflation quickly is very important; inflation escalates rapidly in the face of excess demand and, once entrenched in expectations, is difficult to wring out of the system.

#### 5.4.3 What if the monetary authority could foresee the shock?

We now consider an experiment that illustrates two important features of QPM. We consider the same shock to aggregate demand but this time supposing that the shock is known in advance by the monetary authority and by private agents. The shock is assumed to arrive six quarters in the future, and we allow that information to influence the policy instrument through the usual forward-looking reaction function. This illustrates, again, that QPM has the flexibility to consider various interpretations as to

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what is known and when. It is not meant to be a realistic alternative; it is an unfortunate fact of life for central banks that shocks to demand normally cannot be anticipated.

The results for some key macro variables are shown in Figure 10, where we repeat the information from Figure 9 (the case with no delay in policy response) for comparison. The key point that emerges is that foreknowledge of the shock allows the monetary authority to moderate its cyclical consequences. The magnitudes of the fluctuations are much reduced. There is a small cost, in that some disinflationary pressure has to be put into the system in advance of the arrival of the shock, but overall there is much less cyclical disruption to the economy.

It is instructive to compare these results with those from section 5.3.2 and Figure 8, which consider the consequences of announcing a change of policy in advance. In that case, there is little difference between the two scenarios that emerge, showing that there are no free lunches for the central bank in QPM – that policy changes require intervention to achieve the desired ends and do not happen automatically or more easily through "open mouth" operations. Here, the question is entirely different; we are considering a shock to the system that the policy maker has to fight against. Because of the lagged effect of the instrument on economic activity, and the lagged effect of economic activity on inflation, monetary control is imperfect and delayed. Foreknowledge of the problem to be dealt with is extremely valuable information and therefore allows a better (less cyclically volatile) outcome to be achieved.

## 5.4.4 A temporary increase in aggregate demand in the rest of the world

This shock considers the implications for the domestic economy and domestic monetary policy of a temporary increase in aggregate demand originating in the rest of the world (ROW).<sup>91</sup> Because such a disturbance would have inflationary consequences for the ROW, it can be expected that

<sup>91.</sup> The foreign shock was constructed using stylized facts from the data representing QPM's foreign sector, as well as empirical work done at the Bank and elsewhere. It is intended to represent a typical foreign demand shock. See Hunt (1995) for further details.

# Figure 10: An anticipated future demand shock

Per cent (%) or percentage point (%p), shock minus control (Solid is the contemporaneous shock; dashed is the anticipated future shock)



the foreign monetary authorities would tighten monetary conditions in order to return the sector to its equilibrium path. The shock considered here embodies both the foreign inflationary consequences of the disturbance and the foreign monetary authority's response, which is necessary to return the foreign sector to control.

In this shock, foreign output initially exceeds capacity for an average of 1 per cent for six quarters. Inflation in the foreign output deflator rises to 0.75 percentage points above control, and commodity prices accelerate at an even faster pace. In response, foreign short-term interest rates increase by over 1 percentage point, with roughly half of that increase being transmitted into foreign long-term interest rates. The tightening in policy brings about a subsequent three-year period where output is below capacity. This period of excess supply is sufficient to return the foreign inflation rate to control. Commodity prices decelerate during the period of excess supply, returning to their control level shortly after foreign output returns to control.

Before examining the model's response to this shock, it is worthwhile to provide a thumbnail sketch of the four macroeconomic channels of effect for foreign demand shocks embodied in QPM. The first channel operates through the trade balance: increased output in the ROW increases the demand for exports from Canada. The strengthening export sector results in higher domestic aggregate demand. The second channel operates on the wealth of Canadians through the terms of trade. As a resource-based economy, the increase in commodity prices that frequently accompanies increases in world demand improves Canada's terms of trade, meaning that Canadians need to export less in order to consume a given bundle of imports. This wealth effect stimulates domestic consumption and, to a lesser extent, investment. The third channel of influence is financial, operating from the impact of higher foreign prices and interest rates on Canadian interest rates and the exchange value of the Canadian dollar. Lastly, there is a direct channel of foreign prices, especially but not exclusively commodity prices, on the general price level in Canada. This is so partly because of the impact of higher foreign prices on the cost of imports, some of which are inputs into production, and

partly because some domestic firms, operating in imperfectly competitive markets, have the opportunity to raise prices when the prices of imported competitor products increase.

Through all but the financial channel, the foreign demand shock exerts considerable upward pressure on domestic prices. Consequently, QPM's monetary authority, acting with foresight to maintain or restore a target inflation rate, must tighten monetary conditions in response to this shock. Short-term interest rates climb by a little over 2 percentage points, on average, in the first year of the shock, with long-term rates rising by just over 1 percentage point (solid line in Figure 11). In fact, the differential between Canadian and ROW short-term interest rates actually widens for a brief period of time, a reflection of the extent of the pressure on domestic prices exerted by the shock and the strength of the reaction of domestic policy to those price pressures. Higher short-term interest rates in Canada, relative to the ROW, elicit an increase in the exchange value of the dollar and some strong, albeit short-lived, downward pressure on the general price level via the pass-through of the appreciation into import prices. This limits the inflationary pressure from the shock in the beginning.

Despite the immediate reaction of the monetary authority to limit the development of domestic excess demand, domestic inflation rises to 0.7 percentage points above the target. A period of moderate excess supply lasting just under two years is then necessary to facilitate returning inflation to its target rate. All components of aggregate demand contribute to this excess supply; higher interest rates cause both consumption and investment spending to fall below equilibrium levels. The higher dollar restrains the trade balance, essentially offsetting the stimulative impact of the shock on this sector.

A positive wealth effect arising from the increase in commodity prices is the principal cause of the secondary cycling in this shock. The increase in commodity prices improves the domestic terms of trade, resulting in an improvement in the current account balance. This improved current account cumulates into an NFA position above the desired level.

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#### Figure 11: A shock to foreign demand Per cent (%) or percentage point (%p), shock minus control (Solid is standard monetary reaction function; dashed is alternative) Nominal long bond rate (%p) Nominal 90-day commercial paper rate (%p) 3 1.5 2 1.0 1 0.5 0 0.0 -1 -0.5 00 05 10 00 05 10 Nominal exchange rate (%p) Real exchange rate (%p) (+ is depreciation) (+ is depreciation) 1.0 1.00.5 0.5 0.0 0.0 -0.5 -0.5 -1.0 -1.5 -1.0 00 05 10 00 05 10 Output (%) Consumption (%) 0.4 0.5 0.2 0.0 0.0 -0.2 -0.5 -0.4 -0.6 -1.0 00 05 00 10 05 10 Ratio of net foreign assets to output (%p) Inflation, CPI excluding food and energy (%p) 1.0 1.5 1.0 0.5 0.5 0.0 0.0 -0.5 -0.5 00 05 10 00 05 10

As agents sell foreign assets in response, they are able to finance a temporarily higher level of consumption with the proceeds.<sup>92</sup> While both net exports and investment respond to the easing in monetary conditions, consumption is the key factor keeping output above equilibrium levels during the second period of aggregate excess demand. A second nominal appreciation of the exchange rate, combined with the dissipation of the foreign shock itself, helps keep inflation on track during the third and fourth year of the scenario, in spite of upward pressure on prices coming from a moderate degree of excess demand in Canada.

Most of the cycling in output over the medium term of this scenario is a manifestation of the stock-flow dynamics in QPM. Terms-of-trade shocks change the values of stocks, which implies changes in flows to restore equilibrium. The flows, however, have implications for aggregate demand and inflation to which the monetary authority must respond, and so on. In the long run, domestic monetary conditions, inflation, output, the real exchange rate and wealth all return to their original, steady-state equilibrium levels. The Canadian dollar appreciates, however, by more than half a percentage point, because under the standard policy rule contained in the model, the domestic price level does not increase as much as does the foreign price level.

# 5.4.5 The foreign demand shock with a weaker monetary response

As we have noted, one of the relatively innovative features of QPM is the extent to which it addresses the Lucas critique. Because agents in the model respond to changes in policy regime, QPM is a particularly useful tool for examining alternative policy rules. Two notable features of QPM's response to the foreign shock are the increase in the spread between foreign and domestic interest rates and the long-run appreciation of the currency. To a large extent, these outcomes are a result of the strength of the monetary authority's response to the potentially inflationary consequences

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<sup>92.</sup> This shock being a temporary one, the desired or steady-state ratio of NFA to income is unchanged, meaning that NFA must return to control in the long run. Had there been a permanent change in foreign output or commodity prices, the steady-state level of NFA as a percentage of income would have changed.

of the shock. To demonstrate this, the foreign shock was redone using an alternative policy rule. The alternative rule has the monetary authority raise short-term interest rates less in response to a projected deviation of inflation from its target than it does under the standard policy rule.

The initial increase in short-term interest rates is roughly half a percentage point lower under the alternative policy rule (dashed line in Figure 11). While the spread between domestic and foreign short-term interest rates is lower under the alternative rule, it still widens relative to control. As a result of the milder policy response, more excess demand is allowed to accumulate initially, and the first subsequent period of excess supply is smaller. This additional strength in aggregate demand adds a further 0.2 percentage points to the peak increase in CPI inflation relative to the base-case rule. These additional inflationary pressures mean that interest rates must remain above control longer, since the level of excess supply required in the medium term to return inflation expectations to control is larger. Under the alternative policy rule, there is a slight nominal depreciation of the currency in the long run. This reflects the fact that the alternative rule allows slightly more price level drift than occurs abroad in this shock, reversing the result with the base-case rule.

# 5.5 The importance of initial conditions and the co-ordination of policy

In this subsection, we analyse shocks to government expenditures from differing initial conditions. The analysis extends the above discussion of temporary shocks to aggregate demand. However, the main purpose of the analysis is to establish two new points about QPM's properties. First, there is no such thing as "the" government expenditure multiplier in QPM – the effect of fiscal policy depends on the initial conditions and the time horizon over which one measures the impact. That initial conditions matter to model properties is an important general point. Second, the effect of fiscal policy depends on the nonetary reaction assumed. This gives us the opportunity to introduce the topic of the co-ordination of policy and to stress again the point that the policy reaction functions play an important role in determining the model's dynamic properties.

We discuss these issues by way of two illustrative simulations of QPM. For the first simulation, we begin in steady state and introduce a temporary, expansionary fiscal stimulus in the form of 1 per cent higher government spending for one year, with the shock tapering off to zero by the end of the second year. We constrain taxes initially and delay the monetary response for two quarters.<sup>93</sup> These assumptions both work to minimize any endogenous offsets to the short-run fiscal stimulus. In the second scenario, we conduct precisely the same experiment, except that we introduce the fiscal expansion from initial conditions of excess supply. The control scenario for this second shock is built up through the imposition of a prior autonomous decrease in aggregate demand. The excess supply is made large enough that the fiscal shock does not push the economy back into the region of excess demand on impact.

The results of these experiments are illustrated in Figure 12. The solid line shows the results starting from steady state, the dashed line shows the results starting from excess supply. For output, we also show the cumulative effect of the shock. To begin, the government expenditure multiplier, calculated in the usual way for both sets of initial conditions, is just under one in the first quarter. Thereafter, the effects change and differ between the two cases. In particular, the *cumulative* effects of the government spending shock on output depend dramatically on the initial conditions of the exercise. The cumulative effect of the shock on output is *positive* when initially there is excess supply, whereas it is *negative* when initially there is full employment.

Under initial conditions of full employment, the fiscal authority is working against capacity constraints in conducting a deficit-financed expansion of government expenditures. The resulting excess demand is inflationary, and the monetary authority's target is compromised, triggering a vigorous, albeit delayed, response.

<sup>93.</sup> Both the fiscal and monetary targets are unchanged in these experiments, which means that the constraining of the tax rate in the short run only results in the financing costs of the fiscal expansion being pushed off until later in the scenario. The normal rule has to operate eventually to impose the condition that the cumulative deficit be unchanged (to keep the debt ratio constant).

# Figure 12: A temporary increase in government expenditure

Per cent (%) or percentage point (%p), shock minus control



The picture is much different if the expansion comes when there is excess supply. In this case, inflation begins at a level below the target and the fiscal expansion helps the monetary authority in obtaining its objectives. There must still be some tightening of monetary conditions, relative to control, in order to avoid an escalation of inflation expectations, but the response is much more muted and, as already noted, the fiscal expansion generates a cumulative gain in output.

Had these same shocks been conducted without a delay in the response of monetary policy, the output response to the fiscal stimulus would have been very slightly smaller in the short term. As with any demand shock, delaying the monetary policy response when there is full employment results in larger cumulative output losses than would be realized with immediate response. However, if fiscal stimulus is introduced under conditions of excess supply, delaying the monetary response results in a slightly larger cumulative output gain.

It may seem an obvious point that expansionary fiscal policy will generate more output and less inflation if introduced when there is slack in the economy than when there is full employment, but such obvious points have not always been reflected in the properties of macroeconomic models.

# 5.6 Nominal shocks: more on the nexus of wages, prices and the exchange rate

This subsection examines the model's response to shocks that perturb the nominal values directly, without first affecting the level of output or any other aspects of the real economy. The three cases examined are a shock to the nominal exchange rate, a shock to the output deflator and a shock to the nominal wage. Although each of these shocks is of some interest in its own right in understanding QPM, the main point of the subsection is to describe the interaction of nominal magnitudes in the model.

#### 5.6.1 A shock to the nominal exchange rate

The shock to the nominal exchange rate is a depreciation of just under 2 percentage points in the first quarter. The shock then dissipates by

roughly one half in each subsequent quarter. This shock can be interpreted as a temporary loss of confidence in assets denominated in Canadian dollars.

In response to the shock, the monetary authority must immediately tighten policy, because the exchange disturbance exerts upward pressure on domestic prices through two channels (Figure 13). First, the depreciation in the nominal exchange rate pushes up consumer prices through the import component (and indirect effects on competing goods). Second, the resulting real depreciation stimulates export demand and weakens import demand, thereby generating additional aggregate demand pressure on domestic prices. This excess demand pressure comes from both increased net export demand and increased domestic consumption demand. The improved current account results in a net cumulation of foreign assets, raising holdings above the desired level. Households are induced to increase consumption in order to restore their wealth holdings to equilibrium.

The tightening in policy serves to reduce the net shock to the nominal exchange rate, damping the immediate effects of the shock. The effects of the shock on the real exchange rate are reversed relatively quickly. The increase in the policy instrument is also sufficient to reduce consumption and investment demand directly. Interest rates remain above control for roughly seven years. Output and inflation return to equilibrium in years eight and nine, respectively.

Figure 14 adds some detail not covered above. In particular, it shows what happens to the level of prices and wages in response to the shock to the exchange rate. This picture is included to show that the main channel of nominal response to shocks arising outside the price-wage nexus of the model is through prices first and then wages. The wage response lags the price response initially, in this case, and then catches up. Since the steady-state real wage is not affected by this temporary shock, the nominal levels of the two variables eventually move by the same proportional amounts. This relative timing of price and wage movements

# Figure 13: A shock to the nominal exchange rate



Per cent (%) or percentage point (%p), shock minus control





in QPM reflects research that indicates that this is a stylized fact of the Canadian data. See, for example, Cozier (1991).

## 5.6.2 A shock to the price level

The shock considered here consists of an unexpected increase in the output deflator of just over 0.5 per cent initially, decaying by roughly 50 per cent in each of the following quarters. Shocks to price equations have often been interpreted in the literature as temporary supply shocks. This label tends to come from models without any formal supply side, and it can be misleading. In a model like QPM, where the explicit supply structure permits us to study formally the effects of temporary disturbances to aggregate supply, it is not appropriate to label a shock that affects prices but not supply conditions in the output market as a supply shock. Thus, we prefer to think of this shock as reflecting a more direct disturbance to prices.<sup>94</sup>

It is important to note that although this is a temporary level shock, it can have inflationary consequences (Figure 15). The monetary authority cannot ignore it, but must respond with a vigorous and sustained tightening in policy in order to unwind the impact of the shock on inflation expectations. In addition to the direct price pressures of the shock itself, which have an impact on inflation expectations, there are also secondround price pressures arising from a depreciation in the nominal exchange rate. By the end of the first year, the nominal value of the Canadian dollar has fallen sharply, reflecting the significant increase in the domestic price level. While the initial increase in short-term interest rates prevents the nominal exchange rate from jumping immediately to the value supportable by the domestic price level, continued underlying pressure for further nominal depreciation persists. To contain this depreciation, monetary policy cannot ease too quickly, since a rapid depreciation would contribute to a further increase in inflationary pressures. This is illustrated by the fact that while inflation has returned close to control by the end of the second year, it remains stubbornly above target for several years in spite of persistent excess supply in the economy. After falling approximately 1 per cent below potential in the first two years, output recovers only slowly and does not return to capacity until the seventh year.

# 5.6.3 A shock to the nominal wage level

We conclude this review of the properties of QPM with a shock to the level of wages. In the long run in QPM, the level of *real* wages is determined by the marginal productivity condition of a competitive, neoclassical model of distribution. There can be no permanent shock to real wages by arbitrary

<sup>94.</sup> It is difficult to be precise as to what such a disturbance might be. It could be simply a pricing error. It could be a shock to the *level* of the money supply or demand, with no change in trend money growth or velocity. It could be the aggregate result of a shock to relative prices below the level of the model accounts. This is the type of thing people have in mind when they call such things "supply" shocks.



choice, although many shocks will affect the equilibrium real wage. We can, however, entertain shocks that feature short-term deviations from the marginal product condition, shocks that can be interpreted as disputes over the level of productivity and the warranted real wage, or simply as struggles for income shares. Such issues can be of great importance; an example is the debate over the wage consequences of the introduction of the Goods and Services Tax, where a higher indirect tax rate was introduced well after offsetting (from a revenue perspective) cuts to income tax rates. Moreover, temporary disturbances to wages can have lasting effects on nominal levels.

These points are illustrated in Figure 16. The shock is a temporary increase in nominal wages, all else equal, which raises the real wage sharply at first. The implied increase in production costs puts upward pressure on prices, and the monetary authority responds in the usual way. There is nothing new of particular note in these results. They are included to show that QPM has a description of wage determination that can be used to analyse inflationary shocks coming from wage setting. The typical endogenous response of wages to price pressures, described above and seen in Figure 14, where wages lag prices and are not the main engine of inflationary dynamics, does not prevent us from considering a more standard Keynesian "wage-push" inflationary shock. With the monetary authority acting to limit the deviation of inflation from the target and to ensure that it returns to target when disturbed, the only lasting result of nominal wage push is drift in the price level.

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# 6 CONCLUDING REMARKS

This report describes the dynamic structure and simulation properties of the Bank of Canada's Quarterly Projection Model, QPM. The philosophy of QPM is to build a projection superstructure around an aggregate macro model with a well-defined steady state. The model is thus founded on a highly structured core theory, with complete intertemporal accounting and forward-looking behaviour. Where compromise was necessary, the integrity of the core theory was always given first priority.

The dynamic structure of the model reflects three main elements. First, there is what we have called intrinsic dynamics. This refers to the inherent inertia in the economy stemming from costs of adjustment. We represent such dynamics everywhere in the model using the same basic framework, which embodies a trade-off of the costs of being away from a desired equilibrium and the costs of disequilibrium adjustment. Intrinsic dynamic structure reflects elements of behaviour that are, in principle, immutable to standard aggregate demand management policies and the perceptions of agents, as reflected in their expectations formation rules and their forecasts.

The second element of QPM dynamics arises from expectations themselves. Agents' perceptions of what is driving the current cycle and what is likely to happen in the future play a key role in conditioning behaviour in a forward-looking model. In QPM, expectations formation is partially model-consistent and partially backward-looking. This means that further inertia is added to the model's dynamics from expectations themselves. Yet, in the end, expectations adapt to the assumed policy environment, providing at least a first-level response to the Lucas critique. In particular, because of the forward-looking component, policy makers must act to achieve or sustain a goal.

The third element of dynamics in QPM arises from the incorporation of explicit roles for the policy makers. This is done through the specification of targets for monetary and fiscal policy that will be attained only through use of the instruments of policy to those ends. The required actions are described, in the model, by explicit policy reaction functions. In the case of the fiscal authority, personal tax rates are adjusted to respect the government's intertemporal budget constraint and specific targets for the ratio of its debt to output and the ratio of its expenditures to output. In the case of the monetary authority, short-term rates are varied to achieve an intermediate instrument setting, expressed in terms of the slope of the term structure (short rates minus long rates), with a view to achieving a target rate of inflation within a reasonable period of time. This monetary reaction function is forward-looking, responding to the predicted difference between inflation and the target rate six to seven quarters ahead. The automatic response of the policy instruments to shocks plays a very important role in overall model dynamics. Many examples are provided in the section on model properties.

QPM has not been estimated using traditional econometric methods; it is a calibrated model. Yet it is still a model firmly tied to the data. Considerable effort has been made to ensure that model properties are empirically defensible. QPM has been configured and calibrated to be dynamically stable around its well-defined steady state. It therefore provides clear answers to a wide range of economic questions that can be posed in the form of a shock to the model. Whether the answers are "right" in a fundamental sense is a hard question to answer. Key aspects of the steady state are based on staff judgment. Moreover, although the dynamic structure has been calibrated to produce properties that Bank staff find defensible, there is no clear metric that can be applied to judge the results. On this, time will tell; evaluation of the model's usefulness in its two main roles will be a regular part of its ongoing development.

QPM represents the economy at a highly aggregate level. It is still too early to come to a balanced view as to whether this approach is too simple for the Bank's main model of the economy. Indeed, staff are still working on fleshing out the details of the satellite model structure. However, our first impressions are favourable, in the sense that the staff have not found that the omissions impinge on their ability to create defensible economic projections. In part, this reflects the strong element of judgment that is used for the very short-term part of all such projections. Nevertheless, the model speaks increasingly powerfully as the horizon is extended, and speaks alone for medium- to long-term horizons. Moreover, the model makes a major contribution in analysing risks implicit in a particular scenario. The judgment goes into the baseline. Thereafter, the model provides a wide menu of shocks that can be entertained to assess the sensitivity of the baseline results to particular assumptions. In such analysis, judgment plays no role, other than in the choice of questions to ask.

Experience will no doubt reveal areas of weakness that require attention either to calibration decisions or to more fundamental structural issues. There are a number of areas where we have already identified important limitations in the model. These include the fact that it was built to exhibit neutrality and superneutrality. That is, monetary policy choices have no impact on the real steady-state solution. This does not reflect the considered view of the Bank on this issue. However, it is not clear precisely how one would want to represent such effects in the model, and it was decided to abstract from this issue in the first version of QPM.

Other areas of possible concern include, for example, the implications of the formal one-good paradigm of domestic production in QPM. Terms-of-trade shocks, and particularly the effects of changes in commodity prices, are important macroeconomic phenomena for Canada. We have designed packages of shocks that mimic such disturbances, and we have built into the model some capacity to reflect their effects, but there is no way to handle them formally without a true multisector model. In cases like this, we complement QPM with research focussed on the particular issue and attempt to learn from that research how to reflect the conclusions in QPM's properties.<sup>95</sup> QPM's core structure may ultimately be judged too simple to adequately reflect the Canadian economy. In this case, however, a substantial increase in model complexity would have to be accepted, with consequently higher costs in terms of maintenance of both the model itself and the level of understanding among staff.

<sup>95.</sup> See, for example, Macklem (1993, 1994).

There are aspects of the model's core theory that are limiting for some policy issues. In the analysis of government debt and deficits, for example, Macklem, Rose and Tetlow (1994, 1995) argue that QPM is missing some essential features. They add two of these features (sensitivity of labour supply to the level of income taxation, and an endogenous interest rate "risk premium" that rises with the level of indebtedness) in a version of the model customized for their particular research. That this can be done relatively easily provides one answer to the possible concern about oversimplification. If one tries to put absolutely everything needed for every question into a core model, experience tells us that the "core" rapidly becomes hard to maintain and to understand in day-to-day work. Nevertheless, experience will no doubt lead us to identify areas where more elaborate structure is necessary.

QPM is normally run in what is called "deterministic" mode. That is, simulations do not involve random drawings from hypothetical distributions of disturbances. Like the examples provided in Section 5, most QPM simulation experiments are done with a single shock (or shock sequence, or package of shocks) designed to throw light on a particular issue. Future scenarios are normally constructed from the starting points on the assumption that there will be no new shocks over the projection horizon.

In some cases, this is rather limiting. The real-world policy problem has many dimensions of uncertainty not captured in deterministic analysis. There will be future shocks, and which shocks arrive will matter. Yet, with current technology we are not yet able to do full-scale stochastic simulation analysis on models as complex as QPM, at least not routinely. Limited initiatives are under way to add confidence intervals around deterministic control solutions of QPM. In this area, however, our research is focussing on smaller, simpler models.

Eventually, we will have to face the task of introducing uncertainty into the core macro model in a real way. This will involve more than simply switching to stochastic analysis; it will require a reworking of the fundamental behavioural structure of the model and some method for dealing with learning as part of the dynamic structure.

A seemingly curious aspect of QPM, given that it is a model used by the Bank of Canada, is that money and credit play no explicit role. We hasten to add that, on one level, this is more apparent than real. It is straightforward, for example, to close the circle with respect to money and money growth in the usual way – by specifying a link between inflation and money growth and between the price level and the money stock using a money demand function. But, given current policy practice, with no formal anchor on a nominal level, nothing would be added except an endogenous determination of the money magnitudes. At this level of discussion, then, money is there; staff simply do not pay any explicit attention to it within the model analysis. On another level, there is room for extensions of substance that would identify the role that the financial system plays in facilitating exchange. This has proven difficult to formalize, however, even in terms of abstract research; and the profession is far from able to identify how to do it in working models like QPM.

Models are never finished. There will always be things that can be done better, or features missing that are important in some particular application. Yet, history shows that it is risky to lose sight of the core macro story that must be a feature of any useful policy model. History also tells us that it may not be wise to try to put everything one knows into a general-purpose macro model, simply because the costs of understanding, using and maintaining it increase dramatically with its accounting complexity.

In conclusion, we return to perhaps the key accomplishment of QPM – the combination in one model of sufficient structure to handle fundamental policy analysis and sufficient realism to support quarterly economic projections.

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